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CALCULATION OF RESISTANCE FORCE AT INCOMPRESSIBLE VISCOUS FLUIDS LAMINAR MOTION

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Abstract: *Starting from Newton's viscous friction law for unidirectional movement, this paper presents the calculation of the resistance force to the laminar motion of the viscous and incompressible fluids. Mentioned description can be applied to the demonstration of the Navier-Stokes equations.*

Keywords*: Resistance force, laminar motion, viscous fluids, incompressible fluids*

A problem of both theoretical and practical interest through applications in numerous technical fields and implications in scientific research is the calculation of the resistance force to the movement of viscous incompressible fluids. This calculation was initiated by the French mathematician Claude-Louis-Marie-Henri Navier in 1822, based on a reasoning based not on the action of molecular forces but on arbitrary assumptions [1]. Although he was the first to complete Euler's equations in ideal fluid dynamics with a term that takes into account the internal friction phenomenon, Navier did not recognize the physical significance of viscosity, attributing to his dynamic viscosity coefficient the properties of an intermolecular function. However, we should not be too exacting for Navier's mathematical work, since the inclusion of the friction phenomenon in Euler's equations was a difficult problem from the very beginning, because these equations describe the macroscopic flow velocity of the fluid in while energy dissipation occurs at microscopic level.

In the nearly two hundred years of research in the field of viscous fluid dynamics, various methods of calculating viscosity have been proposed. Most of them are based on tensor calculus and only in singular cases relations based on the principles of general physics have been obtained, using the first impulse theorem – the quantity of motion theorem [2]. The development of a simple method of calculating the strength of laminar motion resistance of incompressible fluids can therefore be regarded as one of the priority problems of modern hydrodynamics.

For the calculation of the strength of the laminar motion resistance of the incompressible viscous fluids, we apply Newton's viscous law to a fluid particle of the shape of an elementary parallelepiped of dimensions dx, dy, dz. For the beginning, examine the unidirectional motion along the *OX* axis (fig. 1). Considering tangential tension τ_{zx} linear with the length, the frictional force exerted between two neighbouring layers, spaced apart from each other, is

$$F_{\mu,zx} = \left(\tau_{zx} + \frac{\partial \tau_{zx}}{\partial z} dz\right) dx\, dy - \tau_{zx} dx\, dy = \frac{\partial \tau_{zx}}{\partial z} dxdydz. \quad (1)$$

According to Newton's viscous friction law, the tangential frictional tension between two neighboring layers of the unidirectional viscous fluid is directly proportional to the linear variation of the velocity in the transverse direction to the general direction of motion, meaning $\tau_{zx} = \mu \partial v_x / \partial z$. In the hypothesis of the constant of the coefficient of dynamic viscosity for the resistance force that is exerted in the *XOZ* plane it is obtained

$$F_{\mu,zx} = \frac{\partial}{\partial z}\left(\mu \frac{\partial v_x}{\partial z}\right) dxdydz = \mu \frac{\partial^2 v_x}{\partial z^2} dxdydz \quad (2)$$

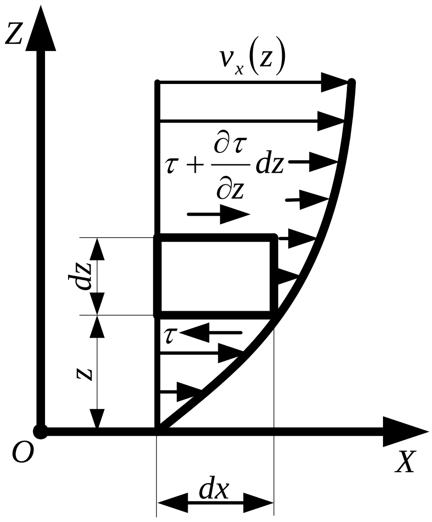

Fig. 1. Resistance strength calculation scheme.

Similar are the expressions of the resistance forces caused by the variation in the amount of motion in the other two planes:

$$F_{\mu,yx} = \mu \frac{\partial^2 v_x}{\partial y^2} dxdydz; \tag{3}$$

$$F_{\mu,xx} = \mu \frac{\partial^2 v_x}{\partial x^2} dxdydz. \tag{4}$$

The resistance force exerted on the *OX* direction is therefore

$$F_{\mu,x} = \mu \left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2} \right) dxdydz, \tag{5}$$

while a mass unit has a resistance force expressed by the mathematical relationship

$$f_{\mu,x} = \frac{F_{\mu,x}}{\rho dxdydz} = \upsilon \left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2} \right), \tag{6}$$

where υ is the kinematic viscosity coefficient.

Similar relationships can be written for the other two speed projections:

$$f_{\mu,y} = \upsilon \left(\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2} \right); \tag{7}$$

$$f_{\mu,z} = \upsilon \left(\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2} \right). \tag{8}$$

Consequently, the strength of resistance relative to a mass unit becomes

$$\vec{f}_{\mu} = f_{\mu,x}\vec{i} + f_{\mu,y}\vec{j} + f_{\mu,z}\vec{k} =$$
$$= \upsilon\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right)\vec{i} + \upsilon\left(\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}\right)\vec{j} + \qquad (9)$$
$$+\upsilon\left(\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right)\vec{k} = \upsilon\Delta v_x\vec{i} + \upsilon\Delta v_y\vec{j} + \upsilon\Delta v_z\vec{k} = \upsilon\Delta\vec{v},$$

where $\vec{i}, \vec{j}, \vec{k}$ are the versors of the coordinate axes, $\Delta\vec{v}$ – the Laplace operator in three dimensions, applied to the vector function $\vec{v}(v_x, v_y, v_z)$, and $\Delta v_x, \Delta v_y, \Delta v_z$ – Laplace scalar operators.

Obtained expression is found in the Navier-Stokes equation for the laminar movement of incompressible viscous fluids [3, 4]. Therefore, the proposed reasoning can be applied to the deduction of this equation.

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