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Averaging Method in Multifrequency Systems with Delay and Nonlocal Conditions

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We consider a system of differential equations of the form [1, 2]

$$\frac{dx}{d\tau} = X(\tau, x_\Lambda, \varphi_\Theta), \quad \frac{d\varphi}{d\tau} = \frac{\omega(\tau)}{\varepsilon} + Y(\tau, x_\Lambda, \varphi_\Theta)$$

with initial conditions, multipoint and boundary integral conditions, for example [3],

$$a(\tau_0) = a_0, \quad \int\limits_{\tau_1}^{\tau_2} \Big[\sum_{j=1}^{s} b_j(\tau, a_\Lambda(\tau))\varphi_{\theta_j}(\tau) + g(\tau, a_\Lambda(\tau), \varphi_\Theta(\tau)) \Big] d\tau = d.$$

Here $0 \le \tau \le L$, $x \in D \subset \mathbb{R}^n$, $\varphi \in \mathbb{T}^m$, $\Lambda = (\lambda_1, \dots, \lambda_p)$, $\Theta = (\theta_1, \dots, \theta_q)$, $\lambda_i, \theta_j \in (0, 1)$, $x_{\lambda_i}(\tau) = x(\lambda_i \tau)$, $\varphi_{\theta_j}(\tau) = \varphi(\theta_j \tau)$, $\varepsilon \in (0, \varepsilon_0]$, $\varepsilon_0 \ll 1$, $0 \leqslant \tau_0 \leqslant L$, $0 \leqslant \tau_1 < \tau_2 \leqslant L$.
The complexity of the research of the problem is the existence of resonances. Resonance condition in point $\tau \in [0, L]$ is

$$\sum_{\nu=1}^{q} \theta_\nu (k_\nu, \omega(\theta_\nu \tau)) = 0, \quad k_\nu \in \mathbb{R}^m, \quad \|k\| \neq 0.$$

Averaging in system (1) is carried out on fast variables φ_Θ on the torus T^m. The averaged problem takes the form

$$\frac{d\overline{x}}{d\tau} = X_0(\tau, \overline{x}_\Lambda,), \quad \frac{d\overline{\varphi}}{d\tau} = \frac{\omega(\tau)}{\varepsilon} + Y_0(\tau, \overline{x}_\Lambda),$$

$$\overline{a}(\tau_0) = a_0, \quad \int\limits_{\tau_1}^{\tau_2} \Big[\sum_{j=1}^{s} b_j(\tau, \overline{a}_\Lambda(\tau))\overline{\varphi}_{\theta_j}(\tau) + g_0(\tau, \overline{a}_\Lambda(\tau)) \Big] d\tau = d.$$

The existence and uniqueness of solution of the problem and the estimation error $\|x(\tau, \varepsilon) - \overline{x}(\tau)\| \le c_1 \varepsilon^\alpha$, where $\alpha = (mq)^{-1}$, $c_1 = const > 0$ of averaging method is obtained.

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