SPIN-SPLITTED STATES AND SPIN HALL EFFECT INDUCED BY POLARIZATION IN SEMICONDUCTORS STRUCTURES

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Abstract: A new terms of SOI induced by interband interaction through the electrical polarization or atomic displacement like optical phonons are proposed and analyzed. These SOI mechanisms have the physical nature in the relativistic quantum mechanics with Lorentz boosts. Some particularities of the electronic states of semiconductor quantum wells (QW) and tunneling characteristics of single barrier heterostrucrure related to new SOI terms are studied. Size quantization states of QW are shown to transform into interface ones under SOI effect. Tunneling characteristics of single barrier heterostrucrure are established to be spin-dependent under SOI induced by the EP. The spin Hall effect (SHE) is analyzed in a two dimensional electron system with the SOI of both intrinsic and EP induced types. New peculiarities of SHE, induced by interband interaction of heavy and light holes through optical like displacements, are studied in p-doped semiconductors in the framework of generalized Luttinger Hamiltonian.

1. INTRODUCTION

The areas of semiconductor physics and microelectronics have been successfully combined for a long time developing and enormous number of devices based on the charge flows and the manipulation of the carrier charge. More recently, the use of the second fundamental attribute of the electrons - spin degree of freedom, as well as the charge, has attracted great interest [1]. As results new field of electronics is developing - spin-dependent electronics or spintronics [1]. The basic concept of spintronics is the manipulation of spin currents, in contrast to mainstream electronics in which the spin of the electron is ignored. Adding the spin degree of freedom provides new effects, new capabilities and new functionalities. At present there is yet several devices working on the basis of spintronics principles. It is enable to mention the giant and tunneling magnetoresistance devices, wich are based on layered structures of ferromagnetic materials. However, despite of some practical achievements of metal magnetoelectronics semiconductor spintronics has at present the most emerging development.

The ability to control the electron spin requires efficient mechanisms of the spin splitting of the electronic states as well as of the spin dependent transport characteristics. Many approaches have been proposed to achieve control of the electron spin degree of freedom using ferromagnetic materials, external magnetic fields and optical excitation [1]. Among the promising methods there is the intrinsic spin-orbit interaction (SOI) in semiconductors, which allows to manipulate the spin by means of electric fields [2] and electric gates [3].

The spin-orbit interaction (SOI) arises as a result of the magnetic moment of the spin coupling to its orbital degree of freedom and being of relativistic nature is enough weeks for an electron in the vacuum. In semiconductor structures due symmetry properties of materials, band structure effects and the SOI becomes stronger and has diverse channels of manifestation [2]. Enhancement of SOI in crystals occurs from two basic sources [2]. The first part appears through the band structure effects and is a result of fast electron motion in a strong electric field near nuclei. As illustration in two-band model of narrow gap semiconductors the equations are similar to a Dirac equation but with the forbidden gap E_g instead of the Dirac gap $2m_0c^2$. Therefore H_{SO} after the reduction to one band model becomes $H_{SO} = \frac{1}{2m_0}\left[\vec{p}\times\hbar\vec{\nabla}\left(\frac{V_{ext}}{2E_g}\right)\right]\vec{\sigma}$. Second type of enhancement and diversification comes from the symmetry of materials and solid-state structures. This leads to new terms in SOI such as Dresselhaus and Rashba ones,which are more frequently exploring for different spintronics applications [1,2].

The above mentioned mechanisms of SOI stem from the electron orbital motion and are related to the 3D spatial angular momentum. At the same time there are angular momentum related to the generators of so-called Lorentz boosts [4] related to 4D angular momentum tensor. This leads to a new attribute of the electron [5] - induced electric moment. In crystals and semiconductor heterostructures one of the manifestation of this fundamental electron properties is the appeareance of new chanels of SOI, which a considered in the first part of the paper.. One such new channel of interaction of the carriers in semiconductor materials and heterostructures are related to electrical polarization appearing in spontaneous way due to of the atomic displacements of two sublattices of semiconductors or due to piezoelectric effect induced by strain. In the two band model of semiconductors a variant of such effect of the electrical polarization on the electronic states of semiconductor heterostructures have been considered in the papers [6, 7].

In the second part of the paper the electronic states and the tunnel characteristics of semiconductor structures (quantum well (QW) and barrier) are studied taking into account the intrinsic Rashba spin-orbit interaction (SOI) and Rashba like SOI induced by the different electrical polarization (EP) of the layers. Both types of SOI terms influence considerably QW size quantization states transforming the last into interface states at some value of the in-plane wave vector. The interface states are shown to occur even in tunnel barrier structure at finite value of the in-plane wave vector [8]. Analyzing the transmission coefficient of a symmetric barrier structure it is show that EP induced SO coupling leads to spin-dependent tunnel characteristics of the structure.

Last part of the paper includes some results deal with the new discovered spin Hall effect (SHE) [9,10] The geometrical structure of the intrinsic SHE is such that for an electric field applied, for example, on the z direction, a y-polarized dissipationless spin current will flow in the x direction . The following macroscopic relation can summarize the electric field induced spin current $J_j^i = \sigma_s \varepsilon_{ijk} E_k$. The extrinsic SHE was considered more two decade ago[11]. However, the real practical interest of SHE as an approach of spin current generation in spintronics devices has appearing recently after the discovery of intrinsic SHE [9,10]. After this, two seminal experiments on the SHE have been done [12,13]. In our paper the SHE is investigated in a two-dimensional electron system with Rashba SOI of both intrinsic and EP induced types. Some new peculiarities of SHE, induced by interband interaction of heavy and light hole bands through electrical polarization or displacement of optical phonon like are revealed in p-doped semiconductors with band degeneracy in the framework of Luttinger Hamiltonian.

2. SPIN-ORBIT INTERACTION INDUCED BY INTERBAND COUPLING THROUGH OPTICAL PHONON LIKE DISPLACEMENTS

In the simplest two band model the equations of the electronic states with incorporated electrical polarization or displacement is written [8]

$$\begin{pmatrix} h_c & h_{cv} \\ h_{vc} & h_c \end{pmatrix} \begin{pmatrix} F_c \\ F_v \end{pmatrix} = \begin{pmatrix} E_g(\vec{r})/2 + Q(\vec{r}) & \vec{\sigma}\,\vec{p} - i\vec{\sigma}\,\vec{U}(\vec{r}) \\ \vec{\sigma}\,\vec{p} + i\vec{\sigma}\,\vec{U}(\vec{r}) & -E_g(\vec{r})/2 + Q(\vec{r}) \end{pmatrix} \begin{pmatrix} F_c \\ F_v \end{pmatrix} = E \begin{pmatrix} F_c \\ F_v \end{pmatrix}, \qquad (1)$$

where $E_g(\vec{r})$ is position dependent band gap of the semiconductor. In the case of heterojunction $Q(\vec{r})$ is the external applied potential, $\vec{p} = ip\vec{\nabla}$ is the momentum operator (p being momentum interband matrix elements), $U(\vec{r})$ is the term related to the above-mentioned electrical polarization or displacement. F_V and F_C are the components of the spinor wave function.

After the reducing to the one band Schrodinger equation (1) become:

$$H_c = h_c + \vec{p}(E - h_v)^{-1}\vec{p} + P\vec{\sigma}\left[\vec{\nabla}(E - h_v)^{-1} \times \vec{p}\right] + +U^2(E - h_v)^{-1} + P\vec{\nabla}\left[\vec{U}(E - h_v)^{-1}\right] -$$
$$- 2\vec{\sigma}\left[\vec{U}(E - h_v)^{-1} \times \vec{p}\right] - iP\vec{\sigma}\left[\vec{\nabla}(E - h_v)^{-1} \times \vec{U}\right] - iP\vec{\sigma}\,\mathrm{rot}\left(\vec{U}(E - h_v)^{-1}\right) \qquad (2)$$

This expression illustrates that even in the two band model including of spin like degree of freedom, through of electrical polarization or deplacement, leads a set of new SOI terms. The term $2\vec{\sigma}\left[\vec{U}(E-h_v)^{-1}\times\vec{p}\right]$ look like traditional SOI term $P\vec{\sigma}\left[\vec{\nabla}(E-h_v)^{-1}\times\vec{p}\right]$, where the electric field related to $\vec{\nabla}(E-h_v)^{-1}$ is replaced by electrical polarization. More unusual look the last term $iP\vec{\sigma}\,\mathrm{rot}\left(\vec{U}(E-h_v)^{-1}\right)$,which describes the SOI induced by the vector of local rotation of the lattices..Also a part of analyzed term $iP\vec{\sigma}\left[\vec{\nabla}(E-h_v)^{-1}\times\vec{U}\right]$ describe the term of SOI induced by rotation like interaction of electrical and polarization fields.

Following [14] and introducing the effective masses of the two band model the Hamiltonian is rewritten for the case of two-dimensional layered heterostructure with variation of the parameters in one z direction and polarization vector $\vec{U}$ oriented in the same direction $\vec{U}=\{0,0,U(z)\}$ takes the form.

$$H_C=-\frac{\hbar^2}{2}\frac{d}{dz}\frac{1}{m(z)}\frac{d}{dz}+\frac{\hbar^2k_\perp^2}{2m(z)}+\frac{\hbar^2U^2}{2P^2m(z)}+\frac{\hbar^2}{2P}\frac{d}{dz}\left(\frac{U}{m(z)}\right)+$$
$$+\left[\frac{\hbar^2}{2}\frac{d}{dz}\left(\frac{1}{m(z)}\right)+\frac{\hbar^2}{P}\frac{U}{m(z)}\right]\cdot\left(k_x\sigma_y-k_y\sigma_x\right) \tag{3}$$

Where k_x and k_y are the wave vectors in the plane of the heterostructure $k_\perp^2=k_x^2+k_y^2$. In this case the energy is measured from the bottom of the band. From (10) it follows that in this case the SOI induced by electrical polarization is Rashba like .

3. SPIN-SPLIT ELECTRONIC STATES AND SPIN-DEPENDENT TUNNELING CHARACTERISTICS IN SEMICONDUCTOR HETEROSTRUCTURES

The electronic states and its energy spectrum in symmetric quantum well with width $2a$ and depth V_0. is investigated on the basis of Hamiltonian (3) taking into account that reduced electrical polarization Q(z)= U(z)/m(z) is oriented along axis z [15].

In (3) the last non diagonal term can be written in diagonal for after the unitary transformation $T=\begin{pmatrix}-e^{j\varphi} & e^{j\varphi}\\ 1 & 1\end{pmatrix}$, where φ is the angle between $\vec{k}_\perp$ and the axis x. The result can be explains as $\left(k_x\sigma_y-k_y\sigma_x\right)\rightarrow k_\perp\sigma_z$

On the basis of (3) and boundary conditions such as $\frac{1}{m}\left[\nabla_z-\frac{U}{P}+k_\perp\lambda\right]\psi=cons$ (where $\lambda=\pm1$ is the quantum number that describes the states with spin "up" and spin "down" in the new spin-coordinate systems) by straightforward calculations we obtain the equation for energy spectrum of QW with thickness $2a$ [15]

$$2\gamma q\kappa\cos 2\kappa a+\left\{\begin{matrix}q^2-\gamma^2\kappa^2-\\-\left[\left(Q_b+k_\perp\lambda\right)-\gamma\left(Q_w+k_\perp\lambda\right)\right]^2\end{matrix}\right\}\sin 2\kappa a=0 \tag{4}$$

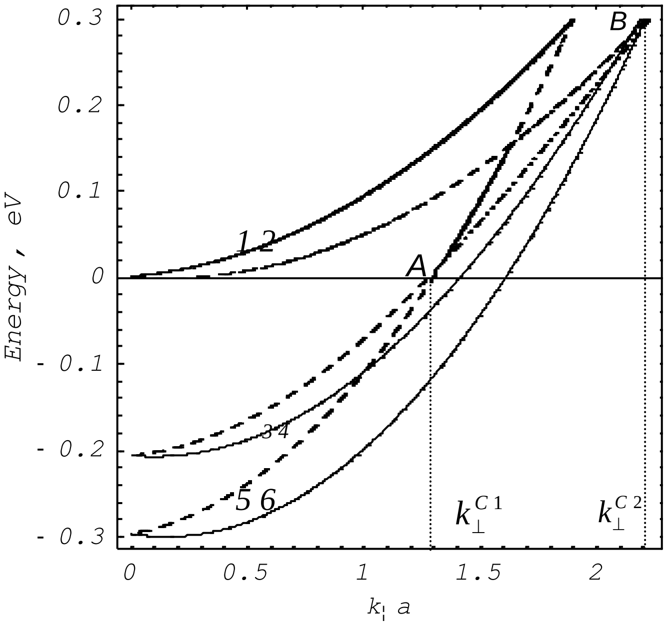

Fig. 1. Quantum well structure
1, 2, 5 and 6 are the bulk dispersion curve; 3 and 4 are an energy level for spin-up (solid line) and spin-down (dotted line) electrons.

where $\gamma = \frac{m_b}{m_w}$ is the mass ratio.

The interface states are described by an exponential behaviour in the well region $\propto e^{\pm\kappa(z+a)}$ and their dispersion relation is similar to (4), where k is replaced by $i\kappa$.

It is important to outline the appearance of spin-orbit coupling contributions in dispersion equation (4) for a symmetric QW without electrical polarization $Q = 0$ of the layers yet.The relation (4) shows a dependence of the electron energy on spin factor λ, both for quantum dimensional states and interface one. To be more explicitly we plot in Fig. 1 the numerical results of some calculation. The effective mass in the well structure is about 0,07 electron mass and the mass ratiois equal to 2. Electrical polarization is equal in magnitude but opposite in sign for well and its borders. Thus $Q_b = -Q_w$=4 10^{-2} nm^{-1}.

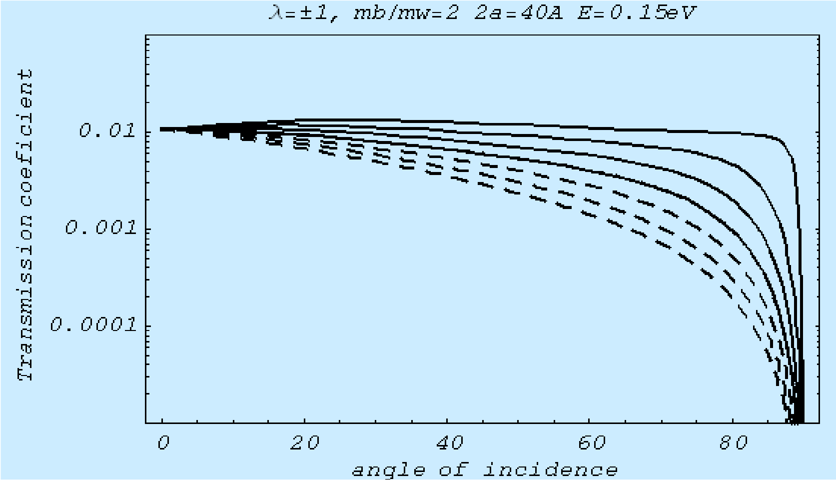

Fig. 2. Variation of the transmission coefficient via angle of incidence of electron beam for barrier structure with height $V_0 = 300\ meV$ and width $2a = 40\ A$. Curves A, B, C and D are plotted for incident energy $E = 450, 350, 250, 150\ meV$ respectively. Dotted lines are for spin down and solid lines for spin up

The interface states for spin down electrons appear from $k_\perp^{C1}$ to $k_\perp^{C2}$ in the range where bulk dispersions intersect (curve AB). For spin up, the situation has mirror symmetry relative to energy axis.

For an symmetrical barrier with width 2a on the basis of (3) an corresponding boundary conditions the transmission coefficient T_C is obtained:

$$T_C = \frac{B^2}{A^2 \sinh^2 2\kappa a + B^2 \cosh^2 2\kappa a} \tag{5}$$

where q and κ are the wave vectors in the well an in the barrier, and the notations

$A = \gamma^2 q^2 - \kappa^2 + [\gamma(Q_w + \lambda k_\perp) - (Q_b + \lambda k_\perp)]^2$ and $B = 2\gamma\kappa q$, (6)

As follows from (5) and (6) the both intrinsic and induced Rashba like SOI lead to the spin-dependent tunneling in an single symetrical barrier heterostructures.

The relations (5) - (6) take into account the space dependent electron effective mass, the motion in the perpendicular and parallel direction to the barrier, the spin of incident electrons and the electrical polarization due to the stress of layers. We are plotted the transmission coefficient via angle of incidence (see Fig. 2)

for a symmetric barrier with height $V_0 = 300\ meV$ and width $2a = 40\ A$ for different energy of incident electrons. The effective mass of incident electrons is 0,067 m_0 and mass ratio $\gamma = 2$. From graphic it is clear that there is a great difference for transmission probability of electrons with up or down spin orientation. This fact can be used to obtain a spin polarization current.

4. SPIN HALL EFFECT INDUCED BY NEW SOI

In the first papers the analysis of intrinsic spin Hall effect (SHE) was done on the basis of semiclassical approach. In two-dimensional n-type systems, studied in [10], the Hall conductivity σ_{xy} in a clean limit is calculated from the Kubo formula. By straightforward calculation following approach [10] it is easy to incorporate the influence of Rashba like SOI induced by electrical polarization, oriented normal to the 2D systems. As follows from (4) in this case the electrical induced SOI leads only to renormalization of Rashba constant and therefore lesds only to the modification of different regimes of SHE.

The intrinsic SHE in p-type systems has been studied [9] in bulk semiconductors on the basis of Luttinger Hamiltonian. Taking into account the interband interaction through optical phonons [16] the SOI induced by electrical polarization or optical like displacement U= (u_x,u_y,u_z) can be incorporated into an effective Luttinger Hamiltonian, which is written as

$$\begin{aligned}\hat{H} = \frac{\hbar^2}{2m^*}\Big[\Big(\gamma_1 + \frac{5}{2}\gamma_2\Big)k^2 - 2\gamma_2\left(k_x^2S_x^2 + k_y^2S_y^2 + k_z^2S_z^2\right) - \\ - 4\gamma_3\left(\{k_x,k_y + u_z\}\{S_x,S_y\} + \{k_y,k_z + u_x\}\{S_y,S_z\} + \{k_z,k_x + u_y\}\{S_z,S_x\}\right)\Big]\end{aligned} \qquad (7)$$

where $S = (S_x, S_y, S_z)$ are spin 3/2 matrices, γ_1, γ_2 and γ_3 are the Luttinger constants, $[S_i,S_j] = \frac{1}{2}(S_iS_j + S_jS_i)$.

Following the recently proposed approach [17] the effective Luttinger Hamiltonian can be transformed in terms of a Clifford algebra $\hat{H} = \gamma_1 k^2 + 2\gamma_3 d_a\Gamma_a$ of Diracmatrices.Where $d_1 = -\sqrt{3}k_zk_y + \sqrt{3}u_x$ $d_2 = -\sqrt{3}k_zk_x + \sqrt{3}u_y$ $d_3 = -\sqrt{3}k_xk_y + \sqrt{3}u_z$

$d_4 = -\frac{\sqrt{3}}{2}\frac{\gamma_2}{\gamma_3}\left(k_x^2 - k_y^2\right)$ $d_5 = -\frac{1}{2}\frac{\gamma_2}{\gamma_3}\left(2k_z^2 - k_x^2 - k_y^2\right)$

Following the approach [17] in spherical approximation and keeping only terms linear in uj we obtain: in analytical form the expression for spin conductance is obtained: in analytical form:

$$\sigma_{ij}^l = \sigma_{ij}^l 0) + \sigma_{ij}^l(u) = \frac{1}{6\pi^2}\varepsilon_{ijl}\left(k_F^H - k_F^L\right) + \frac{1}{6\pi^2}\varepsilon_{ijl}u_l\left(\frac{1}{k_F^L} - \frac{1}{k_F^H}\right) \qquad (8)$$

where k_F^H и k_F^L are the Fermi wave vector of heavy and light holes of bulk semiconductors respectively, ε_{ijl} is the antisymmetric tensor.

5. CONCLUSIONS

Because spin-orbit interactions (SOI) couples electron spins to electric fields and allows electrical and gates manipulation of electron spins and electrical detection of spin dynamics they form the fundamental mechanisms of new effects and the basis of different proposals for new spintronic devices. The induced by interband interaction through the electrical polarization or atomic displacement like optical phonons is shown to generate a new terms of SOI in semiconductor materials and heterostructures in addition to known channels of SOI. They appear as the results of manifestation of Lorentz boosts in materials and appear due to violation of the local crystal symmetry under effect of optical phonon like displacements or electrical polarization.

. Some particularities of the electronic states of semiconductor quantum wells (QW) related to the intrinsic Rashba SOI and Rashba like SOI induced by the electrical polarization (EP) are studied. Size quantization states are shown to transforme into interface ones at some value of the in-plane QW wave vector under SOI effect. Tunneling characteristics of single barrier heterostrucrure are analyzed in the conditions of both types of SOIs and are established to be spin-dependent under induced SOI.

Several aspects deal with the new discovered intrinsic spin Hall effect are revealed. The SHE is investigated in a two dimensional electron system with the SOI of both intrinsic and EP induced types. New peculiarities of SHE, induced by interband interaction of heavy and light holes are studied in p-doped semiconductors with band degeneracy in the framework of generalized Luttinger Hamiltonian. Spin-conductivity induced by the EP is shown to proportional to difference of the inverse values of the hole wave vectors.

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