THE POISSON'S RATIO EFFECT ON THE STRESS DISTRIBUTION OVER THE THICKNESS OF A V-NOTCHED PLATE

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Abstract. *The presence of a stress concentrator in the geometry of a body has the effect of increasing the intensity of the stress state leading to a decrease in the strength of the material. The evaluation of the normal stress and stress concentration factor over the thickness of a V-notch plate plays an important role in life prediction and structure design. The objectives of this study are to highlight and determine the distribution of normal stress and stress concentration factor over the thickness off plate. The present work is based on a number of 25 simulations on rectangular V-notched plates subjected to tensile loading. The determination of the stress state and the stress concentration factor was carried out based on numerical methods with finite element method.*

Keywords*: Poisson's ratio, stress distribution, V notches plate, thickness, stress concentration factor, finite elements.*

Introduction

In general, any sudden change in the geometry of a body produces disturbances in the flow of the stress field, which manifests itself through increases of intensity of the stress state. Disturbances of the stress states are manifested by concentrations of the stress flow at the tip of the geometrical discontinuity, hereafter denoted as the concentrator. These geometrical changes can be of a structural nature such as material voids, defects, microstructural discontinuities, or they can be of a mechanical nature, introduced as a result of manufacturing, or they can be artificial, determined by some material production technologies, casting, or 3D printing. One on the first studies on the effect of stress concentration was carried out in 1898 by Kirsch [1], who conducted a series of tests on uniformly loaded plates with circular or elliptical holes of finite thickness.

Based on these studies, a theoretical concentration factor, Kt, was defined as the ratio of the maximum peak stress of the notch to the nominal applied stress.

A continuing challenge to scientific researchers in the development of analytical solutions of stress distributions in the presence of stress concentrators has occurred since the 19th century, [2]. One of the common approaches in the analysis of stress distribution at the top of a concentrator is based on numerical simulations with the finite element method. Based on theoretical solutions and three-dimensional numerical analyzes, Wei Guo and Wanlin Guo [2], presents a set of explicit equations that give the theoretical stress and specific strain concentration factors, as well as the stress concentration factor for finite thickness plates with elliptical holes.

An important role in the stress state distribution at the top of the concentrators is played by the mechanical behavior of the material, characterised by the longitudinal modulus of elasticity, transverse modulus of elasticity, yield strength, transverse contraction coefficient, Poisson's ratio, and ultimately tensile strength. Research has shown that the thickness of the piece and Poisson's ratio have a substantial contribution to the stress concentration factor. In the case of plates with a central hole and small thickness, the crack starts either from the edges of the hole or from the center of the plate, whereas in the case of plates with a large thickness, the crack starts almost always from the edges of the hole [3]. It was also found that the difference between the maximum value of the stress concentration coefficient (recorded in the mid-plane of the plate thickness) and the value of the stress concentration coefficient recorded on the extreme edges of the thickness, is a monotonically increasing function with plate thickness, and depends on the Poisson's coefficient, [3].

The present work aims to highlight and determine the stress distribution and the stress concentration factor on a rectangular plate with V-notches, subjected to tensile stress. For this study, the influence of the plate thickness and Poisson's ratio is considered.

Method and Materials

Both the determination of the stress state and the determination of the stress concentration factor in a V-notched plate, subjected to tensile stress, were performed on the basis of numerical models with the finite element method. The analysis was performed with the Ansys Workbench 2020 R2 software package, [6].

The geometrical elements of the plate are defined in Table 1 and a sketch of the plate is shown in Figure 1. In order to highlight the effect of plate thickness on the stress state, numerical analysis was performed for 5 plate thicknesses with notches as shown in Table 1.

Table 1

Geometrical Characteristics of the Rectangular Plate

Geometrical element	Notation	Value [M.U]
Width	w	30 [mm]
Length	b	200 [mm]
Distance between notches	h	20 [mm]
Angle between the notch edges	α	45 [°]
Notch radius	R	0.25 [mm]
Thickness	g	1, 3, 5, 7, 15 [mm]

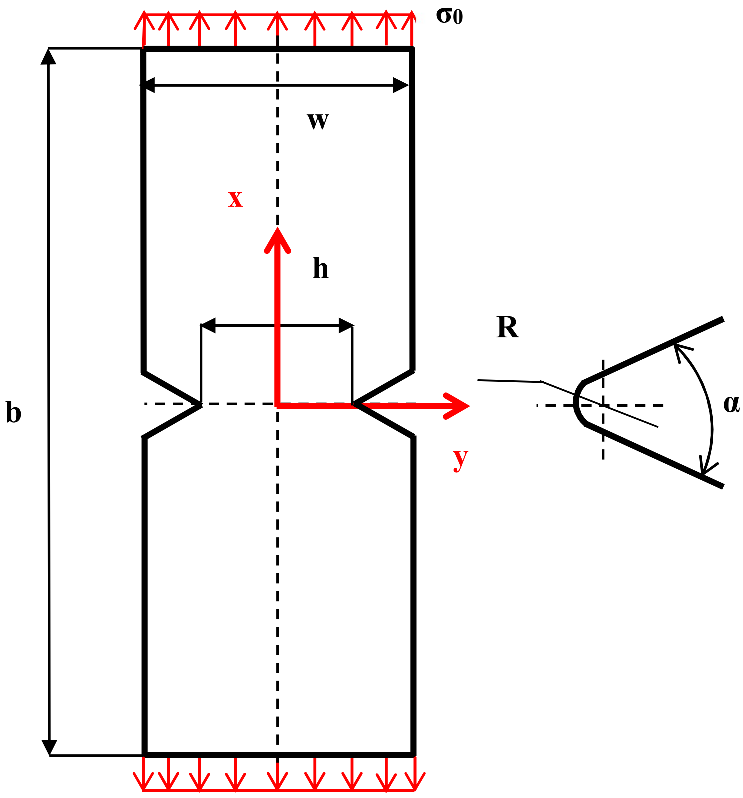


Figure 1. Geometrical characteristics of the rectangular plate with V notches

Also, the effect of the Poisson's ratio on the stress state was highlighted by considering in the numerical analysis materials with different values of Poisson's ratio, but falling under the assumption of linear elasticity, described by Hooke's law. A list of the materials considered in the numerical analysis is given in Table 2 [5].

The tip of the concentrator is discretized by Solid Body, Hex20 elements with a size of 0.1 mm. Figure 2 shows a discretization of the 15 mm thick plate concentrator. Following the analysis, it was found that the maximum stress is given by the vertical component of the normal stress, [4].

Table 2

Types of materials considered in the numerical analysis

Material	Modulus of elasticity, E [MPascal]	Poisson's ratio, ν
Cast iron, 120-90-02	164000	0.23
303 Stainless Steel	193000	0.25
Magnesium Alloy	45 000	0.28
Steel with low Carbon concentration	210 000	0.3
Aluminium Alloy, 6061-T6	69 000	0.35

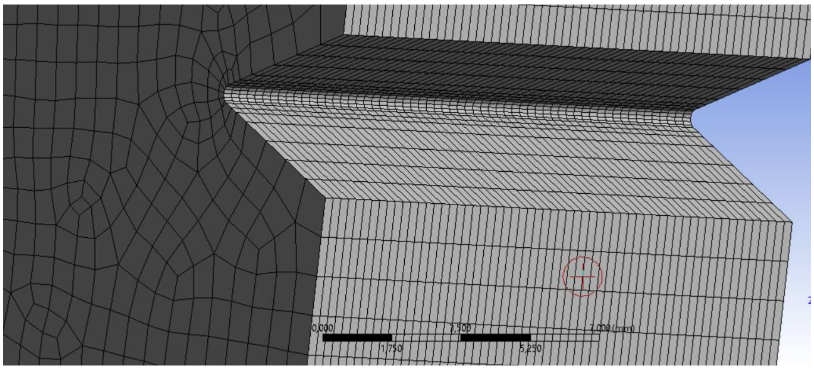


Figure 2. Discretization of the plate at the top of the concentrator

The theoretical stress concentration factor taking into account the maximum normal stress at the concentrator tip and the net normal stress:

$$K_t = \frac{\sigma_{y,max}}{\sigma_{net}} \quad (1)$$

where, $\sigma_{y,max}$ - the value of the maximum normal stress at the peak of the notch V, σ_{net}- the value of the net normal stress which represents the stress in the net section of the plate and is defined by the relation

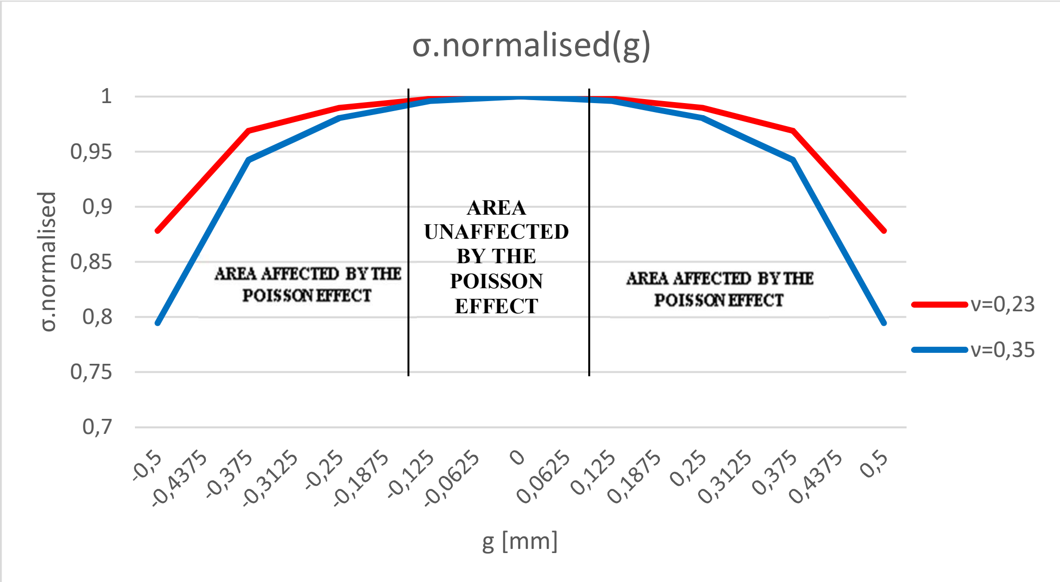
$$\sigma_{net} = \frac{F}{A_{net}} = \frac{F}{g*h} \quad (2)$$

where, F - applied force, A_{net} - net section area.

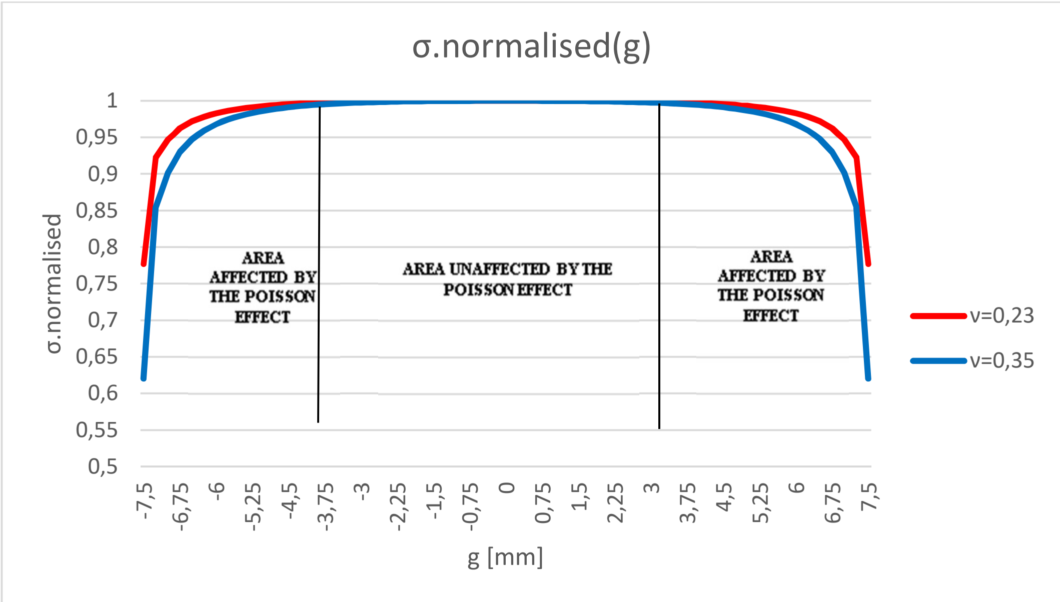
Discussion and Results

The results indicate a parabolic variation of the normal stress across the thickness of the plate, with minimum values on the extreme edges and a maximum value in the mid-plane of the thickness. The normalized stress distribution as a function of Poisson's ratio values and plate thickness is shown graphically in Figure 3 a, b.

On the extreme edges of the concentrator tip, a sudden increase of the normal stress can be observed on thicker plates as opposed to thinner plates, where the stress increases progressively. To highlight this aspect, in addition to the graphical part, the gradient of the normal stress variation per thickness was calculated for a plate with a thickness of 1 mm and 15 mm, respectively.



a)



b)

Figure 3. Normalized stress distribution for a plate thickness of a) 1 mm, b) 15 mm

For the two plates of different thicknesses, the increase in stress is followed by a smoothing over a section of the plate thickness. It can be stated that in this section, the stress is not dependent on the value of the Poisson's ratio. This effect is shown in Figures 4 a, b.

Based on the normal stress values determined in the concentrator area and the normal stress value on the net section, the theoretical stress concentration factor is determined. The distribution of the stress concentration factor per thickness is shown in Figure 5 a, b.

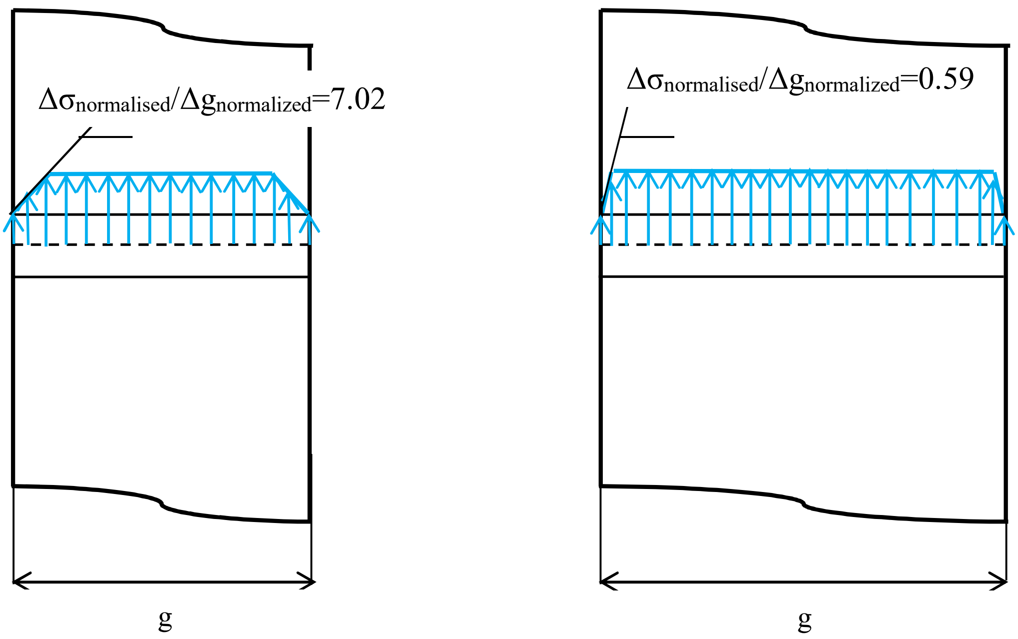
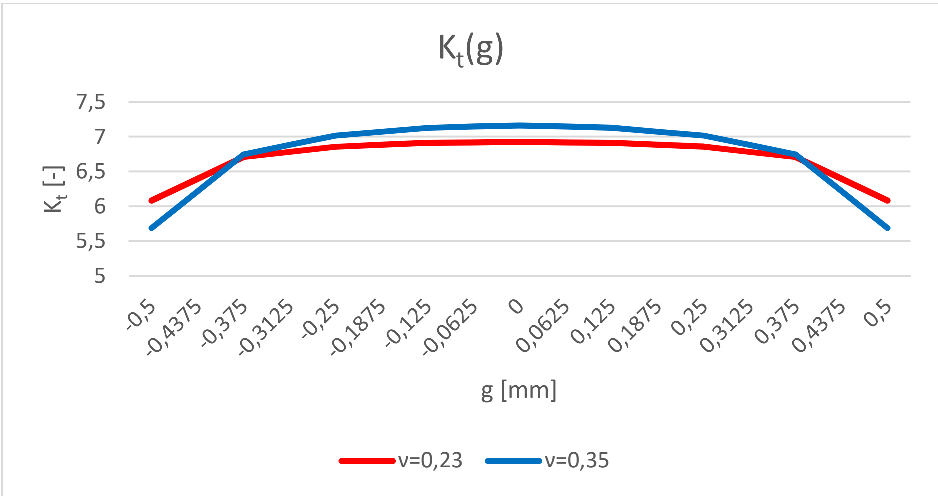


Figure 4. Stress distribution over plate thickness for a) 1 mm and b) 15 mm thick plate



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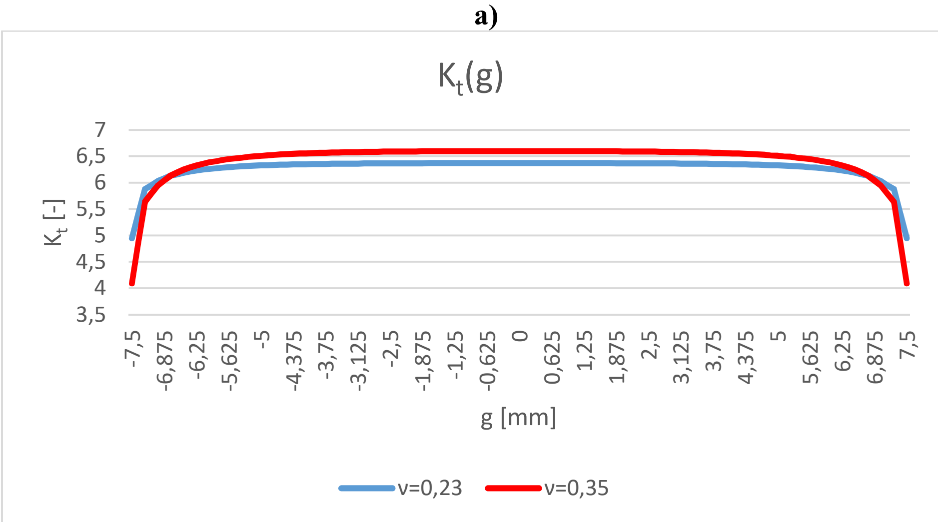

b)

Figure 5. Stress distribution over plate thickness for a) 1 mm and b) 15 mm thick plate

Conclusions

The current study is founded on a number of 25 simulations based on the influence of thickness and transverse contraction coefficient on the stress concentration factor on plates showing a concentrator. The stress distribution is also presented in this research. On the basis of the study, it can be concluded:

1. In the median plane, the normal stress is not dependent on the transverse contraction coefficient. The stress is uniform in this plane with an approximately constant value.
2. The use of materials with a high transverse contraction coefficient leads to a decrease in stress at the tip of the notch.
3. The stress concentration factor distribution is similar to the stress distribution over the thickness of the plate. In the mid-plane, the stress concentration factor reaches its maximum value, thus presenting higher values for a high transverse shrinkage coefficient due to the contraction of the plate thickness.

Acknowledgement

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