Limits of the Solutions to the Initial-Boundary Dirichlet Problem for the Semilinear Klein-Gordon Equation with Two Small Parameters

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Let $\Omega \subset \mathbb{R}^n$ be a bounded open set with smooth boundary $\partial\Omega$. We consider the following singularly perturbed initial-boundary problem

$$\begin{cases} \varepsilon\, \partial_t^2 u_{\varepsilon\delta} + \delta\, \partial_t u_{\varepsilon\delta} + A u_{\varepsilon\delta} + |u_{\varepsilon\delta}|^q\, u_{\varepsilon\delta} = f(x,t), (x,t) \in \Omega \times (0,T), \\ u_{\varepsilon\delta}\big|_{t=0} = u_0(x), \quad \partial_t u_{\varepsilon\delta}\Big|_{t=0}(0) = u_1(x), \quad x \in \overline{\Omega}, \\ u_{\varepsilon\delta}\big|_{x\in\partial\Omega} = 0, \quad t \in [0,T), \end{cases} \tag{$P_{\varepsilon\delta}$}$$

where A is a strong elliptic operator, $q > 0$ and ε, δ are two small parameters.
We study the behavior of solutions $u_{\varepsilon\delta}$ to the problem $(P_{\varepsilon\delta})$ in two different cases:

(i) when $\varepsilon \to 0$ and $\delta \geq \delta_0 > 0$;

(ii) when $\varepsilon \to 0$ and $\delta \to 0$.

We obtain some *a priori* estimates of solutions to the perturbed problem, which are uniform with respect to parameters, and a relationship between solutions to both problems. We establish that the solution to the unperturbed problem has a singular behavior, relative to the parameters, in the neighbourhood of $t = 0$. We show the boundary layer and boundary layer function in both cases.