[7] C. Bujac, N. Vulpe, *Cubic Differential Systems with Invariant Straight Lines of Total Multiplicity Eight possessing One Infinite Singularity*, Qual. Theory Dyn. Syst. **16**(2017), No. 1, 1–30.

Acknowledgement. *The second author is partially supported by FP7-PEOPLE-2012-IRSES-316338 and by the grant 12.839.08.05F from SCSTD of ASM.*

A rational basis of $GL(2,\mathbb{R})$-comitants for the bidimensional polynomial system of differential equations of the fifth degree

Iurie Calin[1,2], Victor Orlov[1,3]

[1]*Institute of Mathematics and Computer Science,*
[2]*Moldova State University,*
[3]*Technical University of Moldova, Chişinău, Republic of Moldova*

e-mail: `iucalin@yahoo.com, orlovictor@gmail.com`

Let us consider the system of differential equations of the fifth degree

$$\frac{dx}{dt} = P_0 + \sum_{i=1}^{5} P_i(x,y), \qquad \frac{dy}{dt} = Q_0 + \sum_{i=1}^{5} Q_i(x,y), \tag{1}$$

where $P_i(x,y),\ Q_i(x,y)$ are homogeneous polynomials of degree i in x and y with real coefficients. The following $GL(2,\mathbb{R})$-comitants [1] have the first degree with respect to the coefficients of the system (1):

$$\begin{gathered} R_i = P_i(x,y)y - Q_i(x,y)x, \quad i = \overline{0,5} \\ S_i = \frac{1}{i}\left(\frac{\partial P_i(x,y)}{\partial x} + \frac{\partial Q_i(x,y)}{\partial y}\right), \quad i = \overline{1,5}. \end{gathered} \tag{2}$$

Using the comitants (2) as elementary "bricks" and the notion of transvectant [2] the following

$GL(2,\mathbb{R})$-comitants of the system (1) were constructed:

$$
\begin{aligned}
&K_{001}=R_0, && K_{002}=(R_0,R_1)^{(1)},\\
&K_{101}=R_1, && K_{102}=S_1,\\
&K_{103}=(R_1,R_1)^{(2)}, && K_{201}=R_2,\\
&K_{202}=(R_2,R_1)^{(1)}, && K_{203}=(R_2,R_1)^{(2)},\\
&K_{204}=((R_2,R_1)^{(2)},R_1)^{(1)}, && K_{205}=S_2,\\
&K_{206}=(S_2,R_1)^{(1)}, && K_{301}=R_3,\\
&K_{302}=(R_3,R_1)^{(1)}, && K_{303}=(R_3,R_1)^{(2)},\\
&K_{304}=((R_3,R_1)^{(2)},R_1)^{(1)}, && K_{305}=((R_3,R_1)^{(2)},R_1)^{(2)},\\
&K_{306}=S_3, && K_{307}=(S_3,R_1)^{(1)},\\
&K_{308}=(S_3,R_1)^{(2)}, && K_{401}=R_4,\\
&K_{402}=(R_4,R_1)^{(1)}, && K_{403}=(R_4,R_1)^{(2)},\\
&K_{404}=((R_4,R_1)^{(2)},R_1)^{(1)}, && K_{405}=((R_4,R_1)^{(2)},R_1)^{(2)},\\
&K_{406}=(((R_4,R_1)^{(2)},R_1)^{(2)},R_1)^{(1)}, && K_{407}=S_4,\\
&K_{408}=(S_4,R_1)^{(1)}, && K_{409}=(S_4,R_1)^{(2)},\\
&K_{410}=((S_4,R_1)^{(2)},R_1)^{(1)}, && K_{501}=R_5,
\end{aligned}
$$

$$
\begin{aligned}
&K_{502}=(R_5,R_1)^{(1)}, && K_{503}=(R_5,R_1)^{(2)},\\
&K_{504}=((R_5,R_1)^{(2)},R_1)^{(1)}, && K_{505}=((R_5,R_1)^{(2)},R_1)^{(2)},\\
&K_{506}=(((R_5,R_1)^{(2)},R_1)^{(2)},R_1)^{(1)}, && K_{507}=(((R_5,R_1)^{(2)},R_1)^{(2)},R_1)^{(2)},\\
&K_{508}=S_5, && K_{509}=(S_5,R_1)^{(1)},\\
&K_{510}=(S_5,R_1)^{(2)}, && K_{511}=((S_5,R_1)^{(2)},R_1)^{(1)},\\
&K_{512}=((S_5,R_1)^{(2)},R_1)^{(2)}.
\end{aligned}
$$

We denote by A the coefficient space of the system (1).

Definition 1. *The set S of the comitants is called a rational basis on $M \subseteq A$ of the comitants for the system (1) with respect to the group $GL(2,\mathbb{R})$ if any comitant of the system (1) with respect to the group $GL(2,\mathbb{R})$ can be expressed as a rational function of elements of the set S.*

Definition 2. *A rational basis on $M \subseteq A$ of the comitants for the system (1) with respect to the group $GL(2,\mathbb{R})$ is called minimal if by the removal from it of any comitant it ceases to be a rational basis.* In [3] was established a method for construction the rational bases of $GL(2,\mathbb{R})$-comitants for the bidimensional polynomial systems of differential equations by using different comitants of the system. In this paper we will present a rational basis of $GL(2,\mathbb{R})$-comitants for the bidimensional polynomial system of differential equations of the fifth degree in the case, when the comitant of the linear part $R_1 \not\equiv 0$. **Theorem.** *The set of $GL(2,\mathbb{R})$-comitants*

$$
\begin{gathered}
\{K_{001},\ K_{002},\ K_{101},\ K_{102},\ K_{103},\ K_{201},\ K_{202},\ K_{203},\ K_{204},\ K_{205},\ K_{206},\\
K_{301},\ K_{302},\ K_{303},\ K_{304},\ K_{305},\ K_{306},\ K_{307},\ K_{308},\ K_{401},\ K_{402},\ K_{403},\\
K_{404},\ K_{405},\ K_{406},\ K_{407},\ K_{408},\ K_{409},\ K_{410},\ K_{501},\ K_{502},\ K_{503},\ K_{504},\\
K_{505},\ K_{506},\ K_{507},\ K_{508},\ K_{509},\ K_{510},\ K_{511},\ K_{512}\}
\end{gathered}
$$

is a minimal rational basis of the $GL(2,\mathbb{R})$*-comitants for the system* (1) *of differential equations of the fifth degree on* $M = \{\, a \in A \mid R_1 \not\equiv 0 \ (K_{101} \not\equiv 0)\}$.

Acknowledgement. *This research is partially supported by the grant 15.817.02.03F from SCSTD of ASM*

Bibliography

[1] K. S. Sibirsky, *Introduction to the Algebraic Theory of Invariants of Differential Equations*. Manchester University Press, 1988.

[2] G. B. Gurevich, *Foundations of the Theory of Algebraic Invariants*. Noordhoff, Groningen, 1964.

[3] Iu. Calin, *On rational bases of* $GL(2,\mathbb{R})$*-comitants of planar polynomial systems of differential equations*. Buletinul Academiei de Stiinte a Republicii Moldova. Matematica, 2003, nr. 2 (42), pp. 69-86.