The Spline analysis of parameters and pollutants dispersion in river surface water

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Abstract: This paper presents the cubic Spline analysis of the physico-chemical parameters of Suceava River. This consideration is the first step in the design of a mathematical model based on differential equation system describing the physico-chemical, biological and environmental phenomena on internal river, according to a minimal set of equations. We start from the data gathered and processed by water labs during different spring - summer – autumn – winter campaigns.

Keywords: Water quality, Spline function analysis, matematical model.

1 Introduction

The relationship between quality and quantity of surface water on the one hand, and people health on the other, represents a pressing issue nowadays, as water is one of the most important environmental factors that contributes to life quality and welfare.

The European Commission has adopted Directive 2000/60/EC (amended by Directive 2008/32/CE), which establishes the framework for Community action in the field of water policy. The general aim of this directive is to establish a framework for the protection and management of surface waters in the European Union, which stipulates the following: reaching ’good’ condition of all water forms in the natural environment of Europe, until 2015; preserving the ’good’ condition and ensuring ’very good’ condition for all water bodies, where there has been already achieved a ’good’ ecological potential for heavily modified and artificial water bodies; compliance with the environmental objectives set by the other Directives in water protected areas [1-2].

The main objectives of the Directive are the following: damage prevention, protection and improvement of aquatic ecosystem condition, taking into account their water requirements, permanent interactions among aquatic ecosystems and terrestrial adjacent ecosystems and wetlands; promoting sustainable water use based on the long-term protection of water resources; enhance protection and improve the condition of aquatic environment by specific measures aimed at progressive emission reduction and loss of hazardous substances in water; groundwater pollution prevention and its progressive reduction; decreasing the adverse effects of dangerous hydro-meteorological phenomena - floods and droughts.

These objectives will be achieved by means of proper management of the aquatic environment of the river basins and the proper combination of limit values and environmental quality standards to control discharges into water bodies [3].

The objective of this study is to assess the quality of Suceava River water, in order to emphasise the effects induced by the discharges of wastewater with various degrees of purification, by defining and tracking the changes of storage tanks characteristics (water surface) in time and space, based on the interaction between surface waters and groundwater.

The dispersion mathematical model of Suceava River aims at providing characterization, analysis and environmental modelling on the basis of physico-chemical indicators of the river current state.

The necessity of modelling resides in the fact that the previous hydrological studies of Suceava River are relatively inconsistent, and they did not succeed in assessing the river water state and quality.

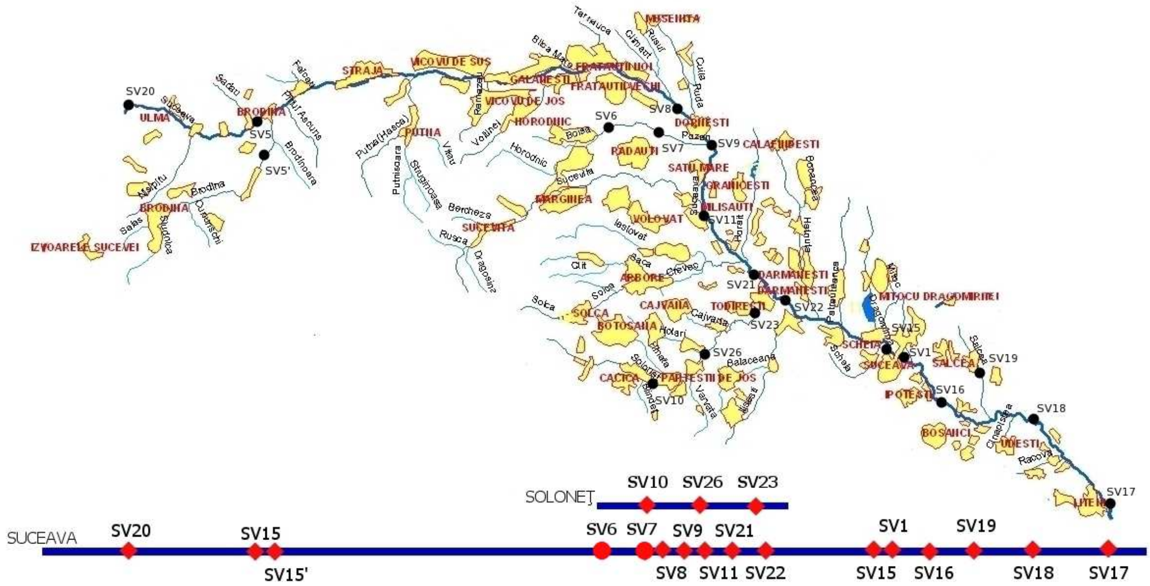

Fig. 1. Sampling sections on Suceava River.

2 Analysis of physico-chemical indicators variation by SPLINE functions interpolation

2.1 Theoretical considerations

Spline functions are piecewise polynomial functions that tie together with a certain number of their derivates forming nodes [4], [10]. In order to approximate $f:[a,b]\rightarrow R$ by a function *S*, we use the following method: the interval $[a,b]$ is divided in *n* subintervals, which correspond to the following points:

$$a = x_0 < x_1 < \ldots x_k < x_{k+1} < \ldots x_n = b \qquad (1)$$

It is necessary for this function to be determined by 2 polynomials $(S_k)_{k=\overline{0,n-1}}$, $S_k:[x_k, x_{k+1}]\rightarrow R$, one of them on each interval in such a way that at the endpoints of relation (1) the *S* function be differentiable for several times. We consider any 2 sections in *S* function where $y = S_k(x), x \in (x_k, x_{k+1})$ and $y = S_{k+1}(x), x \in (x_{k+1}, x_{k+2})$ are equal at the coordination point (x_{k+1}, y_{k+1}).

The result is that the set of functions $(S_k)_{k=\overline{0,n-1}}$ forms a piecewise polynomial curve, which is denoted by $S = (S_k)_{k=\overline{0,n-1}}$. We can notice that instead of approximating the *f* function by a single polynomial on the whole interval $[a,b]$, we approximate this function by *n* polynomials.

Thus, we can obtain an approximated function *S* which can be used in solving different approximation and interpolation problems. The *S* function that we thus obtain is named Spline function. The name Spline comes from the thin rods called 'spline', used by specialists to transform the rectilinear movement in rotation movement. This instrument helps to fit curves through given points. From a practical point of view, the *cubic Spline functions* are the most important ones. They are functions with smooth, continuous curves. When they are used for interpolation, these functions do not have oscillatory behaviour, which is characteristic to high degree polynomials. The cubic Spline functions are easy to calculate and use.

A function $S \in C^2[a,b]$ is called cubic Spline function for a set of points $a = x_0 < x_1 < \ldots x_k < x_{k+1} < \ldots x_n = b$ corresponding to *f*, if there exist *n* cubic polynomials, $S_k(x), k = \overline{0,n-1}$, with the properties:

$$S(x) = S(x_k) = s_{k0} + s_{k1}(x - x_k) + s_{k2}(x - x_k)^2 + s_{k3}(x - x_k)^3,$$
$$x \in [x_k, x_{k+1}], \quad k = \overline{0,n-1}$$

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(2)

- $S(x_k) = y_k = f(x_k), \quad k = \overline{0,n}$ (the cubic Spline function passes through all data points);
- $S_k(x_{k+1}) = S_{k+1}(x_{k+1}), \quad k = \overline{0,n-2}$ (the cubic Spline function is a continuous function);
- $S'_k(x_{k+1}) = S'_{k+1}(x_{k+1}), \quad k = \overline{0,n-2}$ (the cubic Spline function is a smooth function);
- $S''_k(x_{k+1}) = S''_{k+1}(x_{k+1}), \quad k = \overline{0,n-2}$ (the second derivative is continuous).

Provided that the cubic Spline function satisfies the condition $S''(x_0) = S''(x_n) = 0$, then it is called natural Spline function. This condition is based on the solution of the following problem: from all functions $g \in C^2[a,b]$, which satisfy the condition $g(x_k) = y_k, k = \overline{0,n}$, we choose function g that minimises the integral $\int_a^b (g''(x))^2 dx$.

The solution to this problem resides in the natural cubic Spline function and has to display a minimal oscillatory behaviour since $g''(x)$ is small. The condition $S''(x_0) = S''(x_n) = 0$ has the interpretation that $S(f)$ is linear on $(-\infty, x_0)$ and $[x_n, \infty)$.

2.2 Calculus method of the coefficients

The calculus method of the coefficients s_{kj}, $k = \overline{0, n-1}, j = \overline{1, 3}$ for the construction of natural cubic Spline function is:

$$s_{k0} = y_k, k = \overline{0,n-1} \tag{3}$$

$$s_{n2} = s_{n0} = 0 \tag{4}$$

$$s_{k2} = \frac{1}{\lambda_k}\left(b_{k0} - (x_{k+1} - x_k)s_{k+1,2}\right), k = \overline{n-1,1} \tag{5}$$

$$d_{k0} = \frac{3}{h_k h_{k-1}}\left(h_k s_{k-1,0} - (h_{k-1} + h_k)s_{k0} + h_{k-1}s_{k+1,0}\right), k = \overline{0,n-1} \tag{6}$$

$$h_k = x_{k+1} - x_k \tag{7}$$

where:

$$\lambda_1 = 2(x_2 - x_0) \tag{8}$$

$$\lambda_{k+1} = 2(x_{k+2} - x_k) - \frac{(x_{k+1} - x_k)^2}{\lambda_k} \tag{9}$$

$$b_{10} = d_{10} \tag{10}$$

$$b_{k+1,0} = d_{k+1,0} - \frac{(x_{k+1} - x_k)}{\lambda_k} b_{k0}, k = \overline{1,n-2} \tag{11}$$

$$s_{k1} = \frac{y_{k+1} - y_k}{x_{k+1} - x_k} - \frac{x_{k+1} - x_k}{3}\left(2s_{k2} + s_{k+1,2}\right), k = \overline{0,n-1} \tag{12}$$

$$s_{k3} = \frac{s_{k+1,2} - s_{k2}}{3(x_{k+1} - x_k)} \overline{0,n-1} \tag{13}$$

2.3 Calculus models of the coefficients

Calculus models of the coefficients s_{kj} for the following set of points are shown in the table below:

s_{kj}	s_{k0}	s_{k1}	s_{k2}	s_{k3}
s_{0j}	59.85	102.013	0	-6.69016
…	210.35	-23.427	-50.1762	14.2448
	76.75	-53.1937	35.2928	15.3893
s_{n-1j}	60.9	-6.35893	58.3768	-38.9179

$$S(x) = S(x_k) = s_{k0} + s_{k1}(x - x_k) + s_{k2}(x - x_k)^2 + s_{k3}(x - x_k)^3,$$

$$x \in [x_k, x_{k+1}], \quad k = \overline{0,n-1} \tag{14}$$

Using MATHEMATICA, we calculate the s_{kj} coefficients according to relations (3)-(13), we replace them in relation (2) thus obtaining function (14), which approximates the concentration evolution in the monitored sections.

3 Results and discussions

Suceava River is part of Siret River basin. It springs in the north of the country, in Bukovina, and flows into the Siret River at 170 km from its initiation point.

The cubic Spline analysis of the water parameters evolution is realized between SV14 (0 km) and SV2+1000 (5,5 km) sections on Suceava River (see sections in Fig. 1) for the campaigns in 2011.

The sampling aimed to perform analytical and hydro-biological investigations was conducted in accordance with current national and international standards and methodologies [5-8].

The results obtained by using Spline functions for the evolution modelling of physico-chemical indicators gathered from the Suceava River are presented in Fig. 2-4.

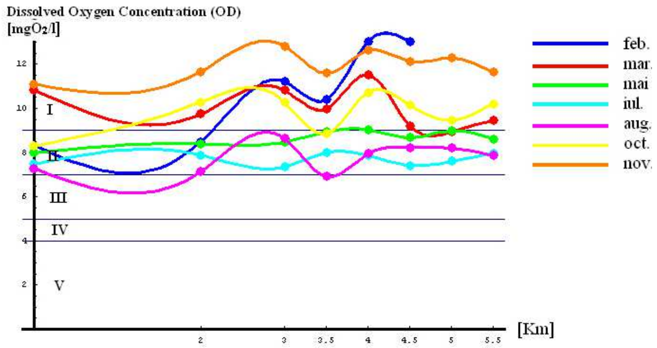

Fig. 2. Evolution of dissolved oxygen concentration (OD).

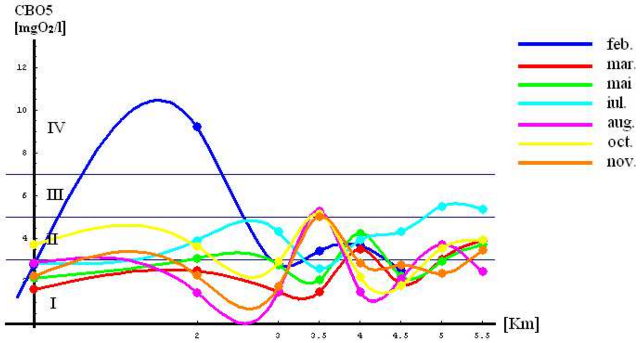

Fig. 3. Evolution of CBO_5 5-day biochemical oxygen demand concentration.

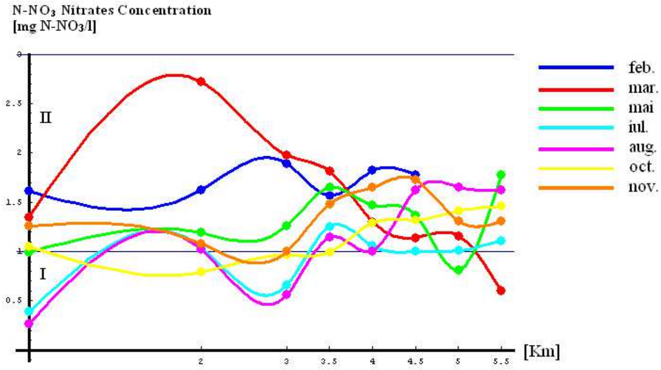

Fig. 4. Evolution of N-NO_3 nitrates concentration.

Figures 2 and 3 show, in terms of organic loading quality (measured as COD, CBO_5), the existence of an ecological 'very good' and 'good' state in most of the analyzed aquatic sections, except for the values recorded in SV14 section in August.

4 Conclusions

Values of determined parameters were compared with the limits established by current regulations, according to MMGA Order No 161/2006 [9] concerning the approval of the Norm regarding reference objectives for preservation of surface water quality.

The analysis of all physico-chemical parameters validate a 'very good' and 'good' state in most sections of Suceva River for all 2011 campaigns.

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