THE APROXIMATE METOD OF REPRESENTATION OF REAL MATERIAL IN THE STRUCTURAL MODEL

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INTRODUCTION

The study [8] was devoted to solution of the problem of representation of a real material in the structural model of micro-heterogeneous medium [1-5]. The experiments were established on the basis of which the rheological functions of subelements can be identified unequivocally.

However, the need to test the thin-walled tubes subjecting to the action of the internal pressure P_i and the axial force F at different constant rates of the gripping device movement and different temperatures narrows the possibilities of use of the published experimental data, which mainly represent results of the thin-walled tubes tests by axial tension at constant rates of the gripping device movement $\dot{d}_{zz} = const$ or by torque at constant rates of change of the torsion angle $\dot{d}_{\varphi z} = const$ (z, r, φ is the cylindrical coordinate system).

We will demonstrate how to with sufficient degree of precision the problem of representation of a real material in the structural model can be solved on the basis of data obtained for the thin-walled tubes by putting to the uniaxial tensile test.

1. THE STATE PARAMETERES

The macroscopic element, being at the initial time in natural state, is subjected to mechanical and thermal action. Material behavior during deformation is supposed to depends significantly on the rate of loading and heating. To describe the micro-heterogeneous medium behavior the macroscopically homogeneous volume element V_0 of the polycrystalline body is considered to be composed of an infinite number of kinematically connected subelements with different thermorheological features. Subelement as the elementary structure identifies the set of all material particles in the interior domain V_0 that have the same irreversible strain tensors.

Within the limits of the examined model let us assume that all types of interactions between subelements in the conglomerate are formed only under the influence of average connections, i.e. material particles in the conglomerate do not deform independently, but only in a coordinated manner. The interaction between two subelements is realized by means of the interactions between material particles which are appertained to the different subelements. This fact is reflected by replacement of the local state parameters in physical equation for subelement on the average values of the whole set.

The thermoviscoplastic properties of subelements within the limits of the structural model [2,5,8] may be determined on the basis of diagrams of the proportional loading $e = e(p, \gamma, \upsilon)$ at different constant values of the state parameters γ and υ.

The state parameter υ describes rheological effects of the subelement, and is expressed by the ratio of the volume variation and its limit possible value, being the same for all subelements

$$\upsilon = \frac{1}{\varepsilon_{0k}} \int_0^1 (\bar{\varepsilon}_0 - \beta \bar{e}_0) d\psi . \qquad (1)$$

or

$$\upsilon = \frac{\varepsilon_0 - \beta e_0}{\varepsilon_{0k}}, \quad \varepsilon_0 = \int_0^1 \bar{\varepsilon}_0 d\psi , \quad e_0 = \int_0^1 \bar{e}_0 d\psi . \qquad (2)$$

Differentiating with respect to time we find the loading conditions at $\upsilon = const$:

$$\dot{\varepsilon}_0 = \beta \dot{e}_0 = \beta \frac{\dot{\sigma}_0}{K} . \qquad (3)$$

Assuming that at the low level of irreversible deformations the elastic volume variation exceeds considerably the irreversible

$$\dot{\varepsilon}_0 \approx \dot{e}_0 + \alpha_T \dot{T} , \qquad (4)$$

we obtain that the rate of temperature's change is proportional to the rate of average stress's change

$$\dot{T} = -\frac{1-\beta}{\alpha_T K} \dot{\sigma}_0 . \qquad (5)$$

Thus, the deformation at $\upsilon = const$ corresponds to isothermal loading if $\beta = 1$.

Rate of irreversible deformation trajectory length's change is the state parameter that reflects the sensitivity of subelement to rate of external action's change:

$$\dot{\bar{\lambda}} = \sqrt{\dot{\bar{p}}_{ij}\dot{\bar{p}}_{ij}}. \qquad (6)$$

Average rate of irreversible deformation's change in subset of subelements, being under loading above the elastic limit, is another state parameter

$$\gamma = \frac{1}{\psi_\lambda}\int_0^1 \dot{\bar{\lambda}}(\psi')d\psi', \qquad 0 \le \psi_\lambda \le 1, \qquad (7)$$

where ψ is the distinctive parameter of subelement and coincides, during the initial loading, with the weight of irreversibly deformed subelements when given subelement exceeded the elastic limit; ψ_λ – summary weights of subelements for which the parameter $\dot{\bar{\lambda}}$ is nonzero.

In monotonous processes throughout the subset of irreversibly deformed subelements an active process of loading occurs, that corresponds to the monotony of the evolution of weight of irreversibly deformed subelements in this process. This means that towards ψ only one separation boundary forms between reversibly $\psi' < \psi \le 1$ and irreversibly $0 \le \psi \le \psi'$ deformed subelements.

Taking into account the law of the admissible trajectories [1,5,7] and the fact that in the monotonous processes the variations $d\bar{p}$ in all subelements have one and the same sign, we can write

$$d\lambda = \int_0^{\psi'} d\bar{\lambda}d\psi, \qquad d\bar{p}\big|_{\psi>\psi'} = 0, \qquad \gamma = \frac{\dot{\lambda}}{\psi'}. \qquad (8)$$

In the monotonous process of deformation along a rectilinear trajectory

$$\gamma = \frac{1}{\psi'}\int_0^{\psi'} \dot{\bar{p}}d\psi = \frac{\dot{p}}{\psi'}, \qquad \dot{\bar{p}}\big|_{\psi>\psi'} = 0. \qquad (9)$$

2. UNIAXIAL TENSILE TESTS AT DIFFERENT RATES OF THE GRIPPING DEVICE MOVEMENT

In the study [8] to calculate the parameter γ on the basis of tests of the thin-walled tubes being under the influence of the internal pressure P_i and the axial force F the following formula was obtained

$$\gamma = \left[\frac{(1-m+a)(2-\zeta)}{6G(a_0+m)} - \frac{\beta(m-a)(1+\zeta)}{3K(a_0+m)}\right]\frac{\dot{t}_{zz}}{a_{zz}} + \frac{m-a}{a_0+m}\frac{\dot{d}_{zz}}{a_{zz}}. \qquad (10)$$

The orientation of the loading trajectory in the space of axial t_{zz} and circumferential $t_{\varphi\varphi}$ stresses we will define by the parameter ζ

$$\zeta = \frac{t_{\varphi\varphi}}{t_{zz}}, \qquad a_{zz} = \frac{2-\zeta}{\sqrt{6(1-\zeta+\zeta^2)}}, \qquad (11)$$

$$t_{zz} = \frac{F}{2\pi Rh} + \frac{P_i R}{2h}, \qquad t_{\varphi\varphi} = \frac{P_i R}{h}, \qquad (12)$$

where h is the tube wall's thickness and R is the average radius of the tube.

In the study [8] it was demonstrated that in the case of the axial tension is impossible to carry out experiment under a constant state parameter γ. Therefore, the solution of the problem of the representation of the real material in the structural model on the basis of experiments conducted by stretching can be achieved only in an approximate way.

In the case of axial tension the circumferential stresses are equal to zero

$$t_{zz} = \frac{F}{2\pi Rh}, \qquad t_{\varphi\varphi} = 0, \qquad (13)$$

$$\zeta = 0, \qquad a_{zz} = \sqrt{2/3}, \qquad (14)$$

for isothermal loading $\beta = 1$, then from (10) we obtain

$$\gamma = \sqrt{\frac{3}{2}}\left[\frac{K-(m-a)(K+G)}{3GK(a+m)}\dot{t}_{zz} + \frac{m-a}{a+m}\dot{d}_{zz}\right]. \qquad (15)$$

Taking into consideration that at $\varepsilon = \varepsilon_{el}$ rate of the axial normal stress change is directly proportional to rate of the axial strain change

$$\dot{t}_{zz} = K(1-2\nu)\dot{d}_{zz}, \qquad (16)$$

we can write the expression for determining the parameter γ in this way

$$\gamma_1 = \sqrt{\frac{2}{3}}\frac{1+\nu}{a_0+m}\dot{d}_{zz}, \qquad (17)$$

where in accordance with the results obtained at [5,6,7] m is the interior parameter of the scheme of kinematic interaction between subelements and depends on the linear hardening coefficient a_0:

$$m = -a_0 + \sqrt{a_0 + a_0^{\,2}} \,. \qquad (18)$$

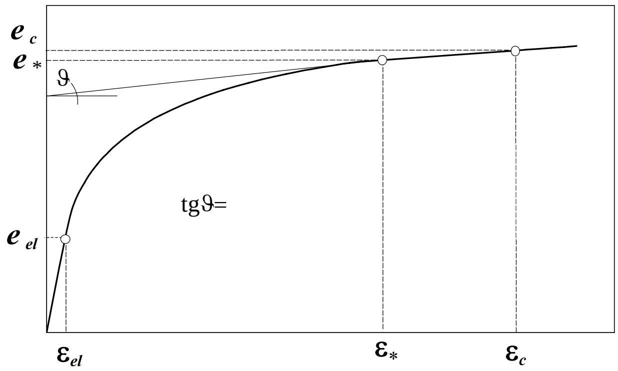

Figure 1. Diagram of the proportional loading of the thin-walled tubes by axial force at constant temperature $T = const$ and constant rate of longitudinal strain change $\dot{d}_{zz} = const$

Let us rebuild the diagram $t_{zz} \sim d_{zz}$ in the space of modules of strain deviators $e \sim \varepsilon$ (figure 1)

$$\sigma = \sqrt{\frac{2}{3}} t_{zz}\,, \quad e = \sqrt{\frac{2}{3}} \frac{t_{zz}}{2G}\,, \quad \varepsilon = \sqrt{\frac{3}{2}} \left(d_{zz} - \varepsilon_0 \right). \quad (19)$$

Assuming that the volume varies elastically, we find

$$\varepsilon = \sqrt{\frac{3}{2}} \left(d_{zz} - \frac{t_{zz}}{3K} \right). \qquad (20)$$

A number of authors [9] found that in the strain diagram $e \sim \varepsilon$ is observed a linear hardening sector, the slope of which does not depend on the temperature and rate of loading.

Linear hardening coefficient $æ$ according to the diagram $e \sim \varepsilon$:

$$æ = \frac{\Delta e}{\Delta \varepsilon} = \frac{\Delta \sigma}{2G \Delta \varepsilon}\,, \qquad (21)$$

or taking into account the relations (19) and (20)

$$æ = \frac{2(1+\nu) æ_{zz}}{3 - (1 - 2\nu) æ_{zz}}\,, \qquad æ_{zz} = \frac{\Delta t_{zz}}{E \Delta d_{zz}}\,. \quad (22)$$

Knowing the hardening coefficient $æ$ in the diagram $e \sim \varepsilon$, we determine the hardening coefficient in the diagram $e \sim p$ $(p = \varepsilon - e)$ within sector $p_* \le p \le p_c$ (where p_* corresponds to the time when all the subelements exceeded the elastic limit $\psi' = 1$, p_c is a measure, starting from which the linear isotropic hardening is broken)

$$a + a_0 = \frac{\Delta e}{\Delta p} = \frac{\Delta e / \Delta \varepsilon}{1 - \Delta e / \Delta \varepsilon} = \frac{æ}{1 - æ}\,, \qquad (23)$$

Within the sector $p_* \le p \le p_c$ of the linear hardening of body's element expression (9) acquires the forme

$$\gamma = \dot{p}\,, \qquad (24)$$

or using the relation (21) we obtain

$$\gamma = \dot{\varepsilon} - \dot{e} = (1 - æ)\dot{\varepsilon}\,. \qquad (25)$$

Differentiating (19)-(20) with respect to time and inserting them into (25) we can express the parameter γ as the function of rate of the gripping device movement $\dot{d}_{zz}$

$$\gamma = \sqrt{\frac{3}{2}} \left(1 - \frac{\Delta t_{zz}}{E \Delta d_{zz}} \right) \dot{d}_{zz}\,, \qquad (26)$$

or introducing (22) the state parameter can be represented in this way

$$\gamma_2 = \sqrt{\frac{3}{2}} (1 - æ_{zz}) \dot{d}_{zz}\,. \qquad (27)$$

From (27) follows that within the sector of linear hardening the experiment under condition $\dot{d}_{zz} = const$ corresponds to loading at $\gamma = const$. This conclusion with a high degree of precision remains valid beyond the sector of linear hardening.

Indeed, for any $\varepsilon > \varepsilon_c$ in (25) can be accepted that $\dot{e} = æ(\varepsilon)\dot{\varepsilon}$, where $æ$ is expressed as a function of ε. Then

$$\gamma = \left[1 - æ(\varepsilon) \right] \dot{\varepsilon}\,. \qquad (28)$$

Further outside of linear hardening section the processes of softening in a material evolve [2], so $æ(\varepsilon)$ is a decreasing function. For most materials within the linear hardening section $æ \approx 10^{-2}$. Therefore, the quantity $æ(\varepsilon)$ compared with unity can be neglected and in this case we may take $\gamma = \dot{\varepsilon}$. Substituting (20) and (22) into (28)

$$\gamma = \sqrt{\frac{2}{3}} (1 + \nu) æ_{zz} \dot{d}_{zz}\,. \qquad (29)$$

Consequently, within and outside the linear hardening section the state parameter γ with precision of the second order is constant.

Within the sector $\varepsilon_{el} < \varepsilon < \varepsilon_*$ of the diagram $e \sim \varepsilon$ the parameter γ in tensile tests at a constant rate of the gripping device movement $\dot{d}_{zz} = const$ undergoes considerable changes. Let us examine how the parameter γ varies from point $\varepsilon = \varepsilon_{el}$, corresponding to the value γ_1 (17) at point $\varepsilon = \varepsilon_*$, corresponding to the value γ_2 (27).

For this purpose the state parameter γ will be expressed by the Poisson's ratio and linear hardening coefficient a, using the relations (18), (22) and (23). In this paper we restrict to examine the case when the coefficients of linear isotropic and kinematic hardening are equal $a = a_0$

$$\gamma_1 = \sqrt{\frac{2}{3}}\frac{1+\nu}{\sqrt{a+a^2}}\dot{d}_{zz}, \tag{30}$$

$$\gamma_2 = \sqrt{\frac{3}{2}}\frac{1+\nu}{1+\nu+3a}\dot{d}_{zz}. \tag{31}$$

Thus, in tensile tests at a constant rate of the gripping device movement $\dot{d}_{zz} = const$ within the sector $\varepsilon_{el} < \varepsilon < \varepsilon_*$ of the diagram $e \sim \varepsilon$ the parameter γ has decreased γ_1/γ_2 times (figure 2)

$$\frac{\gamma_1}{\gamma_2} = \frac{2}{3}\frac{1+\nu+3a}{\sqrt{a+a^2}}. \tag{32}$$

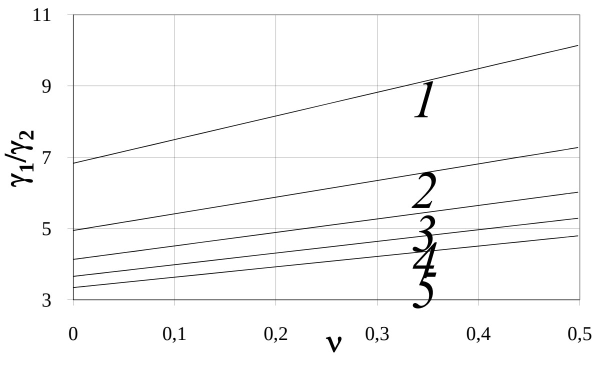

Figure 2. Parameter variation γ within the section $\varepsilon_{el} < \varepsilon < \varepsilon_*$ for the real values of the parameter *a*: ***1** – a=0,01; **2** – a=0,02; **3** – a=0,03; **4** – a=0,04; **5** – a=0,05*

According to the figure 2 the state parameter γ within the range $\varepsilon_{el} < \varepsilon < \varepsilon_*$ is decreased 4-9 times for values of the Poisson's ratio $\nu = 0{,}2 - 0{,}3$. Therefore, the variation cannot be neglected.

Let us study the process of determining of the function $e = e(p, \gamma, \upsilon)$ on the basis of simple tensile tests under different temperatures and different rates of the gripping device movement.

The characteristic values $e_{el} = \varepsilon_{el}$, e_*, ε_* are determined for the each curve $e = e(\varepsilon)$ obtained under conditions $\dot{d}_{zz} = const$ and $T = const$. As a consequence we have the following functions

$$e_{el} = e_{el}(\dot{d}_{zz}, T), \quad e_* = e_*(\dot{d}_{zz}, T), \quad \varepsilon_* = \varepsilon_*(\dot{d}_{zz}, T). \tag{33}$$

To pass from the rate of the gripping device movement $\dot{d}_{zz}$ to another argument – the state parameter γ, we must solve at constant temperature the equation $\gamma_1 = \gamma_2$, which according to (30) and (31) can be written as

$$\frac{\dot{d}_{11}^{(1)}}{\dot{d}_{11}^{(2)}} = \frac{3}{2}\frac{1+\nu_2}{1+\nu_1}\frac{\sqrt{a+a^2}}{1+\nu_2+3a}, \tag{34}$$

where ν depends on the loading conditions

$$\nu_1 = \nu\left(\dot{d}_{zz}^{(2)}\right), \quad \nu_1 = \nu\left(\dot{d}_{zz}^{(2)}\right). \tag{35}$$

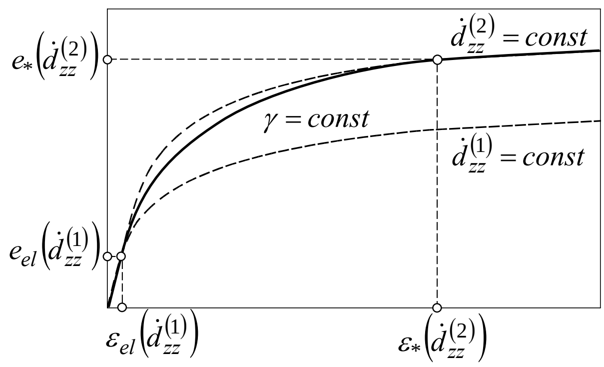

Figure 3. Determining of the characteristic values e_{el}, e_*, ε_* at constant magnitude of the state parameter γ.

The results of solving this equation give us the possibility to find out $e_{el}\left(\dot{d}_{zz}^{(1)}\right)$, $e_*\left(\dot{d}_{zz}^{(2)}\right)$, $\varepsilon_*\left(\dot{d}_{zz}^{(2)}\right)$ for the same value of the parameter γ ($\gamma_1 = \gamma_2$) and $T = const$ (Figure 3).

Taking into account that isothermal loading corresponds to deformation with $\upsilon = const$, functions (33) are written as

$$e_{el} = e_{el}(\gamma, \upsilon), \quad e_* = e_*(\gamma, \upsilon), \quad \varepsilon_* = \varepsilon_*(\gamma, \upsilon). \tag{36}$$

We will approximate the nonlinear section of curve $e \sim p$ within the limits $0 \le p \le p_*$ (other way $\varepsilon_{el} - e_{el} \le p \le \varepsilon_* - e_*$) using the following function

$$e(p,\gamma,\upsilon) = e_{el}(\gamma,\upsilon) + D(\gamma,\upsilon)p^{n(\gamma,\upsilon)}, \quad (37)$$

where e_{el}, D, n at fixed values of the state parameters γ and υ are constant size.

The coefficients of the approximation can be established by studying the conditions at the beginning of the linear hardening sector

$$e\Big|_{p=p_*} = e_{el}(\gamma,\upsilon) + D(\gamma,\upsilon)p_*^{\,n(\gamma,\upsilon)} = e_*(\gamma,\upsilon), \quad (38)$$

$$\frac{\partial e}{\partial p}\Big|_{p=p_*} = D(\gamma,\upsilon)n(\gamma,\upsilon)p_*^{\,n(\gamma,\upsilon)-1} = \frac{æ}{1-æ}. \quad (39)$$

Solving the system (38)-(39) we express the coefficients of approximation $n(\gamma,\upsilon)$ and $D(\gamma,\upsilon)$ in the characteristic values of diagram obtained on the basis of simple tensile tests

$$n(\gamma,\upsilon) = \frac{æ}{1-æ}\frac{\varepsilon_*(\gamma,\upsilon) - e_*(\gamma,\upsilon)}{e_*(\gamma,\upsilon) - e_{el}(\gamma,\upsilon)}, \quad (40)$$

$$D(\gamma,\upsilon) = \frac{e_*(\gamma,\upsilon) - e_{el}(\gamma,\upsilon)}{\left[\varepsilon_*(\gamma,\upsilon) - e_*(\gamma,\upsilon)\right]^{n(\gamma,\upsilon)}}. \quad (41)$$

The accuracy of setting of the characteristic curves $e = e(p,\gamma,\upsilon)$ will depend on the structure of the approximating function (37) and the accuracy of choice the characteristic points of the strain diagram e_{el}, e_*, ε_*.

CONCLUSIONS

In the study [8] it was demonstrated that in the case of the uniaxial tension is impossible to carry out experiment under a constant state parameter γ.

Therefore, elaboration of approximate method for determining the thermorheological properties of subelements on the basis of uniaxial tensile diagrams made for thin-walled tubes under different constant rates of the gripping device movement and temperature levels significantly expands the usability of published experimental data.

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