THE ANALYTICAL DETERMINATION OF THE THERMAL TENSIONS WHILE HEATING A EQUIVALENT CYLINDRICAL STEEL INGOT

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1. INTRODUCTION

The thermo – elastic stresses are building up while heating (cooling) the elastic bodies, with appearance of the thermal gradient within. The building up of theses stresses are brought about by the fact that different parts of the body could not expand uniformly in concordance with temperature. If the thermo – elastic stresses do not exceed the elasticity limits, then the thermal gradient disappears as well as the stress.

The structure stresses appear in products when physical changes take place within the steel, at the same time with the volume change. While heating the steel ingots, the structural changes usually take place at temperatures higher than 550^0C, i.g. when the steel is already in its plastic state. During cooling, the physical changes of the steel could take place also at lower temperatures than 550^0C. In this case the structure stresses are very important, and could surpass the thermal stresses, [1].

The amount and distribution of the thermal stresses which appear, their change during the thermal working, depend on the heating and cooling conditions (rate, symmetry), on the thermo – physical mechanical and plastic characteristics of the product as well as on remanent stress distribution.

In tracing the state of stress of the solid bodies various methods could be followed: computational (analytical and approximate), experimental and physical and mathematical modeling methods.

The analytical determination of the thermal stresses is very complex, because in the general case, the complex resolutions in the theory of non – linear thermal conductivity have to be matched with the resolutions of the non - linear problems of the elasticity and plasticity. Therefore for sorting out the practical problems usually the thermo - elastic stresses are examined, and the thermo - physical and mechanical properties are considered constant, [1]. In theoretical determination of the thermo – elastic stresses we can use formulas and equations of the elastic theory, where are introduced the resolutions of the thermo - conductivity equation that refers to one or another case of the heating, with the subsequent changes. So, non - uniformity of the heating on sections of the parts, determines a non - uniform expansion of the various material layers. The expansion of a layer from the outside of port is slowed down by the inside rigid layers so that the possible but not accomplished expansions are changed in local elastic deformations, to which internal stresses correspond.

2. DETERMINATION OF TENSION AND COMPRESSING STRESSES

The thermal internal stresses appear at cooling the ingot after stripping, when its temperature decreases under the plastic range and enters the elastic range (about 550^0C) as well as at its heating on the whole elastic range of temperature.

The determination of the stresses that appear in the cylindrical bodies in the conditions of heating the furnace $T_{furn.}= 0$ and the initial thermal gradient in the metal is null $\Delta T_0 = 0$, is based on Hook's law as well as the building up mechanism of the thermo – elastic stresses, expressed by the expression of the connection between thermal stresses R_T and the implied corresponding deformations as follow:

$$R_T = \frac{\beta \cdot E}{1-\mu} \cdot [\overline{T} - \theta(r,\tau)] \qquad \text{[MPa]} \qquad (1)$$

where: β – linear thermal expansion coefficient;
$\overline{T}$ - average temperature of the body:
$\overline{T} = T_{centr.} + 0{,}5\Delta T$;
θ(r, τ) -body temperature in the considered element;
E - elastic modulus; **μ** – Poisson's ratio.

For a mill roll of infinite length and radius r (0 < r < R), radial (R_{rad}), tangential (R_θ) and axial (R_x), thermal stresses should be determined by the following expressions:

$$R_{rad} = \frac{\beta E}{1-\mu}\left(\frac{1}{R^2}\int_0^R \theta \cdot r \cdot dr - \frac{1}{r^2}\int_0^R \theta \cdot r \cdot dr\right) \qquad (2)$$

$$R_\theta = \frac{\beta E}{1-\mu}\left(-\theta + \frac{1}{R^2}\int_0^R \theta \cdot r \cdot dr + \frac{1}{r^2}\int_0^R \theta \cdot r \cdot dr\right) \quad (3)$$

$$R_z = \frac{\beta E}{1-\mu}\left(\frac{2}{R^2}\int_0^R \theta \cdot r \cdot dr - \theta\right) \quad (4)$$

The temperature of the body θ(r, τ) is determined from the solutions of the Fourier's differential equation of the heat transfer:

$$T_{s.n.} = \theta_n(R,\tau) = T_{furn0} + (T_{s.0.} - T_{furn0})\sum_{n=1}^{\infty} M_n J_0(z_n) e^{-z_n^2 Fo} \quad (5)$$

$$T_{c.n} = \theta_n(0,\tau) = T_{furn.0} + (T_{s.0} - T_{furn.0})\sum_{n=1}^{\infty} M_n e^{-z_n^2 Fo} \quad (6)$$

where: $M_n = \dfrac{2J_1(z_n)}{z_n[J_0^2(z_n) + J_1^2(z_n)]}$ (7)

$T_{furn.0}$ – furnace initial temperature;
$T_{s.0}$; $T_{c.0}$ – surface initial temperature, respectively of cylinder's axis;
$J_0(z_n)$; $J_1(z_n)$ – Bessel function of 0; 1 order and z_n argument;
$z_n = m_n R$;(m_n – z_n argument solutions) is determined as a multiple solution of the transcendental equation:

$$\frac{J_0(z_n)}{J_1(z_n)} = \frac{z_n}{hR} \qquad n = 1, 2, \ldots, \infty \quad (8)$$

R – equivalent radius of the mill roll, in mm;
$h = \dfrac{\alpha}{\lambda}$ - the relative coefficient of thermal transfer of the steel;
α - heat passing coefficient at the surface of the metal;
λ - thermal conductivity coefficient at the surface in the metal;
hR = Bi – Biot's criterion of the metal heating.
m_n - solution should be determined by secant method on the intersection range with Oz axis of Bessel oscillating functions with argument z_n, whose amplitude decreases with the increase of argument.
$Fo = \dfrac{a\tau}{R^2}$ – Fourier's criterion of heat transfer;
$a = \dfrac{\lambda}{c \cdot \gamma}$ – thermal diffusivity coefficient of metal;
c – metal heat capacity; γ – metal specific weight.

In the case of an initial temperature distribution on the body section after a parabola of order II, when the initial thermal gradient ΔT ≠ 0, at a linear increase of surface temperature with speed w, the thermal stresses should be:

$$R_{rad} = \frac{\beta E}{1-\mu}\cdot\left[\frac{w}{16\cdot a}\left(R^2 - r^2\right) + \left(\frac{wR^2}{a} - 4\Delta T_0\right)\cdot G_r\right] \quad (9)$$

$$R_\theta = \frac{\beta E}{1-\mu}\cdot\left[\frac{w}{16\cdot a}\left(R^2 - 3\cdot r^2\right) + \left(\frac{wR^2}{a} - 4\Delta T_0\right)\cdot G_\theta\right] \quad (10)$$

$$R_z = \frac{\beta E}{1-\mu}\cdot\left[\frac{w}{8\cdot a}\left(R^2 - 2\cdot r^2\right) + \left(\frac{wR^2}{a} - 4\Delta T_0\right)\cdot G_z\right] \quad (11)$$

where: G_2 – function corresponding to axial stresses determined by relation:

$$G_z = \sum_{n=1}^{\infty}\frac{2}{z_n^4}\cdot\left[2 - \frac{z_n^2 J_0\left(z_n \cdot \frac{r}{R}\right)}{J_1(z_n)}\right]\cdot e^{-z_n^2 \cdot F_0} \quad (12)$$

A relation by which the fracturing (cracking) possibilities are estimated for a steel ingot due to the thermal stresses is given by Huber – Mises – Henky's plasticity condition for the spatial state of efforts related to the main orthogonal axes, [4, 5]:

$$(R_z - R_\theta)^2 + (R_\theta - R_{rad})^2 + (R_{rad} - R_z)^2 \le 2R_c^2 \quad (13)$$

where: R_c – ductility limit of steel at the temperature considered of the centre of ingot:

The stresses generated at solidification – cooling of the cast ingot, named remanent stressed as well, considered as belonging to some large sections of the material, specific to the stresses of order I, which appear and are balanced in a large volume of material, at the ingot level.

In order to show the maximum amount the remanent thermal stresses could reach (cooling) in a steel ingot. Such an ingot was sectioned soon after its air solidification – cooling and the remanent stresses of a 0,37m diameter layer were measured. They were of about 250…300 MPa, in the case where the admissible resistance to rupture in this layer was of 800 MPa, [1].

The thermal stresses that appear vary in sign as well as in amount, at cooling they have a contrary sign compared to those that appear at heating the cooling of ingots in a controlled way – in cooling furnaces or pits, on thermos wagon – determines lower cooling rates (about 5…10^{0}C/h), which allow building up of some remanent stresses of low value.

In the case of cooling a solidified ingot, after stripping (fig.1, a), in the initial phase its peripheral zone is cooled faster than the central zone, its volume is diminished (contracting tendency) and tends to compress the central zone, which is hotter and its specific volume is larger. Due to the very reduced compression, practically negligible of the material in the central zone, the peripheral zone would suffer a slight plastic deformation to the entering temperature in the elastic range (about 550^{0}C). Therefore, at the further cooling, the superficial layer would be tensioned by stretching (the stress should have plus sign).

At the same time, the central zone material will suffer a slight compression (practically negligible) overlapped to its cooling contraction, and at passing the elastic temperature range, compression stresses will appear, which will have minus sign [3]. After a temperature decrease in the peripheral zone approaching the environmental temperature, the material of this zone is keeping constant its volume, and the central zone material is cooling on, decreasing its volume as the cooling progresses. In this way, the compressing stresses in the central zone and those of tension stresses in the peripheral zone, are continuously reduced till cancellation (fig.1, a). Due to the contraction of the central zone, which did not cool to the environmental temperature and due to the non-deformation of the peripheral zone, tension stresses are building up in the axial zone (+) as well as compression stresses (-) at the surface of the cylinder. The size of the newly created stresses is directly proportional to the cooling rate, to the thickness of the semi-product and to the physical characteristics of the material. The fracture tendency of the metal and of the alloys, at the same values of the tension stresses, is the more obvious as the plasticity is more decreased, being directly proportional to the values of the tension stresses.

We can notice the fact that within the total cooling time *a'-c* (fig.1, *a*), the larger section *a'-b'* the smaller the remanent stresses at the end of the cooling period, in point *c*.

The decrease of the cooling rate in period *b'-c*, that means after cancellation of the stresses, influences very slightly – in fact negligible - the remanent tensile contractions and stresses, that build up in the central zone. For this reason, it is necessary that the cooling rate to be as low as possible when the surface temperature is under 850^{0}C, so that the point *b'* is as close as possible to *b"*. In other words, for the cracking sensitive metals and alloys because of the thermal stresses, the cooling has to be slowed even from the beginning, within the interval *a'-b'* and not only to the end, in the interval *b'-c'* consequently for these metals and alloys, within the interval 900-20^{0}C, the cooling has to be accomplished at small rates 5…15 ^{0}C/h, usually controlled either in furnace or in cooling pits for ingots.

At cooling the ingots to be heated for plastic deformation, the structural stresses are negligible. At the present time, there is no method of quantitatively tracing and determining in the course of cooling process of the internal stresses, being able only to remark their qualitative aspect and to determine the remanent stresses.

The determination of the thermal stresses depending on steady heating conditions in cylindrical bodies where the temperature distribution is a parabola of order II, is made by the relation:

$$R_T = \frac{\beta \cdot E}{1-\mu} \cdot \Delta T \left(\frac{1}{2} - \frac{r^2}{R^2} \right) \tag{14}$$

If in the relation (14), a replace is operated in the expressions for the ingot surface (T_s) of the ingot and its center (T_c) for determining the thermal gradient ΔT, we obtain the thermal stresses distribution on the product section by the relation:

$$R_{Tn} = \frac{\beta E}{1-\mu}\left(\frac{1}{2} - \frac{r^2}{R^2}\right)\{T_{cupt} + (T_{s.0.} - T_{cupt})\sum_{n=1}^{\infty} M_n J_0(z_n) e^{-z_n^2 Fo} - $$
$$ - [T_{cupt} + (T_{s.0.} - T_{cupt})\sum_{n=1}^{\infty} M_n e^{-z_n^2 Fo} - \Delta T_0 \sum_{n=1}^{\infty} W_n e^{-z_n^2 Fo}]\} \tag{15}$$

The thermal stresses at R_{Tsn} surface and respectively centre R_{Tcn} would be:

$$R_{Tsn} = -\frac{1}{2} \cdot \frac{\beta \cdot E}{1-\mu}\{[T_{cupt} + (T_{s.0.} - T_{cupt})] \cdot (\sum_{n=1}^{\infty} M_n J_0(z_n) e^{-z_n^2 Fo} - $$
$$ - \sum_{n=1}^{\infty} M_n e^{-z_n^2 Fo}) - \Delta T_0 \sum_{n=1}^{\infty} W_n e^{-z_n^2 Fo}\} \tag{16}$$

$$R_{Tc.n.} = +\frac{1}{2} \cdot \frac{\beta \cdot E}{1-\mu}\{[T_{cupt} + (T_{s.0.} - T_{cupt})] \cdot (\sum_{n=1}^{\infty} M_n J_0(z_n) e^{-z_n^2 Fo} - $$
$$ - \sum_{n=1}^{\infty} M_n e^{-z_n^2 Fo}) - \Delta T_0 \sum_{n=1}^{\infty} W_n e^{-z_n^2 Fo}\} \tag{17}$$

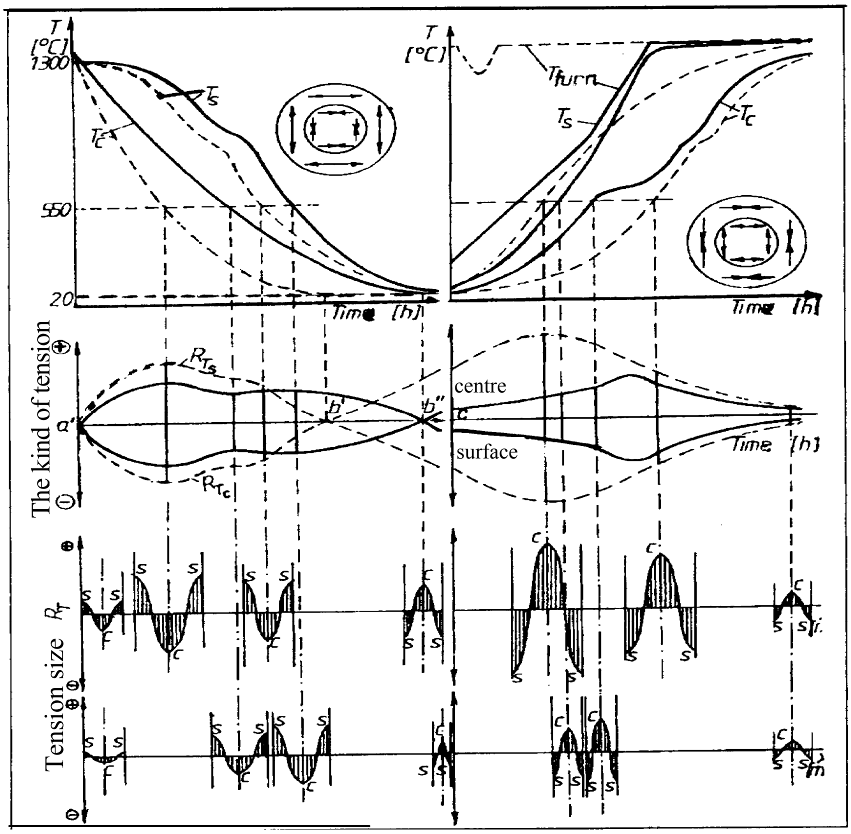
In the field of stationary conditions, the neutral section coordinates are determinate by expression:

$$2J_1(z_n) = z_n \cdot J_0(z_n \cdot \frac{r}{R}) \tag{18}$$

resulting: $\frac{r}{R} = f(Bi)$.

- the axial stresses are negative at surface and positive on the axis of the cylinder;
- the absolute value of the thermal stresses in time, varies similarly to that of the thermal gradient ΔT;

Because the danger is present in the tension stresses, which at heating are produced in the axis of the ingot, in determining the stress state we include in the calculation only the axial thermal stresses R_{Tcn} and remanent stresses.



a b

Figure 1. Variation of the compression and tension stresses in ingot:
a-at cooling the ingot; *b*-at heating the ingot; ---fast cooling (heating); - slow cooling (heating)

For different values of Biot's criterion (Bi = hR), $\frac{r}{R}$ ratio for axial stresses are modified within very narrow limits: $\frac{r}{R}$ = 0.68…0.69.

3. CONCLUSIONS

Because of the analytical solutions (15)…(17) we notice that:

- the absolute maximum value of the thermal stresses can be found either at surface or at the axe of the cylinder;

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