Special elements in octonions algebra over Zp

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Abstract. This paper explores the properties of idempotent and nilpotent elements in octonion algebras over finite fields, specifically Zp, where p is a prime number. Octonion algebras, which extend quaternion algebras, are characterized by their non-associative structure and their dimension of 8. The study begins by defining idempotent elements, where $\sigma^2 = \sigma$, and nilpotent elements, where σ^$z = 0$ for some nonzero integer \(z\). These elements are of particular interest due to their relevance in the algebraic structures of both quaternion and octonion algebras. The paper presents examples and theorems to illustrate the conditions under which elements in octonion algebras retain idempotent or nilpotent properties. Special attention is given to the implications of these findings for the structure of the algebra O/Zp, with potential applications in fields such as coding theory and cryptography. The results demonstrate how algebraic structures like octonions behave under finite fields and their differences from associative algebras, such as quaternions.

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