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ORIGINAL PAPER

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New Green's functions for a thermoelastic unbounded parallelepiped under a point heat source and their application

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Abstract This paper presents new analytical expressions for displacements Green's functions to a steady-state spatial BVP of thermoelasticity for an unbounded parallelepiped, subjected to a unit point heat source. These results are obtained on the base of special structural formulas for displacements Green's functions, which are expressed in terms of respective Green's functions for Poisson's equation. An example of the application of derived new analytical expressions is presented for a particular spatial BVP for a thermoelastic unbounded parallelepiped, subjected to a constant heat source, given inside of a rectangle. Both analytical expressions for displacements Green's functions and thermoelastic displacements in the case of a particular problem are obtained in the form of double infinite series, containing product between exponential and trigonometric functions, which satisfy basic equations, boundary conditions on the marginal strips and vanishes at infinity. The presented example of a new steady-state spatial BVP of thermoelasticity for an unbounded parallelepiped, subjected to a unit point heat source will permit readers to derive the other examples to new analytical expressions for Green's functions. These Green's functions can be applied as kernels in the method of the boundary integral equations to solution of many particular BVP for thermoelastic unbounded parallelepiped. All these analytical results can be used also as some test problems for different numerical methods.

Abbreviations

2D	Two dimensional
3D	Three dimensional
BVP	Boundary value problem
GFs	Green's functions
GFM	Green's function method
GFPE	Green's functions for Poisson equation
TVD	Thermoelastic volume dilatation
HIR	Harmonic integral representations
HIRM	Harmonic integral representations method
MTGFs	Main thermoelastic Green's functions
TSGFs	Thermal stresses Green's functions

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