Regime of Self-trapping in Bose–Einstein Condensate

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***Abstract* — The dynamic of atom–molecular conversion in condition of Bose–Einstein condensate with formation of homonuclear molecule in regime of self-trapping was studied. We received the conditions of appearance of self-trapping effect in the system.**

***Index Terms* — Bose–Einstein condensate, homonuclear molecule, self-trapping.**

Substantial progress has been achieved in recent years in the experimental and theoretical investigation of the properties of atomic Bose–Einstein condensates. Currently, studies of the dynamics of coupled atomic–molecular condensates under conditions of Feshbach resonance or stimulated photoassociation of two atoms into a molecule are of special interest. It is well – known [1] that the dynamic of wave function BEC can be described by the effective equation of mean-field known in the literature as equation of Gross-Pitaevsky, which includes also members of interparticle interaction.

The first experiments in which we can observe BEC of rarefied fallow of alkaline metals [2-4] in ultracold temperatures stimulated further theoretical and experimental investigation of this effect.

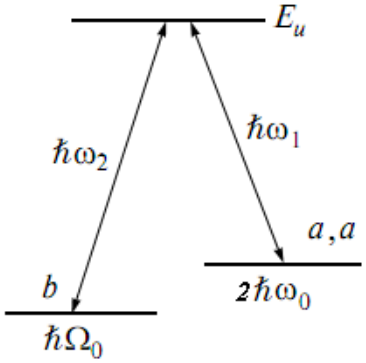

Fig.1 Energetic scheme and quantum transitions in three-level Λ – scheme

In [5, 6] it is studied the process of conversion of two similar atoms into homonuclear molecule in presence of laser emission. It was shown that it takes place as periodic as aperiodic regime of conversion, and peculiarities of time evolution of the system is determined by initial densities of the particles. Earlier was studied the dynamic of tunneling of bose-condensed atoms between two traps. It occurred that taking into account the interatomic interaction in this system the effect self-trapping appeared. The question about existence of this effect in conditions of atom–molecular conversion remains open.

The aim of this work is to explore the effect of self-trapping of system in the process of stimulated Raman atom–molecule conversion with the formation homonuclear molecule as single one-step process. We will study three–level energetic Λ –scheme (Fig. 1). One of the levels corresponds to the basic condition of two free atoms with energy $2\hbar\omega_0$, and the other – to basic condition of double atomic with energy $\hbar\Omega_0$. The third level corresponds to activated condition of molecule E_u. The appearance of molecule from two atoms leads to absorption of light quantum with the energy $\hbar\omega_1$ and radiation of light quantum $\hbar\omega_2$. We use Hamiltonian of interaction H_{int}, describing the effect of induced Raman atomic – molecular conversion as single process under the influence of two short pulses of resonance laser radiation [7] taking into account the processes of elastic interparticle interaction

$$H_{\text{int}} = \hbar g\left(\hat{a}\hat{a}\hat{b}^{+}\hat{c}_1\hat{c}_2 + \hat{a}^{+}\hat{a}^{+}\hat{b}\hat{c}_1^{+}\hat{c}_2\right) + \frac{1}{2}\hbar\nu_1\hat{a}^{+}\hat{a}^{+}\hat{a}\hat{a} + \frac{1}{2}\hbar\nu_2\hat{b}^{+}\hat{b}^{+}\hat{b}\hat{b} + \hbar\nu\hat{a}^{+}\hat{a}\hat{b}^{+}\hat{b}, \quad (1)$$

where $\hat{a}$ and $\hat{b}$ are the boson operators of destruction of atom and molecule, $\hat{c}_1$ and $\hat{c}_2$ are the operators of destruction of photons with frequency ω_1 and ω_2, g is the constant of atom–molecular conversion, and ν_1, ν_2, ν are the constants of interatomic, intermolecular and atom–molecular interaction properly.

Introducing the particle densities $n = |a|^2$, $N = |b|^2$, $f_{1,2} = |c_{1,2}|^2$ and two components of "polarization" $Q = i(aab^{*}c_1c_2^{*} - a^{*}a^{*}bc_1^{*}c_2)$ and $R = aab^{*}c_1c_2^{*} + a^{*}a^{*}bc_1^{*}c_2$, we obtain the system of equation:

$$\begin{aligned}
&\dot{n} = 2gQ,\ \dot{N} = -gQ,\ \dot{f}_1 = gQ,\ \dot{f}_2 = -gQ,\\
&\dot{Q} = (\Delta + (2\nu_1 - \nu)n + (2\nu - \nu_2)N)R +\\
&2gn((4N - n)f_1f_2 + Nn(f_2 - f_1)), \quad (2)\\
&\dot{R} = -(\Delta + (2\nu_1 - \nu)n + (2\nu - \nu_2)N)Q,
\end{aligned}$$

where $\Delta = 2\omega - \Omega + \omega_1 - \omega_2$ is the resonance detuning. Solving the system of the equation (2), we have four independent integrals of motion for particle densities

$$n + 2N = n_0 + 2N_0,\ Q^2 + R^2 = 4n^2 N f_1 f_2,$$
$$f_1 + N = f_{10} + N_0,\ f_2 - N = f_{20} - N_0, \quad (3)$$

where N_0, n_0, f_{10} и f_{20} – the initial density of the particles. Then the main equation, describing the time evolution of molecule densities N, it is convenient to present as equation of oscillation of nonlinear oscillator

$$(dN/dt)^2 + W(N) = 0, \quad (4)$$
$$W(N) = V(N) + U(N),$$
$$V(N) = 16N(1/2 - N)^2 (N_0 - N + f_{10}) \times$$
$$(N_0 - N - f_{20}),$$
$$U(N) = (N - N_0)^2 ((\Lambda_2/2 - \Lambda_1)(N + N_0) + \quad (5)$$
$$+ \Lambda_1 + \delta)^2.$$

Here $\delta = \Delta / g$, $\Lambda_1 = (2\nu_1 - \nu)/g$, $\Lambda_2 = (2\nu - \nu_2)/2g$ is the normalized resonance detuning and coefficients interparticle interaction properly.

There is a special regime of interparticcle interaction, in which we observe the effect of self-trapping when constants of interatomic, intermolecular and atom–molecular interaction which does not equal zero (Fig. 2).

We study further the dynamic of atom–molecular conversion subject to process of interparticle interaction, when resonance detuning equals zero. In this case may happen the effect of self-trapping in the system. When constants of interatomic, intermolecular and atom–molecular interaction does not equal zero, then we can be observed self-trapping (fig. 3). When constants of interaction equals zero this effect cannot be observed. It is natural that "potential" energy includes parameters which determines the presence of self-trapping.

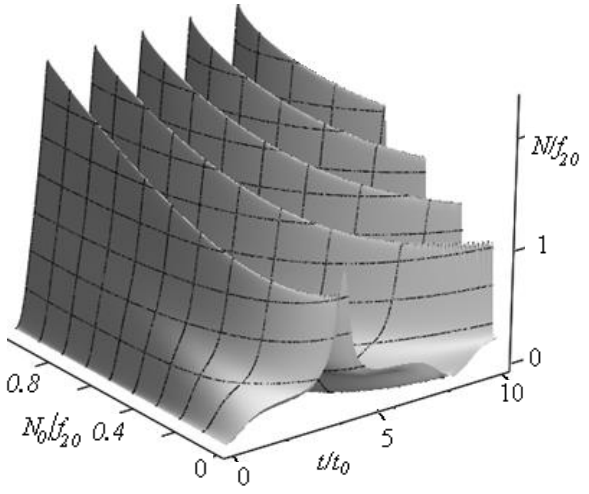

Fig.2 Time evolution of nominolised density of molecules N / N_0 in dependence on value of parameter N_0/f_{20} with $\frac{n_0}{f_{20}} = 0.4, \frac{f_{10}}{f_{20}} = 1, \nu = 1.7, \nu_1 = 1.1, \nu_2 = 1.3$.

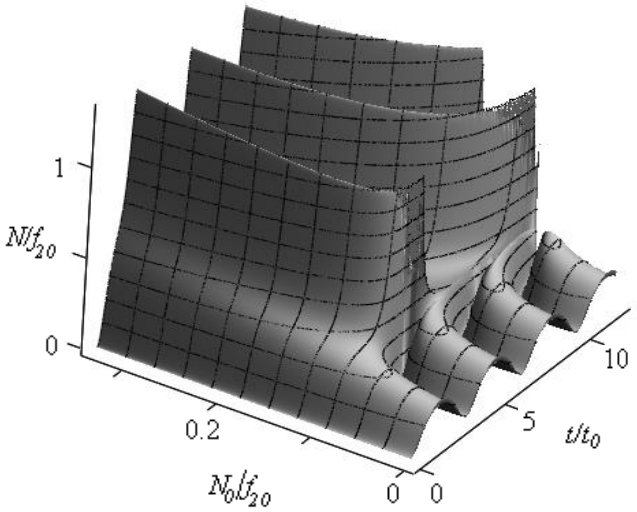

Fig.3 Time evolution of nominolised density of molecules N / N_0 in dependence on value of parameter N_0/f_{20} with $\frac{n_0}{f_{20}} = 0.4, \frac{f_{10}}{f_{20}} = 1, \nu = 0.8, \nu_1 = 1.1, \nu_2 = 1.3$

This normalized molecules' initial concentration which equals zero, the evolution of the system is periodic. If $\frac{N_0}{f_{20}}$ is low, the normalized density of molecules changes periodically in limits from zero to $\frac{n_0}{2}$. Further with the increasing of $\frac{N_0}{f_{20}}$ we can observe the effect of extreme rise of amplitude of oscillation which corresponds to demonstration of self-trapping effect in the system. When with the rising of $\frac{N_0}{f_{20}}$ the evolution of the system becomes periodic again but with larger amplitude which increases with the rise of $\frac{N_0}{f_{20}}$.

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