UNIFIED MATRIX REPRESENTATIONS OF ARCHITECTURE OF A CLASSICAL AND QUANTUM COMPUTERS IN 2^n –DIMENSIONAL COMPUTATIONAL HILBERT SPACE- $H_2^{\otimes n}$.

Enachi Viorel, Gubceac Natalia
State University of Moldova, Department of Theoretical Physics,
Str.Mateevici, 60,Chisinau MD-2009, Republic of Moldova
enachi@usm.md

Abstract. *By using* 2^n *dimensional computational Hilbert space we obtained computers equations for any digital binary computers (or devices) with n -input and m-output .The solution of computers equations is a transformation matrix that represent scalar Boole function in discreet Hilbert space. Also is shown that architecture of reversible and irreversible classical computers can be represented by a transformation matrix, like quantum reversible computers. It is indicate about the possibility of a direct physical implementations of a computer transformation matrix for reversible and irreversible computers*

Keywords: *Boole, computer, matrix, equation, quantum, reversible, irreversible*

I. Introduction

Traditional (classical) Computer science was designed by Alan Turing (1936). He introduced the concept of virtual Turing machine, which was finally replaced (1946) by von Neumann, introducing the notion of stored-program computer, In his computer the Turing machine operations can be replaced by sets of the logic gates. A logic gate is an electronic circuit that implements a logic operation in a classical computers, that transform a state of n-bits input into m-bits output. The logic operation is called Boolean if it acts only on the logic values 0 and 1.In other words, the Boolean function is an application between the logical values of input and output ones: f: $\{0,1\}^n \rightarrow \{0,1\}^m$. For given n and m, there are n input variable $(x_1, x_2...x_n)$ and m multi-output functions $\{f_1(x_1, x_2...x_n), f_2(x_1, x_2...x_n)....f_m(x_1, x_2...x_n)\}$ .In this case a total number of possible sets of truth table is 2^{m2^n} Any function $f_k(x_1, x_2...x_n)$ my be Boolean algebra expression containig input variables . Some truth tables can be classified in terms of information as irreversible (m ≤n) and other (for n=m) as reversible if it maps each input to a unique output. Accordingly traditional computers can be irreversible or reversible. The traditional computers are irreversible.(for n<m).This computers can be considered as the calculations of some functions of the input bits,with the value of the functions written into the output bits. Using Karnaugh map for some truth table it is possible to representing optimally the computer architecture by a lots of logic gates: NOT,AND, OR,etc for irreversible computer and by NOT,TOFFOLI and FREDKIN, etc gates for reversible computer.

However, over time, some problems in physics and computer science arose which could not be efficiently solved with in traditional computing. One of these problems was highlighted (in 1981) by the Nobel Laureate in physics, Richard Feynman, who argued for the impossibility of efficiently simulation of quantum physics processes on traditional computers. Solving the problems of simulations has been proposed by Feynman: the efficient simulation of such processes can occur if a quantum computer were used. The first fundamental works devoted to quantum computation models have been published since 1982 by Bennett, Deutsch, DiVincennzo and others [1]. From mathematical point of view, the difference between traditional computer and quantum computer is that traditional computer is represented by Boolean functions described by scalar Boolean algebra, while the quantum computer is represented by unitary matrices and matrix algebra in Hilbert space. The purpose of this communication is to obtain a matrix representations for traditional computers, such as a usual matrix representations for the quantum computers.

II. Matrix representation of Boole functions.

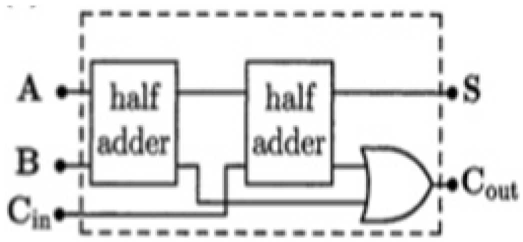
To achieve the intended purpose we use the unifying element that is the representation of classical bits 0 and 1 by a matrix in two-dimensional real Hilbert subspace [2] $: 0 \rightarrow (10)^T; 1 \rightarrow (01)^T$, where T is the transposition operation . Using the Dirac notations, the 0 and 1 bits can be represented as Bra $\langle \ |$ and Ket $| \ \rangle$ vectors like: $\langle 0| = (10), \langle 1| = (01), |0\rangle = (10)^T, |1\rangle = (01)^T$. Using the direct product $\otimes$, it can be defined the multibits Ket (or Bra) vector, for example: $010110 \rightarrow |0\rangle \otimes |1\rangle \otimes |0\rangle \otimes |1\rangle \otimes |1\rangle \otimes |0\rangle$, which is a vector (called the register of memory) in the real 2^6 -dimensional computational Hilbert space: $H_2^{\otimes 6} = H_2 \otimes H_2 \otimes H_2 \otimes H_2 \otimes H_2 \otimes H_2$. Analogically we will also write the direct product of Bra vectors .In general case, both Bra and Ket vectors composed of n- bits will be represented by a matrix in the- 2^n -dimensional space. Dyadic product is also defined for Ket in the- 2^m -dimensional space and the Bra vector in the- 2^n -dimensional space: $|m - bits\rangle \otimes \langle n - bits|$, which is a matrix $2^m \times 2^n$. Because the entire computer is a digital device described by a given Boolean logic function , then it can be considered, from the mathematical point of view, as a transformation matrix $C : H_2^{\otimes n} \rightarrow H_2^{\otimes m}$ determined from the systems of 2^n dyadic equations:

$$C|X\rangle = |f(X)\rangle, \qquad (1)$$

which can be called the dyadic equations of the computers. By first time is obtained unique solution **C** of the dyadic equations (1) of the computers :

$$C = \sum_x |f(x)\rangle \otimes \langle x|, \qquad (2)$$

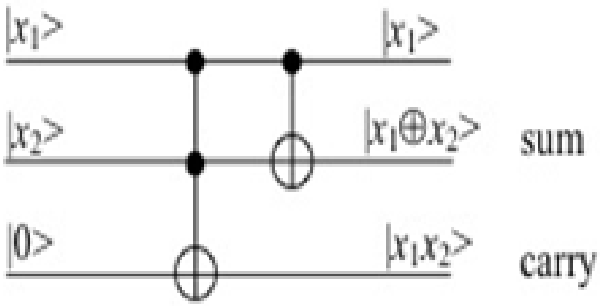
where $x = \{x_1, x_2, ..., x_n\}, \langle x| = \langle x_1| \otimes \langle x_2| \otimes ... \otimes \langle x_n|$ and $|f(x)\rangle = |f_1(x)\rangle \otimes |f_2(x)\rangle \otimes .... \otimes |f_m(x)\rangle$ are the input bits and respectively the output ones expressed by the direct product, the sum is effectuated over all X values and C is the matrix of $2^m \times 2^n$ dimension . Total possible numbers of matrix C is 2^{m2^n} - an enormous number ! Matrix C can serve as a representation matrix of the traditional irreversible computer, if $m \leq n$. Some matrix C, in case that n = m, can represent traditional reversible computer [3] The concept of matrix representation of traditional computers can be used to design computer network architecture representing within the matrix (2) for physical network elements , such as WIRES (C:n=1,m=1); FANOUT(C:n=1,m=2); SWAPING(C:n=2,m=2); AND(C:n=2,m=1); OR (C:n=2,m=1);HALF-ADDER (C:n=2,m=2); FULL –ADDER (C:n=3,m=2); Irreversible TWO- BYTE ADDER (C:n=17,m=9), etc..For classical and quantum reversible computer the TWO-BYTE ADDER is represented by matrix C:n=24,m=24 . For example the architecture for 2-bits irreversible full-adder (Fig.1) consist from two half-adder gate and single Or gate.Full architecture matrix representation is obtained from extended matrix product of a elementary gates matrix obtained from (2) :



$$\rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Fig.1 Architecture and matrix C for irreversible full-adder for two bits

Analogical for classical and quantum reversible half-adder that consist with Control-Not and Control-Control-Not gates can by obtained from (2):



$$\rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

Fig.2 Architecture and unitary matrix C for reversible half-adder for two bits (qubits)

However,the matrix C from Fig.1 and Fig.2 is also the one realization of $2^{2.2^3}$ (Fig.1) and $2^{3.2^3}$ (Fig.2) possible realizations of respective Boole functions .

The network architecture for quantum computers ,that consists from some unitary C matrix and other specific quantum unitary matrix(Hadamard,Pauli,Phase Shift), is represented by the unitary matrix U [4[for entire quantum computer . The inputs in reversible quantum computers,unlike traditional computer, can be superpositions of qubits like: $|Q\rangle = a|0\rangle + b|1\rangle = (a,b)^T$, a and b are complex numbers which satisfy the normalization condition $|a|^2 + |b|^2 = 1$ in the complex Hilbert space (if a = 1 and b = 0 or a = 0 and b = 1, then returns to the concept of bit !). By using qubits it is insured the simultaneous parallel processing of information.

It should be noted however, that the irreversible logic can be easily implemented on quantum systems such as molecules [5]. One way of implementation is the so called QHC (Hamiltonian Quantum Computing) model as opposed to the commonly-QC (quantum computing) model. In this publication [5] is demonstrated theoretically and experimentally the direct implementation of irreversible NOR logic gate on a single molecule (remember that this gate in traditional electronic is composite: NOR = NOT + OR). NOR gate is one of the 16 possible achievement of the matrix C for n = 2 and m = 1. The further research will show how far (n> 2, m> 1) will reach the direct implementation of a transformation matrix C in quantum system such as the molecules or crystals. Other promissing technology for direct implementations of some unitary transformation matrix C is a photon- holograme method in a piece of glass[6].

III. Conclusions

The traditional and quantum digital computers at this moment can be classified as (irreversible / reversible) and can be represented by a transformation matrix (between different / into same) discret Hilbert space . The exposed material could be recommended as being useful for computer scientists and engineers in the field of research in digital nanoelectronics to facilitate the transition from the traditional computing concept to the quantum one.

IV. Refeences

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