Boundary Value Problem Solution Existence For Linear Integro-Differential Equations With Delays

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We consider the following boundary value problem

$$y''(x) = \sum_{i=0}^{n}\Bigg(a_i(x)\,y(x-\tau_i(x)) + b_i(x)\,y'(x-\tau_i(x)) \tag{1}$$

$$+\sum_{j=0}^{1}\int_a^b K_{ij}(x,s)\,y^{(j)}(s-\tau_i(s))\,ds\Bigg) + f(x),$$

$$y^{(j)}(x) = \varphi^{(j)}(x),\ j=0,1,\ x\in[a^*;a],\ y(b)=\gamma, \tag{2}$$

where $\tau_0(x)=0$ and $\tau_i(x),\ i=\overline{1,n}$ are continuous nonnegative functions defined on $[a,b]$, $\varphi(x)$ is a continuously differentiable function given on $[a^*;a]$, $\gamma\in R$, $a^*=\min\limits_{0\le i<n}\left\{\inf\limits_{x\in[a;b]}(x-\tau_i(x))\right\}$. We introduce the sets of points determined by the delays $\tau_1(x),\dots,\tau_n(x)$:

$$E_i=\Big\{x_j\in[a,b]:\ x_j-\tau_i(x_j)=0,\ j=1,2,\dots\Big\},\ E=\bigcup_{i=1}^{n}E_i.$$

A function $y(x)$ is called a solution of (1)-(2) if it satisfies the equation (1) on $[a;b]$ (with the possible exception of a set E) and the conditions (2).
In this work coefficient conditions for the existence of a solution of the boundary value problem for linear integro-differential equations with many delays, which are efficient for verification in practice, are researched [1].
Approximation of the boundary value problem (1)-(2) solution using spline functions with defect 2 was investigated in [2].

Bibliography

[1] Cherevko, I., Dorosh, A., *Boundary Value Problem Solution Existence For Linear Integro-Differential Equations With Many Delays*, Carpathian Math. Publ., **10** (2018) 1, pp. 65–70.

[2] Cherevko, I., Dorosh, A., *Approximation of boundary value problem solutions for linear integro-differential equations with many delays*, Bukovynsky Math. J., **5** (2017) 3–4, pp. 77–81. (in Ukrainian)