Reflective functors and factorization structures

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In the $C_2\mathcal{V}$ category of locally convex Hausdorff vector topological spaces, some relationships between reflective functors and factorization structures are examined.

Theorem 1. *Let $r : C \to \mathcal{R}$ be a reflective functor, and $(\mathcal{P}, I)$ - a factorization structure or a right factorization structure in category C. We examine the following conditions:*

(1) Subcategory $\mathcal{R}$ is $\mathcal{P}$-reflective.

(2) Subcategory $\mathcal{R}$ is closed in relation to I-subobjects.

(3) $\mathcal{U}(\mathcal{R}) \subset \mathcal{P}$, where $\mathcal{U}(\mathcal{R}) = r^X | X \in |C|$.

(4) $r(\mathcal{P}) \subset \mathcal{P}$.

(5) Subcategory $\mathcal{R}$ is closed in relation to $\mathcal{P}$-factorobjects.

(6) In category $\mathcal{R}$ the pair ($\mathcal{R} \cap \mathcal{P}$, $\mathcal{R} \cap I$) is a factorization structure.

Then $1 \Leftrightarrow 2 \Leftrightarrow 3 \Rightarrow 4$; $3 \Rightarrow 6$, $5 \Rightarrow 6$.
If $\mathcal{P} \subset \mathcal{E}_P$, then $4 \Leftrightarrow 5$.

Corollary 1. [1] *In the $C_2\mathcal{V}$ category, any reflector functor is an epifunctor.*

Corollary 2. *Let $\mathcal{R}$ be the reflective subcategory of the $C_2\mathcal{V}$ category. Then the following statements are equivalent: 1. Subcategory $\mathcal{R}$ is a variety ($\mathcal{R}$ is closed relative to Ef - factorobjects). 2. The $r : C_2\mathcal{V} \to C_2\mathcal{V}$ functor is exactly to the right. 3. The $r : C_2\mathcal{V} \to C_2\mathcal{V}$ functor is an E_f -functor.*

In the $C_2\mathcal{V}$ category, any non-zero reflective subcategory is monoreflective. Thus, it is epireflective and $\mathcal{M}_u$–reflective. So, if $(\mathcal{P}, I)$ is a factorization structure with $\mathcal{P} \subset E_p$ and $\mathcal{R}$ is $\mathcal{P}$-reflective subcategory, then $\mathcal{R} = C_2\mathcal{V}$.

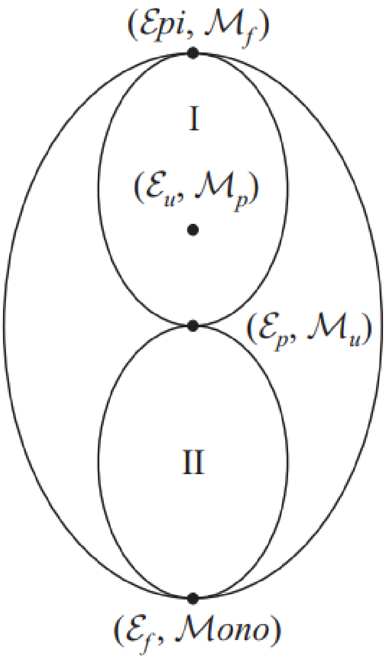

Figure 1. Diagram of factorization structures that verify the relationship $\mathcal{P} \subset E_p$ and $\mathcal{R} = C_2\mathcal{V}$.

So the factorization structure, for which $\mathcal{R}$ is a $\mathcal{P}$-reflective subcategory and is not a trivial subcategory: $\mathcal{R} = C_2\mathcal{V}$ is located in sublattice (I), in the diagram in Figure 1. The factorization structures in sublattice (II) verify the $\mathcal{P} \subset E_p$ relation, for which conditions 4 and 5 are equivalent (Theorem 1).

References

[1] Botnaru D., Structures bicatégorielles complementaires, ROMAI J., 2009, v.5, nr.2, p.5-27

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