Contributions of Controllers Tuning in the Multiple-Loop Feedback Control System with Two Contours with Inertia

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***Abstract*—A tuning algorithm of linear controllers *P, PI, PID* in multiple-loop feedback control systems is proposed in this paper. The control objects consist of two subprocesses, which are described by dynamical models with internal (second order). The controllers in the internal contour and in the external contour are tuning according the maximal stability degree method. In the internal contour is used controllers P and *PI,* in the external contour is used controllers *P, PI, PID.* There are using the iterative procedure, for determinate the optimal parameters of controllers *P, PI, PID.* The tuning algorithm of controllers represents an algebraic method, which consists of two stages. On first stage the numerical value of the optimum stability degree of designed control system is determined. On second stage the numerical value of tuning dynamic parameters of controllers are determined from algebraic expressions. The obtained results are compared with the results which are obtained using the other known methods.**

***Index Terms*—maximal stability degree method, multiple-loop feedback control system, tuning of controllers.**

I. INTRODUCTION

At the projecting of multiple-loop control systems are used many tuning methods of typical controllers: frequency method, criteria (of modulus) method etc. Frequency method is accompanied with difficulties of calculating [1,2,3]. Criteria (of modulus) method becomes unacceptable when the control processes are slowly, and they have big time constants and this reduce the performances of entire system [1,2,3]. To bypass these above-cited inconveniences in the paper is proposed to use of the maximal stability degree method for tuning of typical controllers *P, PI, PID* for a class of control objects' models with inertia, which are connected in cascade, represented by two subprocesses and, as result with two regulating loops.

II. THE TUNING ALGORITHM OF CONTROLLERS

The multiple-loop feedback control system is represented by two contours: internal contour with controller's transfer function $H_{R2}(s)$ and subprocess $H_{F2}(s)$, and external contour with controller's transfer function $H_{R1}(s)$ and subprocess $H_{F1}(s)$ (fig. 1). The tuning of controllers is recommended to realize first in the internal contour then in the external contour.

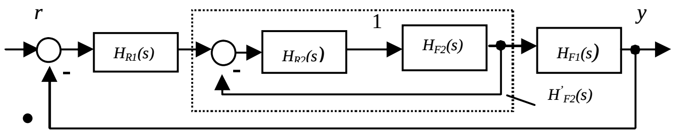

Figure 1. The multiple-loop feedback control system.

The control object consists of two inertial subprocesses with the transfer functions:

$$H_{F1}(s) = \frac{k_1}{(T_1 s+1)(T_2 s+1)} = \frac{k_1}{g_0 s^2 + g_1 s + g_2}, \qquad (1)$$

$$H_{F2}(s) = \frac{k_2}{(T_3 s+1)(T_4 s+1)} = \frac{k_2}{l_0 s^2 + l_1 s + l_2}, \qquad (2)$$

with $T_3 < T_1, T_2$, $T_4 < T_1, T_2$.

In expressions (1) and (2) we have the notations: k_1, k_2 are transfer coefficients of subprocesses; T_1, T_2, T_3, T_4 are time constants of respective subprocesses and $g_0 = T_1 T_2$;

$g_1 = T_1 + T_2; g_2 = 1; l_0 = T_3 T_4;\ l_1 = T_3 + T_4; l_2 = 1.$

III. THE TUNING CONTROLLERS IN THE INTERNAL CONTOUR

Is implementing the tuning of controller with transfer function (t.f.) $H_{R2}(s)$ from internal contour to the subprocess with t.f. $H_{F2}(s)$. Suppose that *P* and *PI* controllers are used.

***P* controller** is tuning to the object with transfer function (2), applied the maximal stability degree method (M.S.D.) and tuning parameters of controller are determined from relations [4,5,6]

$$k_{p2} = \frac{1}{k_2}(-l_0 J^2 + l_1 J - l_2). \qquad (3)$$

In the relation (3) J is the maximal stability degree and which is chose from the following condition $J>0$. The maximal stability degree J is choosing from the consideration as the duration of the transition process will has the sated values.

The t.f. of internal contour with *P* controller:

$$H'_{F2}(s)=\frac{H_{R2}(s)H_{F2}(s)}{1+H_{R2}(s)H_{F2}(s)}=\frac{n_0}{h_0s^2+h_1s+h_2}, \tag{4}$$

where $n_0=k_{p2}k_2;\ h_0=T_3T_4;\ h_1=T_3+T_4;\ h_2=1+k_{p2}k_2.$

***PI* controller** is tuning to the object with the transfer function (2), applied the M.S.D. method and tuning parameters of controller are determined from relations [4,5,6]

$$k_{p_2}=\frac{1}{k_2}(-3l_0J^2+2l_1J-l_2), \tag{5}$$

$$k_{i2}=\frac{J^2}{k_2}(-2l_0J+l_1)\cdot$$

We can obtain the values of parameters $k_{p2},\ k_{i2}$, changing the *J>0* value, for that the performances of control system are sated.

The t.f. of internal contour with *PI* controller is

$$H'_{F2}(s)=\frac{H_{R2}(s)H_{F2}(s)}{1+H_{R2}(s)H_{F2}(s)}=\frac{n_0s+n_1}{h_0s^3+h_1s^2+h_2s+h_3}, \tag{7}$$

where $n_0=k_2k_{p2};n_1=k_2k_{i2};h_0=T_3T_4;h_1=T_3+T_4;$ $h_2=1+k_{p2}k_2;h_3=k_{i2}k_2.$

IV. THE TUNING CONTROLLERS IN THE EXTERNAL CONTOUR

The structure block scheme of *external contour* is represented in the figure 2 *a, b*.

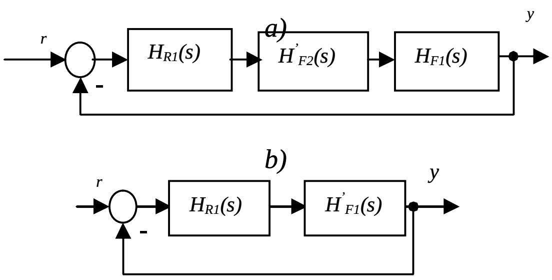

Figure 2. Structure block schema of external contour

For tuning of *P, PI, PID* controllers in the external contour it is necessary to determine the equivalent transfer function of object (4) with *P* controller in the internal contour with the t.f. of subprocess (1)

$$H'_{F1}(s)=H'_{F2}(s)H_{F1}(s)=\frac{k}{a_0s^4+a_1s^3+a_2s^2+a_3s+a_4}, \tag{8}$$

where $k=k_{p2}k_2k_1;\ a_0=T_1T_2T_3T_4;$

$a_1=T_3T_4(T_1+T_2)+(T_3+T_4)T_1T_2;$

$a_2=T_3T_4+(T_3+T_4)(T_1+T_2)+(1+k_{p2}k_2)T_1T_2;$

$a_3=T_3+T_4+(1+k_{p2}k_2)(T_1+T_2);\ a_4=1+k_{p2}k_2.$

For object with t.f. (8) *P, PI, PID* controllers can be tune applied the M.S.D. method using the relation from [4,5,6]:

Control system with P controller:

$$k_{p1}=\frac{1}{k}(-a_0J^4+a_1J^3-a_2J^2+a_3J-a_4). \tag{9}$$

Control system with PI controller:

$$k_{p1}=\frac{1}{k}(-5a_0J^4+4a_1J^3-3a_2J^2+2a_3J-a_4), \tag{10}$$

$$k_{i1}=\frac{J^2}{k}(-4a_0J^3+3a_1J^2-2a_2J+a_3). \tag{11}$$

Control system with PID controller:

$$10a_0J^2-4a_1J+a_2=0\ , \tag{12}$$

$$k_{p1}=\frac{1}{k}(15a_0J^4-8a_1J^3+3a_2J^2-a_4), \tag{13}$$

$$k_{i1}=\frac{J^3}{k}(6a_0J^2-3a_1J+a_2),$$

$$k_{d1}=\frac{1}{k}(10a_0J^3-6a_1J^2+3a_2J-a_3). \tag{15}$$

(6)

Values of k_{p1},k_{i1},k_{d1} parameters are obtained, made the variation of *J* value, for that the performances of control system are sated.

For tuning of *P, PI, PID* controllers in the external contour it is necessary to determine the equivalent t.f of object (7) with *P* controller in the internal contour with the t.f. of subprocess (1)

$$H'_{F1}=H'_{F2}(s)H_{F1}(s)= =\frac{b_0s+b_1}{a_0s^5+a_1s^4+a_2s^3+a_3s^2+a_4s+a_5}, \tag{16}$$

where $b_0=k_1k_2k_{p2};\ b_1=k_1k_2k_{i2};\ a_0=T_1T_2T_3T_4;$

$a_1=T_3T_4(T_1+T_2)+(T_3+T_4)T_1T_2;$

$a_2=T_3T_4+(T_3+T_4)(T_1+T_2)+(1+k_{p2}k_2)T_1T_2;$

$a_3=T_3+T_4+(1+k_{p2}k_2)(T_1+T_2)+k_2k_{i2}T_1T_2;$

$a_4=1+k_2k_{p2}+k_2k_{i2}(T_1+T_2);\ a_5=k_{i2}k_2.$

For object with t.f. (8) *P, PI, PID* controllers can be tune applied the M.S.D. method using the relations from [4,5,6]:

Control system with P controller:

$$k_{p1}=\frac{a_0J^5-a_1J^4+a_2J^3-a_3J^2+a_4J-a_5}{b_1-b_0J}. \tag{17}$$

Control system with PI controller:

$$k_{p1}=\frac{-d_0J^6+d_1J^5-d_2J^4+d_3J^3-d_4J^2+d_5J-d_6}{(b_1-b_0J)^2}, \tag{18}$$

where $d_0=5a_0b_0,\ d_1=6a_0b_1+4a_1b_0,$

$d_2=5a_1b_1+3a_2b_0, d_3=4a_2b_1+2a_3b_0,$

$d_4=3a_3b_1+a_4b_0, d_5=2a_4b_1,\ d_6=a_5b_1.$

$$k_{i1}=\frac{-a_0J^6+a_1J^5-a_2J^4+a_3J^3-a_4J^2+a_5J}{b_1-b_0J}+k_pJ. \tag{19}$$

Control system with PID controller:

$$k_d=\frac{d_0J^7-d_1J^6+d_2J^5-d_3J^4+d_4J^3-d_5J^2+d_6J-d_7}{2(b_1-b_0J)^4}, \tag{20}$$

where $d_0 = 5a_0b_0^3; d_1 = 68a_0b_0^2b_1 + 12a_1b_0^3;$

$$d_2 = 78a_0b_0b_1^2 + 42a_1b_0^2b_1 + 6a_2b_0^3;$$
$$d_3 = 30a_0b_1^3 + 50a_1b_0b_1^2 + 22a_2b_0^2b_1 + 2a_3b_0^3;$$
$$d_4 = 20a_1b_1^3 + 28a_2b_0b_1^2 + 8a_3b_0^2b_1;$$
$$d_5 = 12a_2b_1^3 + 12a_3b_0b_1^2;$$
$$d_6 = 6a_3b_1^3 + 2a_4b_0b_1^2 - 2a_5b_0^2b_1;$$
$$d_7 = 2a_4b_1^3 - 2a_5b_0b_1^2;$$

$$k_{p1} = \frac{-d_0J^6 + d_1J^5 - d_2J^4 + d_3J^3 - d_4J^2 + d_5J - d_6}{(b_0 - b_1J)^2} + 2k_{d1}J, \quad (21)$$

where $d_0 = 5a_0b_0; d_1 = 6a_0b_1 + 4a_1b_0;$

$$d_2 = 5a_1b_1 + 3a_2b_0;$$
$$d_3 = 4a_2b_1 + 2a_3b_0;$$
$$d_4 = 3a_3b_1 + a_4b_0; d_5 = 2a_4b_1; d_6 = a_5b_1;$$

$$k_{i1} = \frac{-a_0J^6 + a_1J^5 - a_2J^4 + a_3J^3 - a_4J^2 + a_5J}{b_1 - b_0J} - k_{d1}J^2 + k_{p1}J. \quad (22)$$

Values of k_{p1}, k_{i1}, k_{d1} parameters are obtained, varied the J value, for that the performances of control system are sated. The procedure of determine of the optimal coefficients k_p, k_i, k_d from expressions (9)-(11), (13)-(15), (17)-(22) for that the control system will have the sated performance, is the difficult procedure.

It is proposed the following procedure with iterations for determine the optimal values of parameters k_p, k_i, k_d from relations (9)-(11), (13)-(15), (17)-(22) which are represented the dependences of the maximal stability degree *J*.

The variable *J* is changing and it is made the curves $k_p = f(J)$, $k_i = f(J)$, $k_d = f(J)$ for the respectively controller and object. After, the sets of values of the k_p, k_i, k_d parameters get for the optimal and quasioptimal value of *J*. For each set of values of the k_p, k_i, k_d parameters it is making the simulation and it is determined the transition process for that the obtained performance corresponding the sated performance.

In some case the tuning of the *PID* controller in the external contour to the model of object (16) become so difficult. In this case when the *PID* controller in the external contour can't be tuned it is proposed to use the following procedure. For the equivalent object (16) from external contour obtained the transition process and using the identification procedure it is determine the equivalent object's model. For this equivalent model it is use the maximal stability degree method to tune *PID* controller. With this calculations the tuning procedure of controllers in the multiple-loop feedback control system with inertia (fourth order) is finished.

V. APLICATIONS AND COMPUTER SIMULATION

To show the efficiency of the proposed algorithm for tuning of the typical controllers in the multiple-loop feedback control system with inertia (fourth order) using the presented relations was examine an example with the object models which has the following parameters: $H_{F2}(s)$- $k_2=0.2$, $T_3=5$, $T_4=2$ and $H_{F1}(s)$ – $k_1=0.5$, $T_1=10$, $T_2=20$.

The *P, PI* controllers were tuning in internal contour using the maximal stability degree method, which permitted to obtain the high performance, varied values of *J* and choosing the k_{p2} and k_{p2}, k_{i2} values for respectively controllers.

The *P, PI* and *PID* controllers were tuning in external contour using the maximal stability degree method, which permitted to obtain the high performance, varied values of *J* and choosing the respectively values of the *P, PI, PID* controllers.

The computer simulation have been made in MATLAB and the simulation diagram of multiple-loop feedback control system is presented in figure 3.

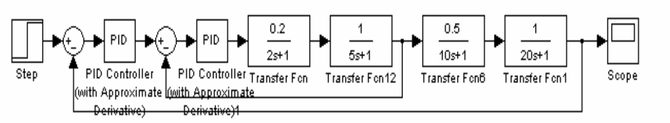

Figure 3. Simulation diagrams of the control system.

In some case using the maximal stability degree method doesn't give the positive results tuning the *PID* controller in the external contour and *PI* controller in the internal contour (equivalent model of the external contour is presented in the relation (16)). In this case it is obtained the transition process of equivalent object (16) with *PI* controller in the internal contour (for this case the transition process is presented in the fig.4). For this transition process it is made identification of equivalent object's models with inertia (first order) and time delay with parameters *k=0,5, T= 46,66 s, τ =10 s.* It was tuning the *PID* controller to this object, using the maximal stability degree method and Ziegler-Nichols method (curves 4, 5 fig. 5 *b*).

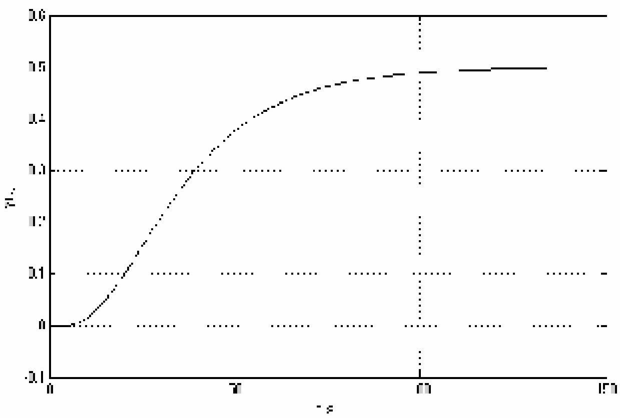

Figure 4. The transition process of equivalent object (16).

In the fig. 5 are presented the transition processes of the multiple-loop feedback control system for external contour:

a) – transition process in the external contour with *P* controller in the internal contour ($k_{p2}=1.125$): external contour with *P* controller ($k_{p1}=2.9$) - curve *1*; with PI controller with optimal parameters ($J_{opt}=0.04$, $k_{p1}=6.75$, $k_{i1}=0.283$ or $T_i=3.533$ s) – curve *2*; with *PID* controller with optimal parameters ($J_{opt}=0{,}12$, $k_{p1}=36.991$, $k_{i1}= 1.433$ or $T_i=0.697$ s, $k_d=$ 226.71) – curve *3*, but curves 4, 5 for suboptimal values of the J=0.05, J=0.06 respectively; b) – transition process in the external contour with PI controller in the internal contour (Jopt=0.23, kp2=3.165, ki2= 0.6348 or Ti2=1.575 s): external contour with P controller (kp1=0.141) - curve 1; with PI controller with optimal parameters (Jopt=0.05, kp1=0.68, ki1= 0.034 or Ti=29.411 s) - curve 2; with PI controller tuning with Ziegler-Nichols method with critical parameters kcr=12.5, Tcr=47 s (kp1=5.625, ki1= 0.026 or Ti=37.6 s) - curve 3; with PID controller tuning with maximal stability degree method to the equivalent object (16) (Jopt=0.04, kp1=1.4515, ki1= 0.0496 or Ti1=20 s, kd1= 9.73)) - curve 4; with PID controller tuning with maximal stability degree method to the object with transition process from fig. 4 (Jopt=0.1362, kp1=6.999, ki1= 0.0256 or Ti1=39 s, kd1= 17.393) - curve 5; with PID controller tuning with Ziegler-Nichols method with critical parameters kcr=12.5, Tcr=47 s (kp1=9.375, ki1= 0.035 or Ti1=28.2 s, kd1= 4.7) - curve 6.

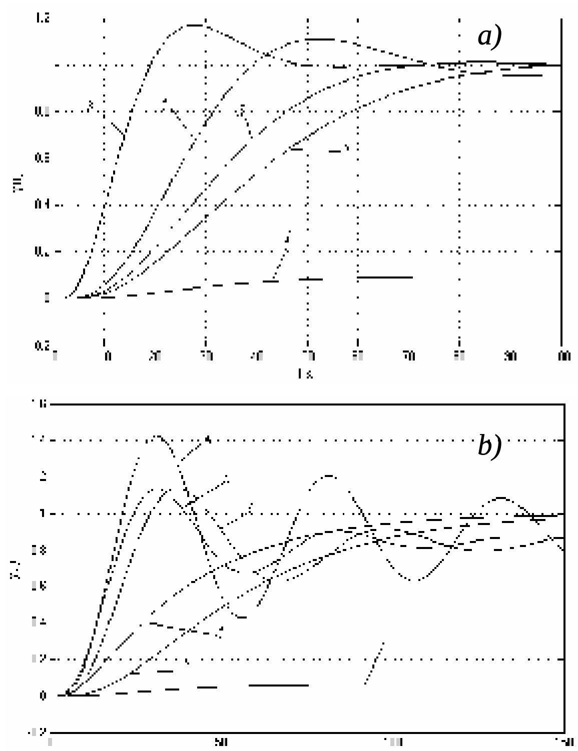

Figure 5. The transition process of the multiple-loop feedback control system.

Analysing the obtained results we can mention from fig. 5 *a*, *b* the following: tuning the *PI* and *PID* controller to the model's object (8), using the maximal stability degree method the controllers' parameters get the optimal values; tuning the *PI* controller to the model's object (16) using the maximal stability degree method the controllers' parameters get the optimal values, but tuning the *PID* controller to the model's object (16) using the maximal stability degree method it was made in two case. In the first case it was made the calculations in conformity with proposed procedure, but in the second case it was using the transition process of equivalent model (16) presented in the fig. 4 and getting the approximation model as model with inertia (first order) and time delay, it was tuning the *PID* controller using the maximal stability degree method and Ziegler-Nichols method in the multiple-loop feedback control system. Control system with *PID* controller tuning with the maximal stability degree method has the higher results than control system with *PID* controller tuning with Ziegler-Nichols method.

VI. CONCLUSIONS

As a results of obtained results after tuning the *P, PI, PID* controller to the multiple-loop feedback control system with object's models (1), (2) with known parameters, the following conclusion can be made:

The tuning of *P, PI* controller in the internal contour in conformity with the maximal stability degree method permitted to obtain the high results varying the value of the *J>0* and choosing the parameters of the respectively controllers for obtain the sated performance of the internal contour. For the *PI* controller it is obtained the optimal values.

The tuning of *P, PI, PID* controllers in the external contour using the maximal stability degree method permitted to obtain the high results varying the value of the *J>0* and obtained the optimal value and the suboptimal values of the *J*, and choosing the set of the values of controllers' parameters for obtain the sated performances.

REFERENCES

[1] V. Ia. Rotach. Teoria avtomaticheskogo upravlenia termoenergheticheskimi protzessami. – Moskva: Energoatomizdat, 1985.-292 s.

[2] V. A. Lukas Teoria avtomaticheskogo upravlenia. – Moskva: Nedra, 1990.- 416 s.

[3] I. Dumitrache şi al. Automatizări electronice. - Bucureşti: EDP, 1993.- 660 p.

[4] B. Izvoreanu, I. Fiodorov, F. Izvoreanu. The Tuning of Regulator for Advance Delay Objects According to the Maximal Stability Degree Method / In: Proceedings of the 11th International Conference on Control Systems and Computer Science (CSCS-11), Bucureşti, 1997, V.I. – pp. 179-184.

[5] Izvoreanu B., Cojuhari I. Contribuţii la acordarea regulatoarelor tipizate în sisteme de reglare în cascadă cu două contururi cu inerţie / În: Materialele Conferinţei Tehnico - Ştiinţifice a Colaboratorilor, Doctoranzilor şi Studenţilor UTM, Chişinău, 2007, V.I.

[6] Izvoreanu B., Fiodorov I., Cojuhari I. Tuning of Controllers to the Third Order Advance Delay Objects / In: Proceedings of the 5th International Conference on Microelectronics and Computer Science (ICMCS-2007), Chişinău, V.I. – pp. 250-253, 2007.