OPTIMAL SYNTHESIS AFTER GEOMETRY IN ANTENNA TECHNIQUE

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1. INTRODUCTION

1.1. Antenna optimal synthesis after geometry

The antenna synthesis problem, generally speaking, is a problem of finding such a spatial distribution of electromagnetic field sources, that generate the desired radiation pattern. It means the determination of the *geometry structure* of a radiating system, as well as the *current distribution* on this structure.

The basic electrodynamic relation, connecting the involved functions is the integral equation of the first kind

$$\widehat{A}z = f, \qquad (1)$$

where $z = z(x), x \in G$ is the current distribution in the region *G,* occupied by antenna; $f = f(\omega), \omega \in \Omega$ is antenna pattern as a function of spherical angles $\{\theta, \varphi\}$. The integral operator $\widehat{A}$ is determined by the antenna geometry *G.* Equation (1) is just the equation that is traditionally used as the model of antenna synthesis problem, being reduced to a standard mathematically abstracted *inverse problem for current distribution* along the fixed geometry and solved then by Tihonov's regularization methods [1-3]. However, the current distribution can't be an independently variating function physically. As such functions can be only antenna geometry and excitation function, i.e. incident electromagnetic field or voltage of $\delta -$functional generator. The equation (1) does not contain the excitation function and thus, the based on it model must be treated physically incomplete. The result is that the practical realization of the obtained current distribution remains to be a separate not at all simple engineering problem. As to *optimal* antenna synthesis, some formulations of quasi-optimal or optimal synthesis were proposed [3], but all of them were obtained within the framework of regularization method and are like a many-parametric variation method, which is not properly speaking an optimization method. Furthermore, in view of mathematical difficulties, the deviation of synthesized pattern f from the desired one f_0 is considered in the least mean square sense (L_2 norm), which is not only one of practical importance. The type of closeness to desired function must be an engineering decision since it will govern the performance of the antenna being synthesized. From the practical standpoint, more important is the closeness estimation in sense of difference $/ f - f_0 / |$for all directions (L_∞, norm). As a deficiency of the based on equation (1) approach also must be considered the unjustified difficulties, arising via necessity to carry out the regularization of mathematically instable inverse problem

1.2. A new statement of antenna optimal synthesis 1y geometry problem

As it was pointed above, the relation (1) is not enough for optimal synthesis by geometry problem formulation because it does not contains the mechanism of antenna excitation that is why we first of all complete it with integral equation of the type

$$\widehat{B}z = \varphi, \qquad (2)$$

It explicitly describes the relationship between current distribution function $z = z(x), x \in G$ and excitation operator function $\varphi = \varphi(x), x \in G$. The integral operator $\widehat{B}$ is determined by the physical part of problem and the geometrical form. In case of thin wire antennas (2) may be the well-known Hallen's or Poclington's equation [4]. In general case it is an integral relation between incident electromagnetic field and induced current distribution.

The system of equations (1) - (2) is thus physically complete, since describes both the excitation and radiation processes, including geometry. Basing on it we can correctly formulate different statements of antenna optimal synthesis problem: optimal pattern synthesis by geometry, optimal pattern synthesis by excitation, combined statements.

Consider further the optimal synthesis by geometry. Let $\rho_i(t)$ be a set of continuous parametric functions describing the geometric form - axial line for thin wire structures, contour of revolution for rotational symmetry shells. The problem is to find a set of $\rho_i(t)$ such that for a given excitation mechanism the corresponding antenna pattern will possess desired characteristics. To give it a standard form of optimal control problem the following system of differential equations is introduced

$$\begin{cases} \dfrac{d\rho_i}{dt} = u_i(t), \alpha \le t \le \beta; \\ \rho_i(\alpha) = \rho_i^0, i = 1,2,... \end{cases} \quad (3)$$

This system plays part of state equations dynamical controi system respect to functions $\rho_i(t)$, as the state or phase variables. Totality of quantities $\{u_i(t), \rho_i^0\}$ is declared as control, since it uniquely determines the functions $\rho_i(t)$, i.e. the geometrical form of radiating system. Knowing $\rho_i(t)$, one can derived all the electrodynamtc characteristics, using equations (1) - (2) in direct calculations. From the optimal control theory point of view, the operator equations (l) - (2) play role of the bond equations. Note that they are integral equations, but no inverse problem arises here. The optimal synthesis problem is formulated thus under the scheme: on the multitude of system (3) solutions to find the extremum of functional F_0 under conditions $F_k \le 0$, or

$$\begin{cases} F_0[u_i(t), \rho_i^0] \to extremum; \\ F_k[u_i(t), \rho_i^0] \le 0, k = 1,2,... \end{cases} \quad (4)$$

As a quality functional F_0 any expressions derived from synthesized and desired pattern can be chosen. The restrictions $F_k \le 0$ also can be of any kind concerning the pattern, as we D as the current distribution or geometricaL dimensions. Concrete expressions of F_0, F_k bond equations (1)-(2) and state equations (3) allow us to express the variations of these functionalize on geometrical form by the variations on control $\{u_i(t), \rho_i^0\}$. Thus, different concrete problems can be resolved in strict accordance with engineering desires: minimization of sideiobes and main beam area by arbitrary geometrical restrictions, beam-peak maximization by any restrictions on pattern and s.a.

The formulated problem of optimal control can be easy reduced to a straight non-linear probliem of mathematical programming in functional space and numerically solved using the consecutive linearization method [5]. this approach both closeness estimation between synthesised and desired pattern can be admitted: in L_2 norm and in L_∞ norm.

2. OPTIMAL SYNTHESYS OF ROTATIONAL SYMMETRY SHELLS

2.1. Problem formulation

Consider for certainty the rotational symmetry problem of excitation of a conducting shell magnetic dipole irradiator as shown Figure 1. The non-zero field components in this case of E - polarization are E_φ, H_r, H_z. The shell is described by rotated contour Γ in vector-parametrical form

$$\vec{r}(t) = \vec{i}_\rho \rho(t) + \vec{i}_z \xi(t), \alpha \le t \le \beta. \quad (4)$$

The pattern of such radiating system is

$$f(\omega) = f^{(i)}(\omega) + f^{(s)}(\omega), \omega \in \{\vartheta, \varphi\} \quad (5)$$

where $f^{(i)}(\omega)$ is the known pattern of irradiator, $f^{(s)}(\omega)$ the pattern of induced on shell current *distribution* $j(t)$

$$f^{(s)}(\omega) = \int_\alpha^\beta \Phi[\rho(t), \xi(t), \dot{\rho}(t), \dot{\xi}(t), \omega] j(t) dt. \quad (6)$$

Here $\Phi[...]$ is a known complex function and $j(t)$ is the solution of integral equation [6]:

$$\int_\alpha^\beta K(t,\tau) j(t) dt = \hat{\varphi}(\tau), \alpha \le \tau \le \beta. \quad (7)$$

$$K(t,\tau) = i\rho(t)s(t)\sqrt{\dot{\rho}^2(t) + \dot{\xi}^2(t)} \int_a^b \frac{e^{-ikL}}{L} \cos\varphi d\varphi, \quad (8)$$

$$L = \{\rho^2(t) + \rho^2(\tau) - \rho(t)\rho(\tau)\cos\varphi + [\xi(t) - \xi(\tau)]^2\}^{1/2}, \quad (9)$$

where $k = 2\pi/\lambda$ is the wave number, λ - wave length. The excitation function $\hat{\varphi}(\tau)$ for magnetic dipole (round current frame) is

$$\hat{\varphi}(\tau) = \frac{\rho(t)}{R_0^2}\left(k - \frac{i}{R_0}\right) e^{-ikR_0}, \quad (10)$$

$$R_0 = \{\rho^2(t) + [\xi(\tau) - H]^2\}^{1/2}, \quad (11)$$

where H is the coordinate z value of dipole. The function $s(t)$ in (8) is determined by the edge condition for ***E*** - polarization [6]

$$s(t) = \frac{1}{\sqrt{(t-\alpha)(\beta - t)}}. \quad (12)$$

From expressions (5)-(11) it is seen that adjusting the contour Γ, i.e. variating the functions $\rho(t), \xi(t)$ provides the variation of the pattern $f(\omega)$. Naturally arises the possibility of choosing such a contour Γ, that $f(\omega)$ would be optimal in some sense.

Let us discuss the kind of conditions and restrictions that may occur in problem of optimal synthesis by geometry from engineering standpoint. First group of restrictions is concerned to geometric structure. For example, the inequalities

$$\max_t /\rho(t)/ \le K_\rho, \max_t /\xi(t)/ \le K_\xi, \quad (13)$$

$$\max_t /\dot{\rho}(t)/ \le K_{\dot{\rho}}, \max_t /\dot{\xi}(t)/ \le K_{\dot{\xi}}, \quad (14)$$

restrict the spatial position and the curvature of the contour Γ correspondingly. Coefficients K_ρ, K_ξ, $K_{\dot{\rho}}$, $K_{\dot{\xi}}$ are the given quantities. It assumed that $\rho(t), \xi(t)$ have continuous derivate not turning into zero

$$\begin{cases} \rho^-(t) \le \rho(t) \le \rho^+(t), \\ \xi^-(t) \le \xi(t) \le \xi^+(t), \alpha \le t \le \beta \end{cases} \quad (15)$$

keep the contour Γ in some space corridor limited by curves $\{\rho^-(t), \xi^-(t)\}$ and $\{\rho^+(t), \xi^+(t)\}$.

The end points α, β of contour Γ can be fixed or not fixed. For the free end point the restrictions on its possible position should be specified

$$\begin{cases} \rho^-(\alpha) \le \rho(\alpha) \le \rho^+(\alpha), \\ \xi^-(\alpha) \le \xi(\alpha) \le \xi^+(\alpha). \end{cases} \quad (16)$$

At last, to avoid electrical contact between the conducting shell and the irradiator the minimum distance (11) between them must be required to be more then or equal to a given quantity $\boldsymbol{K_H > 0}$

$$\min_t R_0(t) \ge K_H, \alpha \le t \le \beta. \quad (17)$$

The second group of restrictions is related to antenna pattern form. They can be of a large diversity and be applied to a part ofpattern coinciding with main beam area Ω_M or with sidelobes area Ω_S as well as to the total pattern. The more natural for practice are the restriction

$$\max /f(\omega)/ < K_S, \omega \in \Omega_S, \quad (18)$$

$$\max /f(\omega)/ > K_M, \omega \in \Omega_M, \quad (19)$$

where K_S , K_M are the given quantities. Last restriction can be substituted for more rigorous one

$$\max // f(\omega)/ - f_0(\omega)/ < K_0, \omega \in \Omega_M, \quad (20)$$

where $f_0(\omega)$ is the needing function. The restrictions can be also applied to different functions on pattern such as directivity or antenna gain.

As quality functionals which must be minimized we can choose one from foHowing expressions

$$F_0 = \int_{\Omega_M} | f(\omega) |^2 d\omega, \quad (21)$$

$$F_0 = \int_{\Omega_S} || f(\omega) - f_0(\omega) |^2 d\omega, \quad (22)$$

$$F_0 = \max_{\omega \in \Omega_M} || f(\omega) | - f_0(\omega) \quad (23)$$

The physical sense ofthem is obvious.

Finally formulate a possible statement of pattern optimal synthesis for geometry. On the totality of solutions of the differential equations system

$$\begin{cases} \dfrac{d\rho}{dt} = u(t), \\ \rho(\alpha) = \rho^0, \end{cases} \begin{cases} \dfrac{d\xi}{dt} = v(t), \\ \xi(\alpha) = \xi^0, \end{cases} \alpha \le t \le \beta, \quad (24)$$

with $\rho(t), \xi(t)$ being the components of vector-parametrical form of contour Γ, to minimize the functional

$$F_0[u, v, \rho^0, \xi^0] = \int_{\Omega_S} | f(\omega, u, v, \rho^0, \xi^0) |^2 d\omega, \quad (25)$$

under the conditions:

$$F_1[u,v,\rho^0,\xi^0] = K_M - \max_{\omega \in \Omega_M} | f(\omega,u,v,\rho^0,\xi^0) | \le 0, \quad (26)$$

$$F_2[u,v,\rho^0,\xi^0] = \rho(\beta) - \rho^1 = 0, \quad (27)$$

$$F_3[u,v,\rho^0,\xi^0] = \xi(\beta) - \xi^1 = 0, \quad (28)$$

$$F_4[u,v,\rho^0,\xi^0] = \max_t \rho(t) - K_\rho \le 0, \quad (29)$$

$$F_5[u,v,\rho^0,\xi^0] = -\max_t \rho(t) < 0, \quad (30)$$

$$F_6[u,v,\rho^0,\xi^0] = \max_t | \xi(t) | - K_\xi \le 0, \quad (31)$$

$$F_7[u,v,\rho^0,\xi^0] = -\min_t R_0(t) + K_1 \le 0 \quad (32)$$

if $\{u, v, \rho^0, \xi^0\} \in \Delta$, where Δ is determined by the conditions

$$\begin{cases} | u(t) | \le K_{\dot{\rho}}, \\ | \rho^0 | \le K_{\rho^0}, \end{cases} \begin{cases} | v(t) | \le K_{\dot{\xi}}, \\ | \xi^0 | \le K_{\xi^0}, \end{cases} \quad (33)$$

The so formulated optimal synthesis problem in form of optimal control problem can be reduced to a non-linear problem of mathematical programming in functional space. For numerical solution the sequential linearization method [7] is used.

2.2. Calculation of functionals variations

The explicit form of functionals F_i allows relatively easy to deduce the expressions for δF_i in form of a linear functional on all components of variation of trajectory $\{\delta u, \delta v, \delta\rho^0, \delta\xi^0, \delta\rho, \delta\xi, \delta j, \delta f\}$. These expressions must be transformed by substitution the terms with $\delta\rho, \delta\xi, \delta j, \delta f$ for others equal them and containing only the variations of control $\{\delta u, \delta v, \delta\rho^0, \delta\xi^0\}$ This is possible since the variations $\delta\rho, \delta\xi, \delta j, \delta f$ are completely determined through $\{\delta u, \delta v, \delta\rho^0, \delta\xi^0\}$ by virtue of:

- "*Equations in variations*"

$$\begin{cases} \dfrac{d\delta\rho}{dt} = \delta u(t), \\ \delta\rho(\alpha) = \delta\rho^0, \end{cases} \quad \begin{cases} \dfrac{d\delta\xi}{dt} = \delta v(t), \\ \delta\xi(\alpha) = \delta\xi^0, \end{cases} \alpha \le t \le \beta, \quad (34)$$

$$\begin{aligned} \delta f^S(\omega) = &\int_\alpha^\beta \Phi(t,\omega)\delta j(t)dt + \int_\alpha^\beta \Phi_\rho(t,\omega) j(t)\delta\rho(t)dt + \\ &+\int_\alpha^\beta \Phi_\xi(t,\omega) j(t)\delta\xi(t)dt + \int_\alpha^\beta \Phi_u(t,\omega) j(t)\delta u(t)dt + \\ &+\int_\alpha^\beta \Phi_v(t,\omega) j(t)\delta v(t)dt \end{aligned} \quad (35)$$

- "*Lagrangian identity*"

$$\begin{aligned} &\int_\alpha^\beta \left\{ (\Psi_\rho \frac{d\delta\rho}{dt}) + (\Psi_\xi \frac{d\delta\xi}{dt}) + (\frac{d\Psi_\rho}{dt}\delta\rho) + (\frac{d\Psi_\rho}{dt}\delta\rho) \right\} dt = \\ &= (\Psi_\rho\delta\rho)\Big|_\alpha^\beta + (\Psi_\xi\delta\xi)\Big|_\alpha^\beta, \end{aligned} \quad (36)$$

- "*Integral equation in variations*"

$$\begin{aligned} &\int_\alpha^\beta \{K(t,\tau)\delta j(t) + Q_1(t,\tau)\delta\rho(t) + Q_2(t,\tau)\delta\xi(t)\} dt + \\ &+ Q_3(\tau)\delta\rho(\tau) + Q_4(\tau)\delta\xi(\tau) + \\ &\int_\alpha^\beta \{Q_5(t,\tau)\delta u(\tau) + Q_6(t,\tau)\delta v(\tau)\} dt - \\ &- P_\rho(\tau)\delta\rho - P_\xi(\tau)\delta\xi = 0, \alpha \le \tau \le \beta, \end{aligned} \quad (37)$$

which is obtained by variation (7) under procedure analogous that described in [8]. Here ω belongs the range of definition for $f(\omega)$ and

$Q_i, i = 1,2,3,4,5,6, P_\rho, P_\xi$ are

$$\begin{aligned} Q_1(t,\tau) &= K_{\rho_t}(t,\tau) j(t), \\ Q_2(t,\tau) &= K_{\xi_t}(t,\tau) j(t), \\ Q_3(\tau) &= \int_\alpha^\beta K_{\rho_\tau}(t,\tau) j(t)dt, \\ Q_4(\tau) &= \int_\alpha^\beta K_{\xi_\tau}(t,\tau) j(t)dt, \\ Q_5(t,\tau) &= K_u(t,\tau) j(t), \\ Q_6(t,\tau) &= K_v(t,\tau) j(t), \\ P_\rho(\tau) &= \varphi_{\rho_\tau}(\tau), \\ P_\xi(\tau) &= \varphi_{\xi_\tau}(\tau), \end{aligned} \quad (38)$$

where the indices $\rho_t, \rho_\tau, \xi_t, \xi_\tau$ mean partial derivates with respect to corresponding function in kernel $K(t,\tau)$ or in right part of $\varphi_{\rho_\tau}(\tau)$.

The technique of calculation the functional derivates is more convenient to demonstrate for the functional (25) as an example. In this case the calculations embrace all typical chain elements of dependences and are realized in following order:

- Having $\{u, v, \rho^0, \xi^0\}$ calculate $\rho(t), \xi(t)$ from (24);
- Solving (7) than determine $j(t)$;
- Further calculate $f(\omega)$ using (5) and;
- Finally functional $F[u, v, \rho^0, \xi^0]$ using (25).

The calculations of functional variations are realized in invers order:

1. First, the straight variation of (25) gives

$$\delta F_0[u,v,\rho^0,\xi^0] = \int_{\Omega_S} \{\bar{f}(\omega)\delta f(\omega) + f(\omega)\delta\bar{f}(\omega)\} d\omega \quad (39)$$

Obviously, if is enough in the following to express through $\delta u, \delta v, \delta\rho^0, \delta\xi^0$ only the first term in (39), since the second one is obtained by analogy with it.

2. Variating (5), we obtain the equation in variations $(\delta f(\omega) = 0)$

$$\begin{aligned} \delta f(\omega) = &\int_\alpha^\beta \Phi(t,\omega)\delta j(t)dt + \int_\alpha^\beta \Phi_\rho(t,\omega) j(t)\delta\rho(t)dt + \\ &+\int_\alpha^\beta \Phi_\xi(t,\omega) j(t)\delta\xi(t)dt + \int_\alpha^\beta \Phi_u(t,\omega) j(t)\delta u(t)dt + \\ &+\int_\alpha^\beta \Phi_v(t,\omega) j(t)\delta v(t)dt \end{aligned} \quad (40)$$

We are interesting though in integral on $\bar{f}(\omega)\delta f(\omega)$. It is obviously that

$$\int_{\Omega_S} \bar{f}(\omega)\delta f(\omega)d\omega = \int_\alpha^\beta \hat{\Phi}(t)\delta j(t)dt + \int_\alpha^\beta \hat{\Phi}_\rho(t)\delta\rho(t)dt + \\ + \int_\alpha^\beta \hat{\Phi}_\xi(t)\delta\xi(t)dt + \int_\alpha^\beta \hat{\Phi}_u(t)\delta u(t)dt + \int_\alpha^\beta \hat{\Phi}_v(t)\delta v(t)dt, \tag{41}$$

where

$\hat{\Phi}(t) = \int_{\Omega_S} \bar{f}(\omega)\Phi(t,\omega)\, j(t)d\omega,$ and s.o.

3. The following step is to transform $\int_\alpha^\beta \hat{\Phi}(t)\delta j(t)dt$ into integral on $\delta\rho, \delta\xi, \delta u, \delta v$. It is achieved by using the "integral equation in variations" (37). Multiplying (37) by a function $P(t)$, which will be defined bellow, integrating on *t,* changing the order of integration, denoting *t* through τ and vice versa, we obtain the expression

$$\int_\alpha^\beta R_0(\tau)\delta j(\tau)d\tau + \int_\alpha^\beta R_\rho(\tau)\delta\rho(\tau)d\tau + \\ \int_\alpha^\beta R_\xi(\tau)\delta\xi(\tau)d\tau + \int_\alpha^\beta R_u(\tau)\delta u(\tau)d\tau + \\ + \int_\alpha^\beta R_v(\tau)\delta v(\tau)d\tau = 0, \tag{42}$$

which we name the "*Lagrangian identity*" for integral equation in variations (37). Here

$$R_0(\tau) = \int_\alpha^\beta K^*(t,\tau)P(t)dt, K^*(t,\tau) = \bar{K}(\tau,t), \\ R_\rho(\tau) = \int_\alpha^\beta Q_1(t,\tau)P(t)dt + Q_3(\tau)P(\tau) - P_\rho(\tau)P(\tau), \\ R_\xi(\tau) = \int_\alpha^\beta Q_2(t,\tau)P(t)dt + Q_4(\tau)P(\tau) - P_\xi(\tau)P(\tau), \\ R_u(\tau) = \int_\alpha^\beta Q_5(t,\tau)P(t)dt, R_v(\tau) = \int_\alpha^\beta Q_6(t,\tau)P(t)dt. \tag{43}$$

Now concretize the choice of $P(t)$ talking it as a solution of integral equation

$$\int_\alpha^\beta K^*(t,\tau)P(t)dt = \Phi(\tau), \alpha \le \tau \le \beta. \tag{44}$$

Then (42) obviously gives the expression of integral on $\delta j(t)$ to be found, through the integrals on $\delta\rho, \delta\xi$ and $\delta u, \delta v$

$$\int_\alpha^\beta \hat{\Phi}(t)\delta j(t)dt = -\int_\alpha^\beta R_\rho(\tau)\delta\rho(\tau)d\tau - \\ -\int_\alpha^\beta R_\xi(\tau)\delta\xi(\tau)d\tau - \int_\alpha^\beta R_u(\tau)\delta u(\tau)d\tau - \\ -\int_\alpha^\beta R_v(\tau)\delta v(\tau)d\tau. \tag{45}$$

Making the substitution of $\int_\alpha^\beta \hat{\Phi}(t)\delta j(t)dt$ for that from (41) we obtain

$$\int_{\Omega_S} f(\omega)\delta f(\omega)d\omega = \int_\alpha^\beta \hat{R}_\rho(\tau)\delta\rho(\tau)d\tau + \\ \int_\alpha^\beta \hat{R}_\xi(\tau)\delta\xi(\tau)d\tau + \int_\alpha^\beta \hat{R}_u(\tau)\delta u(\tau)d\tau + \\ + \int_\alpha^\beta \hat{R}_v(\tau)\delta v(\tau)d\tau, \tag{46}$$

where

$\hat{R}_\rho(\tau) = \hat{\Phi}_\rho(\tau) - R(\tau),$ and s.o.

In the same way for $\int_{\Omega_S} f(\omega)\overline{\delta f(\omega)}d\omega$ we obtain

$$\int_{\Omega_S} f(\omega)\overline{\delta f(\omega)}d\omega = \int_\alpha^\beta \hat{R}^*_\rho(\tau)\delta\rho(\tau)d\tau + \\ \int_\alpha^\beta \hat{R}^*_\xi(\tau)\delta\xi(\tau)d\tau + \int_\alpha^\beta \hat{R}^*_u(\tau)\delta u(\tau)d\tau + \\ + \int_\alpha^\beta \hat{R}^*_v(\tau)\delta v(\tau)d\tau, \tag{47}$$

Here $\hat{R}^*_\rho(\tau) = \overline{\hat{R}^*_\rho(\tau)}$ and s.o. From (46), (47) and (39) we have

$$\delta F[\delta u, \delta v, \delta\rho^0, \delta\xi^0] = \int_\alpha^\beta R(\tau)\delta\rho(\tau)d\tau + \\ + \int_\alpha^\beta \Xi(\tau)\delta\xi(\tau)d\tau + \int_\alpha^\beta U(\tau)\delta u(\tau)d\tau + \\ + \int_\alpha^\beta V(\tau)\delta v(\tau)d\tau, \tag{48}$$

where

$$R(\tau) = 2\,\mathrm{Re}\,\hat{R}_\rho(\tau), \Xi(\tau) = 2\,\mathrm{Re}\,\hat{R}_\xi(\tau), \\ U(\tau) = 2\,\mathrm{Re}\,\hat{R}_u(\tau), V(\tau) = 2\,\mathrm{Re}\,\hat{R}_v(\tau), \tag{49}$$

4. The final step of transformations consists in expressing integrals with $\delta\rho, \delta\xi$ through integrals on $\delta u, \delta v, \delta\rho^0, \delta\xi^0$.

Using the equation in variations (34), Lagrangian identity (36) and specifying functions $\Psi_\rho(t), \Psi_\xi(t)$ as the solutions of the systems

$$\begin{cases} \dfrac{d\Psi_\rho(t)}{dt} = -R(t), \\ \Psi_\rho(\alpha) = 0, \end{cases} \quad \begin{cases} \dfrac{d\Psi_\xi(t)}{dt} = -\Xi(t), \\ \Psi_\xi(\alpha) = 0, \end{cases} \tag{50}$$

on the interval $\alpha \le t \le \beta$ obtain the final expression for δF

$$\delta F[\delta u, \delta v, \delta\rho^0, \delta\xi^0] = \int_\alpha^\beta W_u(\tau)\delta u(\tau)d\tau + \int_\alpha^\beta W_v(\tau)\delta v(\tau)d\tau + a\delta\rho^0 + b\delta\xi^0, \tag{51}$$

where

$$\begin{cases} W_u(t) = \Psi_\rho(t) + U(t), \\ a = \Psi_\rho(\alpha) \end{cases} \quad \begin{cases} W_v(t) = \Psi_\xi(t) + V(t), \\ b = \Psi_\xi(\alpha) \end{cases} \tag{52}$$

All calculations are making of course on a not "perturbed trajectory" $\{ u, v, \rho^0, \xi^0, j, f \}$.

The derivatives evaluation for other functional being by Freshe differentiable are making by the some scheme.

As to functionals being only by Gato differentiable (by directions in functional space) the approximation described in [5,8] is used. The main elements of this approximation are directional derivatives of functionals like

$$F[u, v, \rho^0, \xi^0] = | f(\omega^*) |, \tag{53}$$

where ω^* is a point in range of definition of $f(\omega^*)$. The evaluation of derivatives for such functional differs from above described scheme only in that the equation (44) must be solved with the right part of the form

$$A(t) \to \frac{f(\omega^*)}{| f(\omega^*) |} F(t, \omega^*), \tag{54}$$

The solvability of formulated problem of mathematical programming in functional space is proofing by references to theorems of extremum theory. That is about existence. As to uniqueness, in similar problems it is not essential: even if the solution is not unique, we satisfy with any optimal one.

3. NUMERICAL RESULTS

The formulated problem was numerically solved using the programme, described in detail in [7]. Its adaptation is connected with discrete approximation of the continuous problem. As a result a set of FORTRAN programmes was elaborated which allows to synthesize the radiating surfaces of revolution by any conditions on geometry form and on antenna pattern.

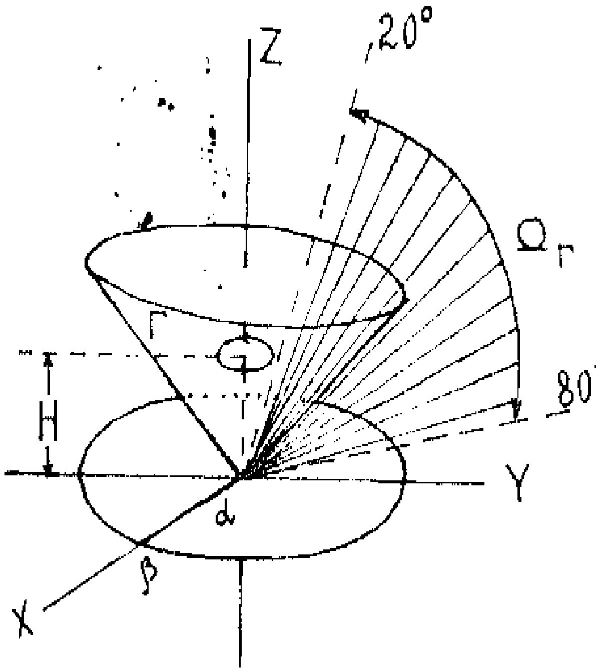
Figure 1, a shows the geometry. As a primary radiator a magnetic dipole is chosen, but it is not a serious restriction since any excitation with axial symmetry can be used. The function $\varphi(\tau)$ in right part of (7) will be changed only. Figure 1,b – pattern, Figure 1,c - geometry.

In **Figure 2** are represented the results of synthesis a surface having the pattern of a disk. So, the main bim is demanded to be in range $\Omega_M = [20^0, 80^0]$.The end points of contour Γ are fixed at $\{\rho(\alpha), \xi(\alpha)\} = (0;0)$; $\{\rho(\beta), \xi(\beta)\} = (1;0)$.

So as the control is the set $\{u(t), v(t)\}$. Initial control $\{u^0(t), v^0(t)\}$ satisfies (30) and initial geometric form is a cone with cone angle **45°**. The synthesized surface is very close to that of a disk and synthesized pattern is also close to a disk pattern. The result was achieved on 11-s iteration of the sequential linearization method.

As the second example (**Figure 3**) was chosen the problem (23)-(33) with fixed end points of contour Γ. The purpose was to synthesize the surface with pattern in region $\Omega_M = [150^0, 170^0]$ Initial control $\{u^0(t), v^0(t)\}'$ satisfies (30). In **Figure 3a** and **Figure 3b** are represented the pattern and the corresponding contours Γ after 1-t, 6-s and 12-s iteration. The synthesized pattern satisfies the conditions and the synthesized surface has a smooth character so that easy can be reproduced technically.

In **Figure 4** are represented the results of contour synthesis with free end point $|\rho(\alpha), \xi(\alpha)\}$.The control in this case is $\{u^0(t), v^0(t), \rho^0, \xi^0\}$ The desired pattern must be in region $\Omega_M = [4^0, 24^0]$ The initial control satisfies (33). In **Figure 4a** and **Figure 4b** are represented the patterns and contours after 1-t, 10-s and 12-s iteration correspondingly. The obtained surface as in previous case is smooth enough. The represented results demonstrate the wide possibilities of the proposed here technique for solving the antenna pattern optimal synthesis for geometry problem.



a.

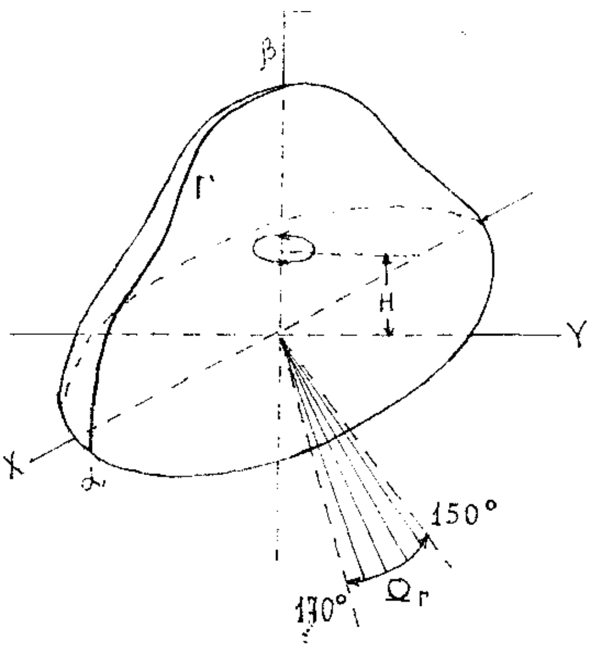
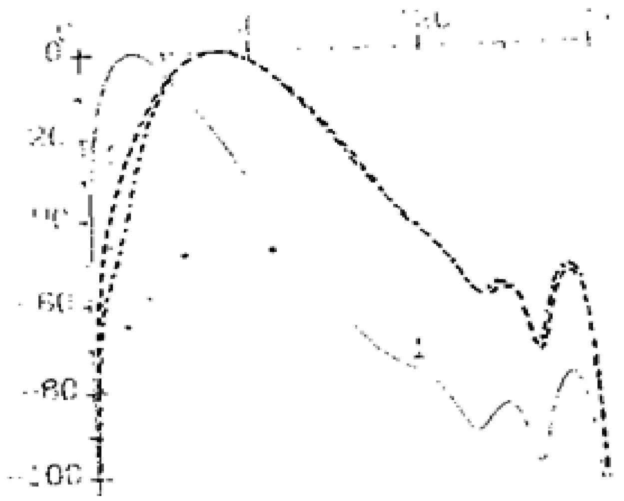
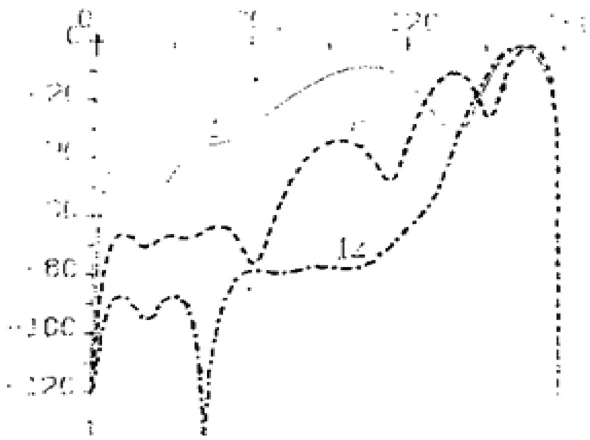


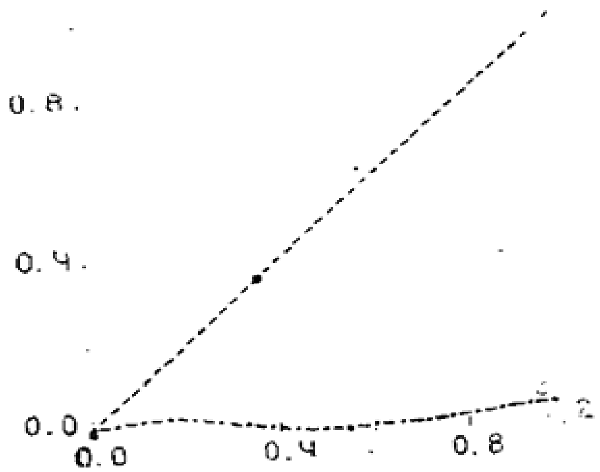
Figure 2. The results of synthesys a surface having a pattern of a disk.



b.

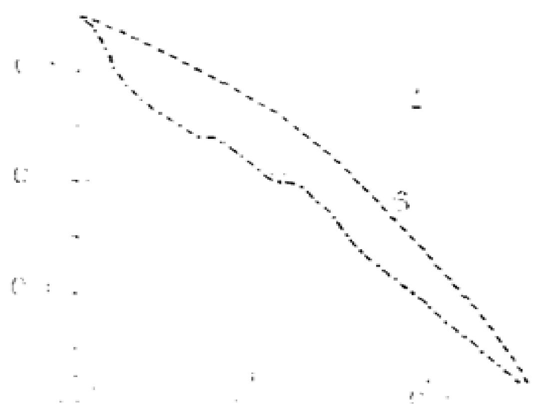


a.



c.

Figure 1. Initial surface area has been used the surface area of the cone: *a – magnetic dipole; b – pattern; c - geometry*.



b.

<u>Figure 3</u>. Magnetic dipole under the surface with fixed point of the coutour Γ : *a – pattern; b – geometry.*

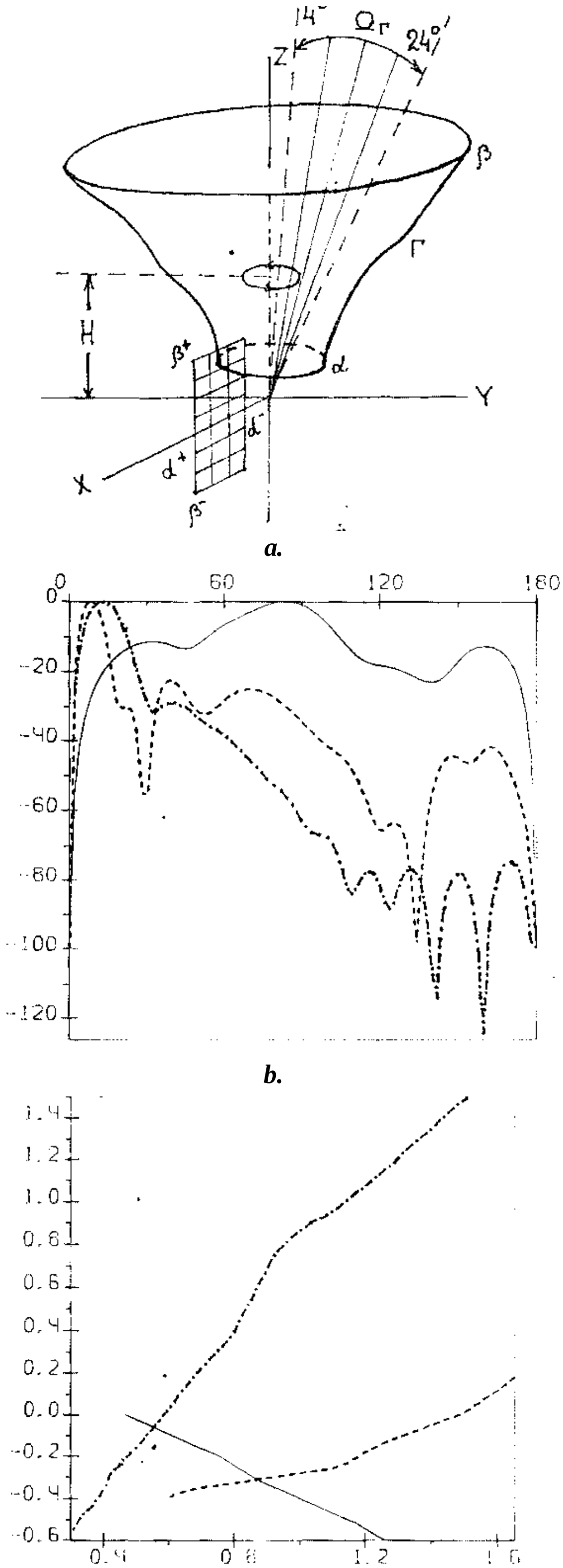


Figure 4. *a - geometry with free end point of contour Γ ; the results of synthesys a surface: b – pattern; b - geometry.*

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