Topological configurations of singularities for quadratic differential systems

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In [1-7] the authors proved that there are at least 1879 (and at most 1880) different geometric configurations of singularities of quadratic differential systems in the plane. This classification is completely algebraic and done in terms of invariant polynomials and it is finer than the classification of quadratic systems according to the topological classification of singularities.

The long term project is the classification of phase portraits of all quadratic systems under topological equivalence. A first step in this direction is to obtain the classification of quadratic systems under topological equivalence of local phase portraits around singularities.

In this paper we extract the local topological information around all singularities from the 1879 geometric equivalence classes. We prove that there are exactly 208 topologically distinct global topological configurations of singularities for the whole quadratic class. From here the next goal would be to obtain a bound for the number of possible different phase portraits, modulo limit cycles.

Bibliography

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