Some properties of Neumann quasigroups

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Main concepts and definitions can be found in [1, 4, 6].

Definition 1. *Quasigroup* $(Q, \cdot)$ *is unipotent if and only if* $x \cdot x = a$ *for all* $x \in Q$ *and some fixed element* $a \in Q$.

Definition 2. *Quasigroup* $(Q, \cdot)$ *has right unit element (a right unit) if there exists element* e *(unique) such that* $x \cdot e = x$ *for all* $x \in Q$.

Definition 3. *A quasigroup* $(Q, \cdot)$ *is said to be Neumann quasigroup if in this quasigroup the identity*

$$x \cdot (yz \cdot yx) = z \tag{1}$$

holds true [3, 5, 2], [7, p. 248].

In the articles [3, 7, 2] the following result is pointed out.

Theorem 1. *If quasigroup* $(Q, \cdot)$ *satisfies the following identity*

$$xy \cdot z = y \cdot zx, \tag{2}$$

then (13)*-parastrophe of this quasigroup satisfies Neumann identity (1).*

Notice that the identity (2) has the following identity as its (13)-parastrophe : $(x \cdot yz) \cdot xy = z$.

Theorem 2. *If quasigroup* $(Q, \cdot)$ *satisfies the identity (2), then this quasigroup is an abelian group.*

Theorem 3. *Any Neumann quasigroup* $(Q, \cdot)$ *is isotope of an abelian group* $(Q, +)$ *of the form* $x \cdot y = x - y$.

Corollary 1. *Any Neumann quasigroup* $(Q, \cdot)$ *is unipotent and has right unit element.*

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