Voltage regulators with limited capacity power supply and Non-Euclidean geometry

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***Abstract* — The restriction of load power, two–valued regulation characteristic, and interference of several loads is observed in power supply systems with limited power of voltage source. The approach for interpretation of changes or "kinematics" of load regimes is presented by using the conformal and hyperbolic plane. This geometrical interpretation allows to base the definition of operating regime parameters. Results can be useful for electric circuit theory education and for voltage coordinated control of given loads. Non-Euclidean geometry is a new mathematical apparatus in the electric circuit theory, adequately interprets "kinematics" of circuit, provides the validation for the introduction and definition of the proposed concepts.**

***Index Terms* — cross ratio, limited voltage source, projective transformations, regulated characteristics, stereographic projection.**

I. INTRODUCTION

In power supply systems with limited voltage source power, the restriction of load power, two–valued regulation characteristic, and interference of several loads is observed [1]. Distributed power supply systems, autonomous or hybrid power supply systems on the basis of solar cells, fuel elements, and accumulators can be examples of such systems [2].

At present time, a digital control of voltage converters is used. One way of the digital control performance is a predictive technique. In one switching period, the duty cycle for the next switching cycle is calculated, based on the sensed or observed state and the input/ output information [3], [4]. Also, a feed– forward control method improves a load regulation dynamics of converter [5].This method calculates the required duty ratio variation by the predicted load current. Therefore, it is necessary to take into account the internal resistance of power supply, to carry out analysis of the load interference and obtain relationships for definition of regime parameters at the possible coordinated predictive control for preset load regimes.

The approach for interpretation of changes or "kinematics" of load regimes is presented by using the conformal and hyperbolic plane [6]. In the present paper, the base of this approach is developed.

II. BASIC MODEL OF VOLTAGE REGULATORS. DISPLAY OF CONFORMAL GEOMETRY

We consider a power supply system with two idealized voltage regulators VR_1, VR_2 , and loads R_1, R_2 in Fig.1. The regulators define transformation ratios

$$n_1 = \frac{V_1}{V},\ n_2 = \frac{V_2}{V}. \tag{1}$$

An interference of the regulators on load voltages V_1, V_2 is observed by internal resistance R_i .

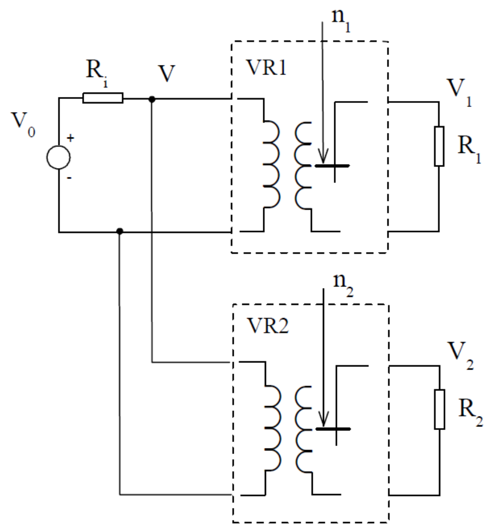


Fig.1 Power supply system with two regulators and loads

The equation of this circuit at change of parameters n_1, n_2 has the view

$$\frac{R_i}{R_1}(V_1)^2 + \frac{R_i}{R_1}(V_2)^2 + \left(V - \frac{V_0}{2}\right)^2 = \frac{(V_0)^2}{4}. \tag{2}$$

This expression represents a sphere by coordinates in Fig.2. For simplification of drawing, the axes V_1, V_2 are superposed. In turn, the variables n_1, n_2 are resulted from a stereographic projection of sphere's points from the pole $0,0,0$ [7]. These variables define the conformal plane $n_1 n_2$ [8].

In the plane $V_1 V_2$ we have an area of voltage change. This area is defined by the internal area of circle (ellipse) and corresponds to the sphere's equator in Fig.3(a).
In this case $V = V_0 / 2$. Taking into account (2), we obtain the equation of this circle

$$\frac{R_i}{R_1}(V_1)^2 + \frac{R_i}{R_2}(V_2)^2 = \frac{(V_0)^2}{4}. \qquad (3)$$

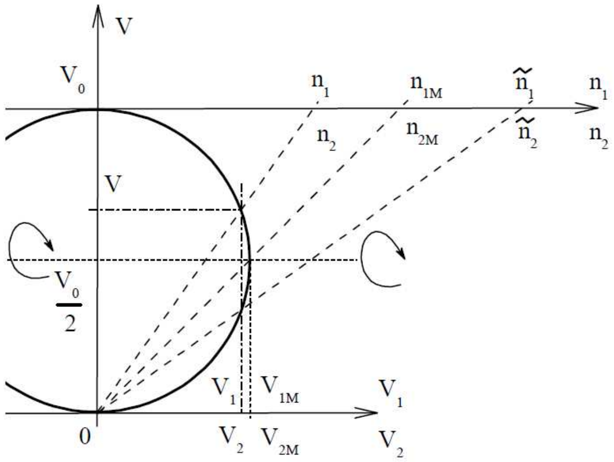


Fig.2 Stereographic projection of sphere's points $V(V_1, V_2)$ on the plane $n_1 n_2$

In the plane $n_1 n_2$ this equation has the form

$$\frac{R_i}{R_1}(n_1)^2 + \frac{R_i}{R_2}(n_2)^2 = 1. \qquad (4)$$

Then, the maximum load voltages and transformation ratios

$$V_{1M} = \pm\frac{V_0}{2}\sqrt{\frac{R_1}{R_i}}\ ,\ V_{2M} = \pm\frac{V_0}{2}\sqrt{\frac{R_2}{R_i}}\,. \qquad (5)$$

$$n_{1M} = \pm\sqrt{\frac{R_1}{R_i}}\ ,\ n_{2M} = \pm\sqrt{\frac{R_2}{R_i}}\,. \qquad (6)$$

Let, for example, the regime $V_1 = const$ (there are lines L_1) be supported at the expense of n_1, n_2 changes. Then, the circular section L_1 of sphere and corresponding circle L_1 in the plane $n_1 n_2$ turn out. Let us obtain the equation of this circle L_1 . Taking into account (1)

$$(V_1)^2 = (n_1)^2 V^2,\ (V_2)^2 = (n_2)^2 V^2.$$

Then, by (2)

$$V = \frac{V_0}{1 + \frac{R_i}{R_1}(n_1)^2 + \frac{R_i}{R_2}(n_2)^2}. \qquad (7)$$

Therefore, we have

$$V_1 = \frac{n_1 V_0}{1 + \frac{R_i}{R_1}(n_1)^2 + \frac{R_i}{R_2}(n_2)^2}. \qquad (8)$$

Thus, the equation of circle L_1 has the view

$$\frac{R_i}{R_2}(n_2)^2 + \frac{R_i}{R_1}(n_1)^2 - n_1\frac{V_0}{V_1} + 1 = 0. \qquad (9)$$

Points of intersection of the circle L_1 with the axis n_1 correspond to equation (9) as $n_2 = 0$. Then

$$(n_1)^2 - n_1\frac{R_1}{R_i}\frac{V_0}{V_1} + \frac{R_1}{R_i} = 0. \qquad (10)$$

Therefore, in the plane $n_1 n_2$ the circles correspond to the parallel straight lines of the plane $V_1 V_2$ for different values V_1 .

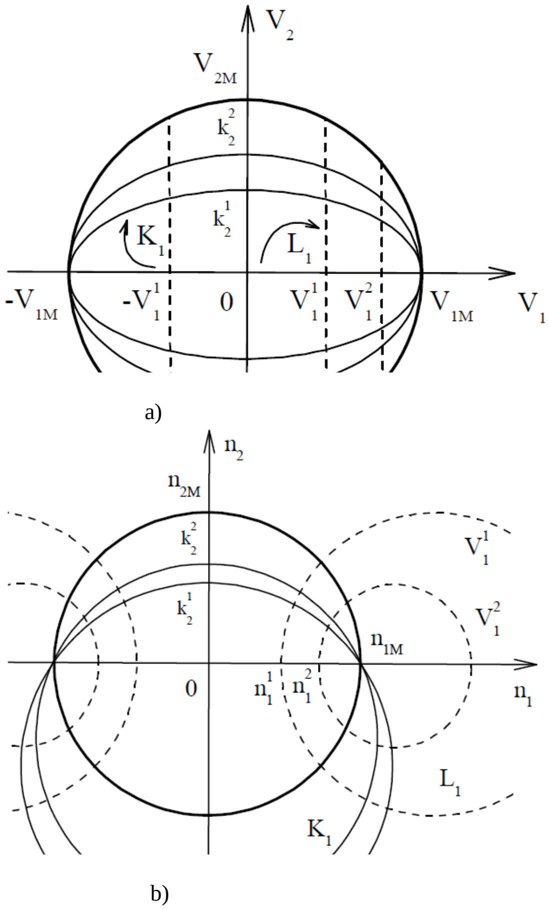


Fig.3 Correspondence the plane $V_1 V_2$ – (a) to the conformal plane $n_1 n_2$ – (b) for $V_1 = const$

Such family of circles describes the regime $V_1 = const$ as a rotation group of sphere in Figs.2, 4. This motion of points $n^1 \rightarrow n^2$ has the two fixed points $\pm n_{1M}$.

Let the set of initial points be situated on sphere's equator for different values V_1. The rotation of the equatorial plane of sphere about its diameter , as shown by arrows in Fig.2, gives the circle family K_1 with parameters k_2^1, k_2^2 , and so on in Figs.3,4.

In turn, in the plane $V_1 V_2$ the projected circles k_2^1, k_2^2 give the family of ellipses K_1 , as shown in Fig.3(a). By Fig.5(a), we have

$$V_2 = k_2\left(V - \frac{V_0}{2}\right) = n_2 V\,, \qquad (11)$$

where k_2 is an angular coefficient. Therefore, we get

$$\frac{R_i}{R_1}(V_1)^2+\left(\frac{R_i}{R_2}+\frac{1}{(k_2)^2}\right)(V_2)^2=\frac{(V_0)^2}{4}. \qquad (12)$$

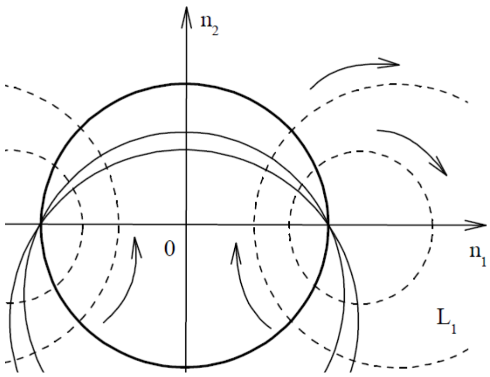


Fig. 4 Regime $V_1 = const$ as the rotation group of sphere

Next, we obtain the equation of corresponding circles in the plane n_1n_2 . From (11) it is follows

$$\frac{V_0}{V}=2\frac{k_2-n_2}{k_2}.$$

Using (7), we get the required equation of circle K_1

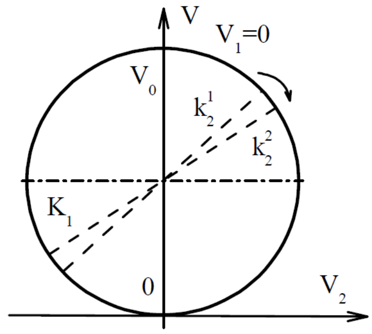
$$\frac{R_i}{R_2}(n_2)^2+\frac{R_i}{R_1}(n_1)^2+2\frac{n_2}{k_2}-1=0. \qquad (13)$$

The moving of points of circles K_1 is shown in Fig.5(b). Such moving, as a hyperbolic transformation with the two fixed points $\pm n_{1M}$, has form (5).

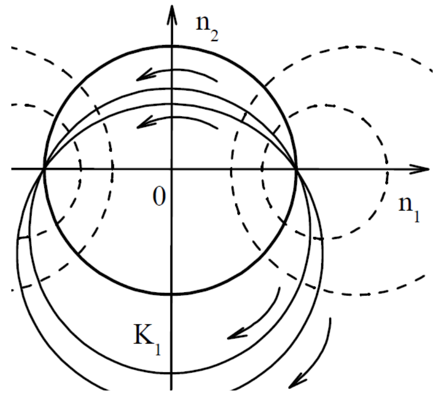
On some step of switching cycle at an increase of parameters n_1, n_2 , a running point can pass over the equator and the voltage V_2 is going on step down that is inadmissible. Therefore, it is better to use such groups of transformations or movements of points in the plane n_1n_2 , when it is impossible to move out the running point over the circle or equator of sphere . So, we must decrease the next values n_1, n_2 by some rule. In this sense, we come at hyperbolic geometry [9]. There is Poincare's model in the plane n_1n_2 . The corresponding circle carries the name of absolute and defines infinitely remote border. The equation of absolute conforms to (4). The arcs of circles L_1 are the straight lines of this model. In turn, the arcs of circles K_1 are the lines. A distance between these lines is constant. The straight lines L_1 are orthogonal to equidistance lines K_1 .

In turn, the Beltrami- Klein model is other hyperbolic geometry model in the plane V_1V_2 in Fig.3(a). The lines L_1 , K_1 of this model have the same sense.

Now, we can obtain requirement expressions or rules of regime parameters and their changes.



a)



b)

Fig. 5 Circle family K_1 with parameters k_2^1, k_2^2 ; rotation of the equatorial plane of sphere about its diameter in the coordinate plane V, V_2 – (a), moving of points of circles K_1 – (b)

III. CASE OF ONE LOAD

We consider our power supply system with one load, as $n_2 = 0$, $V_2 = 0$. Thus, a regime change goes only on the axes V_1 , n_1 . Points of initial n_1^1 and subsequent n_1^2 regime form a segment $n_1^1n_1^2$. A moving of this segment for different initial points and the conformity of variables n_1 , V_1 are shown in Fig.6. It is obvious that Euclidean length of segment $n_1^1n_1^2$ decrease, while this segment approaches to the fixed points $\pm n_{1M}$ of absolute. But, a projective transformation has an invariant in the form of cross ratio for four points [10], [11].

In the given case, there are the two fixed points, as base points, the point of initial n_1^1 , and subsequent n_1^2 regime. Then, the cross ratio m_n^{21} , which corresponds to a regime change, has the form

$$m_n^{21}=(-n_{1M}\ n_1^2\ n_1^1\ n_{1M})=\frac{n_1^2+n_{1M}}{n_{1M}-n_1^2}\div\frac{n_1^1+n_{1M}}{n_{1M}-n_1^1}. \qquad (14)$$

The value of this cross ratio is constant for different initial points, as it is shown in Fig.6. The cross ratio m_n^1 for the initial regime n_1^1 relatively to $n_1 = 0$

$$m_n^1 = (-n_{1M}\ n_1^1\ 0\ n_{1M}) = \frac{n_1^1 + n_{1M}}{n_{1M} - n_1^1}. \qquad (15)$$

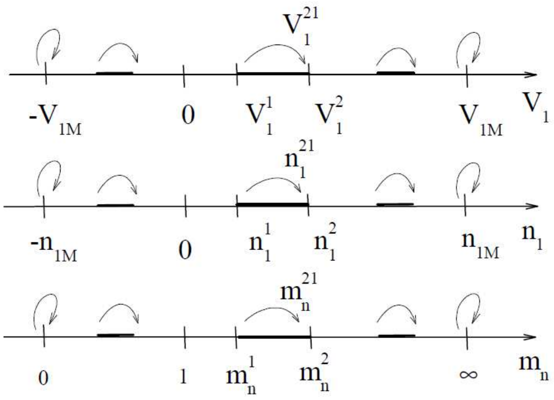


Fig. 6 Moving of segment for variables V_1, n_1

The value m_n^1 determines a non-uniform projective coordinate of the value n_1^1. Hence, the point $n_1 = 0$ is a unite point. In the same way, for subsequent regime n_1^2

$$m_n^2 = (-n_{1M}\ n_1^2\ 0\ n_{1M}) = \frac{n_1^2 + n_{1M}}{n_{1M} - n_1^2}. \qquad (16)$$

Let us obtain the expression of transformation $n_1^1 \to n_1^2$. Using normalized values, we get (14) in the form

$$m_n^{21} = (-1\ \bar{n}_1^2\ \bar{n}_1^1\ 1) = \frac{\bar{n}_1^2 + 1}{1 - \bar{n}_1^2} \div \frac{\bar{n}_1^1 + 1}{1 - \bar{n}_1^1}. \qquad (17)$$

Then, the subsequent value

$$\bar{n}_1^2 = \left(\bar{n}_1^1 + \frac{m_n^{21} - 1}{m_n^{21} + 1} \right) \div \left(1 + \bar{n}_1^1 \frac{m_n^{21} - 1}{m_n^{21} + 1} \right). \qquad (18)$$

We can introduce the value $n_1^{21} = \dfrac{m_n^{21} - 1}{m_n^{21} + 1}$.

Finally we obtain

$$\bar{n}_1^2 = \frac{\bar{n}_1^1 + n_1^{21}}{1 + \bar{n}_1^1 n_1^{21}},\ n_1^{21} = \frac{\bar{n}_1^2 - \bar{n}_1^1}{1 - \bar{n}_1^2 \bar{n}_1^1}. \qquad (19)$$

There is the required property of this transformation. If the initial value $\bar{n}_1^1 = 1$, then the subsequent value $\bar{n}_1^2 = 1$ for various values n_1^{21}. Therefore, *there is a strong reason to introduce the value of transformation ratio change as* n_1^{21}.

Let us now consider the variable V_1. Similarly to (15), (16), the cross ratios

$$m_V^1 = (-V_{1M}\ V_1^1\ 0\ V_{1M}) = \frac{V_1^1 + V_{1M}}{V_{1M} - V_1^1},\ m_V^2 = \frac{V_1^2 + V_{1M}}{V_{1M} - V_1^2}.$$

Using normalized values, similarly to (19), we get

$$\bar{V}_1^2 = \frac{\bar{V}_1^1 + V_1^{21}}{1 + \bar{V}_1^1 V_1^{21}},\ V_1^{21} = \frac{\bar{V}_1^2 - \bar{V}_1^1}{1 - \bar{V}_1^2 \bar{V}_1^1}.$$

Thus, *there is a strong reason to introduce the value of voltage change as* V_1^{21}.

IV. CONCLUSION

Geometrical interpretation allows basing the definition of operating regime parameters. The concrete kind of circuit and character of regime determines system parameters; arbitrary expressions are excluded.

Obtained expressions are generalized for two or more loads. From the methodological point, the presented approach is applied for a long time in other scientific domains, as mechanics, biology.

REFERENCES

[1] D. Yahia et al., "Deliverable-power dependence on distribution-line resistance and number of loads in low-voltage DC distribution system," IEEJ Trans. Electrical and Electronic Engineering, No.7, pp. 23–30, 2012.

[2] X. Lui, P. Wang, and P. Loh, "A Hybrid AC/ DC Micro grid and its coordination control, " IEEE Trans. Smart Grid, Vol. 2, No. 2, pp. 278-286, 2011.

[3] J. Chen, et al., "Predictive Digital Current Programmed Control," IEEE Trans. Power Electronics, Vol. 18, No. 1, pp. 411-419, 2003.

[4] S. Chattopadhyay and S. Das, "A Digital current-mode control technique for DC–DC converters," IEEE Trans. Power Electronics, Vol. 21 No. 6, pp. 1718-1726, 2006.

[5] S. Chae, et al., "Digital predictive feed-forward controller for a DC–DC converter in plasma display panel," IEEE Trans. Power Electronics, Vol. 23 No. 2, pp. 627-634, 2008.

[6] A. Penin, "Analysis of regimes of voltage regulators with limited capacity voltage sources. Geometrical approach," WSEAS Trans. Circuits and Systems, Issue 1, Vol. 12, 2013. http://www.wseas.org/multimedia/journals/circuits/2013/55-251.pdf

[7] Encyclopedia Wikipedia "Stereographic projection," A http://en.wikipedia.org/wiki/Stereographic_projection

[8] Encyclopedia Wikipedia "Conformal geometry," http://en.wikipedia.org/wiki/Conformal_geometry

[9] Encyclopedia Wikipedia "Hyperbolic geometry", http://en.wikipedia.org/wiki/Hyperbolic_geometry

[10] Jr. A. Frank, Schaum's outline of theory and problems of projective geometry, McGraw-Hill, 1967.

[11] A. Penin, "Projective geometry method in the theory of electric circuits with variable parameters of elements," International Journal of Electronics Communications and Electrical Engineering, Issue 2, Vol. 3 pp. 18-34, 2013. http://www.ijecee.com/uploads/displayVolumeIssue/V-3-I-2-ID-2.pdf