COLLECTIVE ELEMENTARY EXCITATIONS OF TWO-DIMENSIONAL MAGNETOEXCITONS IN THE BOSE-EINSTEIN CONDENSATION STATE WITH WAVE VECTOR $k = 0$

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Abstract

The collective elementary excitations of the two-dimensional magnetoexcitons in a state of Bose-Einstein condensation (BEC) with wave vector $\vec{k} = 0$ were investigated in terms of the Bogoliubov theory of quasiaverages. The starting Hamiltonian of the electrons and holes lying on the lowest Landau levels (LLLs) contains the supplementary interactions due to the virtual quantum transitions of the particles to the excited Landau levels (ELLs) and return back. As a result, the interaction between the magnetoexcitons with $\vec{k} = 0$ does not vanish and their BEC becomes stable as regards the collapse. The energy spectrum of the collective elementary excitations consists of two exciton-type branches (energy and quasienergy branches) each of them with energy gap and roton-type section, the gapless optical plasmon branch, and the acoustical plasmon branch, which reveals the absolute instability in the range of small wave vectors.

1. Introduction

Properties of atoms and excitons are dramatically changed in strong magnetic fields, such that the distance between Landau levels $\hbar\omega_c$, exceeds the corresponding Rydberg energies R_y and the magnetic length $l = \sqrt{\hbar c / eH}$ is small compared to their Bohr radii [1, 2]. Even more interesting phenomena are exhibited in the case of two-dimensional (2D) electron systems due to the quenching of the kinetic energy at high magnetic fields, with the representative example being integer and fractional Quantum Hall effects [3-5]. The discovery of the FQHE [6-8] changed fundamentally the established concepts about charged elementary excitations in solids [5]. The notion of the incompressible quantum liquid (IQL) was introduced in [7] as a homogeneous phase with the quantized densities $\nu = p / q$, where p is an integer and $q \neq 1$ is odd having charged elementary excitations with a fractional charge $e^* = \pm e / q$. These quasiparticles were named anyons. A classification for free anyons and their hierarchy were studied in [9, 10]. An alternative concept to hierarchical scheme was proposed in [11], where the notion of composite fermions (CF) was introduced. The CF consists of the electron bound to an even number of flux quanta. Within the frame of this concept, the FQHE of electrons can be physically understood as a manifestation of the IQHE of CFs [11]. The statistics of anyons was determined in [10, 12]. It was established that the wave function of the system changes by a complex phase factor $\exp[i\pi\alpha]$, when the quasiparticles are interchanged. For bosons $\alpha = 0$, for fermions $\alpha = 1$, and for anyons with $e^* = -e/3$, their statistical charge is $\alpha = -1/3$. As was shown in [13], there were no soft branches of neutral excitations in IQL. The energy gap Δ for formation of a quasielectron-quasihole pair has the scale of Coulomb energy $E_Q = e^2 / \varepsilon l$, where ε is the dielectric constant of the background. However, delta was found to be small $\Delta \simeq 0.1 E_Q$. The lowest branch was called magnetoroton [13]; it can be modelled as a quasiexciton [5]. As was mentioned in [5], the traditional methods and concepts based either on the neglecting of the electron-electron interaction or on self-consistent approximation are inapplicable to IQL. In a strong magnetic field, the binding energy of an exciton increases from R_y to I_l.

There are two small parameters of the theory. One of them determines how strong the magnetic field strength H is, and it verifies whether the starting supposition of a strong magnetic field is fulfilled. This parameter is expressed by the ratio $I_l / \hbar\omega_c < 1$. Here I_l is the magnetoexciton ionization potential, ω_c is the cyclotron frequency $eH / \mu c$ calculated with the reduced mass μ and the magnetic length l. Another small parameter has a completely different origin and is related with the concentration of the electron-holes(e-h) pairs. In our case, it can be expressed as a product of the filling factor $\nu = \mathrm{v}^2$ and of another factor $(1 - \mathrm{v}^2)$ which reflects the Pauli exclusion principle and the phase-space filing (PSF) effect. This compound parameter $\mathrm{v}^2(1 - \mathrm{v}^2)$ in the case of Bose-Einstein condensed excitons can take the form $u^2\mathrm{v}^2$, where u and v are the Bogoliubov transformation coefficients and $u^2 = (1 - \mathrm{v}^2)$. The two small parameters will be used below. However, in the case of FQHE, the filling factor $\nu = \mathrm{v}^2$ basically determines the underlying physics and it can not be changed arbitrarily. Instead of the perturbation theory on the filling factor ν, the exact numerical diagonalization for a few number of particles $N \leq 10$ proved to be the most powerful tool in studies of such systems [5]. The spherical geometry for these calculations was proposed [10, 14], considering a few number of particles on the surface of a sphere with the radius $R = \sqrt{S} l$, so as the density of the particles on the sphere to be equal with the filling factor of 2DEG. The magnetic monopole in the center of the sphere creates a magnetic flux through the sphere $2S\phi_0$, which is multiple to the flux quantum $\phi_0 = 2\pi\hbar c / e$. The angular momentum L of a quantum state on the sphere and the quasimomentum k of the FQHE state on the plane obey the relation $L = Rk$. Spherical model is characterized by continuous rotational group, which is analogous to the continuous translational symmetry in the plane.

The properties of the symmetric 2D electron-hole (e-h) system (i.e., $h = 0$), with equal concentrations for both components, with coincident matrix elements of Coulomb electron-electron, hole-hole, and electron-hole interactions in a strong perpendicular magnetic field also attracted much attention during last two decades [15-22]. A hidden symmetry and the multiplicative states were discussed in many papers [19, 23, 24]. The collective states such as the Bose-Einstein condensation (BEC) of two-dimensional magnetoexcitons and the formation of metallic-type electron-hole liquid (EHL) were investigated in [15-22]. The search for Bose-Einstein condensates has become a milestone in the condensed matter physics [25]. The remarkable properties of super fluids and superconductors are intimately related to the existence of a bosonic condensate of composite particles consisting of an even number of fermions. In highly excited semiconductors, the role of such composite bosons is taken on by excitons, which are bound states of electrons and holes. Furthermore, the excitonic system has been viewed as a keystone system for exploration of the BEC phenomena, since it allows to control particle densities and interactions *in situ*. Promising candidates for experimental realization of such system are semiconductor quantum wells (QWs) [26], which have a number of advantages compared to the bulk systems. The coherent pairing of electrons and holes occupying only the lowest Landau levels (LLLs) was studied using the Keldysh-Kozlov-Kopaev method and the generalized random-phase approximation [20, 27]. The BEC of magnetoexcitons takes place in a single exciton state with wave vector *k*, supposing that the high density of electrons in the conduction band and of holes in the valence band was created in a single QW structure with size quantization much greater than the Landau quantization. In the case $k \neq 0$, a new metastable dielectric liquid phase formed by Bose-Einstein condensed magnetoexcitons was revealed [20, 21]. The importance of the excited Landau levels (ELLs) and their influence on the ground states of the systems was first noticed by the authors of papers [16-19]. The influence of the ELLs of electrons and holes was discussed in detail in papers [21, 22]. The indirect attraction between electrons (e-e), between holes (h-h), and between electrons and holes (e-h) due to the virtual simultaneous quantum transitions of the interacting charges from LLLs to the ELLs is a result of their Coulomb scattering. The first step of the scattering and the return back to the initial states were described in the second order of the perturbation theory.

Das Sarma and Madhukar [28] have investigated theoretically the longitudinal collective modes of spatially separated two-component two-dimensional plasma in solids using the generalized random phase approximation. It can be implemented in semiconductors heterojunctions and superlattices. The two-layer structure with two-component plasma is discussed below. It has long been known that two-component plasma has two branches of its longitudinal oscillations. The higher frequency branch is named optical plasmon (OP). Here the two carrier densities of the same signs oscillate in-phase and their density fluctuation operators $\hat{\rho}_{e,1}(\vec{Q})$ and $\hat{\rho}_{e,2}(\vec{Q})$ form an in-phase superposition

$$\hat{\rho}_{OP}(\vec{Q}) = \hat{\rho}_{e,1}(\vec{Q}) + \hat{\rho}_{e,2}(\vec{Q}).$$

In the case of opposite signs of electron and hole charges, they oscillate out-of-phase and their charge density fluctuation operators $\hat{\rho}_e(\vec{Q})$ and $\hat{\rho}_h(\vec{Q})$ combine in out-of-phase manner

$$\hat{\rho}_{OP}(\vec{Q}) = \hat{\rho}_e(\vec{Q}) - \hat{\rho}_h(-\vec{Q}).$$

The lower frequency branch is named acoustical plasmon (AP). Now the carriers of different signs oscillate in-phase, whereas the carriers of the same signs oscillate out-of-phase. Their charge density fluctuation operators combine in the form

$$\hat{\rho}_{AP}(\vec{Q}) = \hat{\rho}_{e,1}(\vec{Q}) - \hat{\rho}_{e,2}(\vec{Q}); \;\; \hat{\rho}_{AP}(\vec{Q}) = \hat{\rho}_e(\vec{Q}) + \hat{\rho}_h(-\vec{Q}).$$

The optical and acoustical branches of two-component electron plasma have the dispersion relations in the long wavelength region as follows

$$\omega_{OP}(q) \sim \sqrt{q}; \;\; \omega_{AP}(q) \sim q; \;\; q \to 0.$$

The plasmon oscillations in one-component system on the monolayer in a strong perpendicular magnetic field were studied by Girvin, MacDonald, and Platzman [13], who proposed the magnetoroton theory of collective excitations in the conditions of the fractional quantum Hall effect (FQHE). The FQHE occurs in low-disorder, high-mobility samples with partially filled Landau levels with filling factor of the form $\nu = 1/m$, where m is an integer, for which there is no single-particle gap. In this case, the excitation is a collective effect arising from many-body correlations due to the Coulomb interaction. Considerable progress has recently been achieved toward understanding the nature of the many-body ground state well described by Laughlin variational wave function [7]. The theory of the collective excitation spectrum proposed by [13] is closely analogous to the Feynman's theory of superfluid helium [29]. The main Feynman's arguments lead to the conclusions that, on general grounds, the low lying excitations of any system will include density waves. As regards the 2D system, the perpendicular magnetic field quenches the single particle continuum of kinetic energy leaving a series of discrete highly degenerate Landau levels spaced in energy at intervals $\hbar\omega_c$. In the case of filled Landau level $\nu = 1$ because of Pauli exclusion principle, the lowest excitation is necessarily the cyclotron mode in which particles are excited into the next Landau level. In the case of FQHE, the LLL is fractionally filled. The Pauli principle no longer excludes low-energy intra-Landau-level excitations. For the FQHE case, the low-lying excitations, rather than the high-energy inter-Landau-level cyclotron modes, are of the primary importance [13]. The spectrum has a relatively large excitation gap at zero wave vector $kl = 0$; in addition, it exhibits a deep magneto-roton minimum at $kl \sim 1$ quite analogous to the roton minimum in helium. The magneto-roton minimum becomes deeper and deeper with decreasing filling factor ν in the row 1/3, 1/5, 1/7; it is the precursor to the gap collapse associated with the Wigner crystallization, which occurs at $\nu = 1/7$. For largest wave vectors, the low lying mode crosses over from being a density wave to becoming a quasiparticle excitation [13]. The Wigner crystal transition occurs slightly before the roton mode goes completely soft. The magnitude of the primitive reciprocal lattice vector for the crystal lies close to the position of the magneto-roton minimum. The authors of [13] suggested also the possibility of pairing of two rotons of opposite momenta leading to the bound two-roton state with small total momentum, as it is known to occur in helium. In difference from the case of fractional filling factor, the excitations from a filled Landau level in the 2DEG were studied by Kallin and Halperin [30].

Fertig [31] investigated the excitation spectrum of two-layer and three-layer electron systems. In a particular case, the two-layer system in a strong perpendicular magnetic field with filling factor $\nu = 1/2$ of the LLL in the conduction band of each layer was considered. Inter-layer separation z was introduced. The spontaneous coherence of two-component two-dimensional (2D) electron gas was introduced.

Fertig has determined the energy spectrum of the elementary excitations within the frame of this ground state. In the case of z = 0, the lowest-lying excitations of the system are the higher energy excitons.

Because of the neutral nature of the $\vec{k} = 0$ excitons, the dispersion relation of these excitations is given in a good approximation by $\hbar\omega(k) = E_{ex}(k) - E_{ex}(0)$, where $E_{ex}(k)$ is the energy of exciton with wave vector $\vec{k}$. This result was first obtained by Paquet, Rice, and Ueda [19] using a random phase approximation (RPA). In the case z = 0, the dispersion relation $\omega(k)$ vanishes as k^2 for $k \to 0$, as one expects for Goldstone modes. As was shown by Fertig [31], for z > 0, $\omega(k)$ behaves as an acoustical mode $\omega(k) \sim k$ in the range of small k, whereas in the limit $k \to \infty$ $\omega(k)$ tends to the ionization potential $\Delta(z)$.

In the region of intermediate values of k, when $kl \sim 1$, the dispersion relation develops the dips as z is increased. At certain critical value of $z = z_{er}$, the modes in the vicinity of the minima become equal to zero and are named soft modes. Their appearance testifies that the two-layer system undergoes a phase transition to the Wigner crystal state.

The similar results concerning the linear and quadratic dependences of the dispersion relations in the range of small wave vectors q were obtained by Kuramoto and Horie [32], who studied the coherent pairing of electrons and holes spatially separated by the insulator barrier in the structure of the type coupled double quantum wells (CDQW).

2. Hamiltonian of the supplementary interaction

The Hamiltonian of the Coulomb interaction of the electrons and holes within the frame LLLs has the form

$$\hat{H} = \frac{1}{2}\sum_{\vec{Q}} W_{\vec{Q}} \left[\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q}) - \hat{N}_e - \hat{N}_h \right] - \mu_e \hat{N}_e - \mu_h \hat{N}_h + \hat{H}_{\text{suppl}}, \tag{1}$$

where $W_{\vec{Q}}$ is the Fourier transform of the Coulomb interaction within the frame of LLLs, $\hat{N}_e$ and $\hat{N}_h$ are the operators of the numbers of electrons and holes on the LLLs. They are determined below. $\hat{H}_{\text{suppl}}$ is the supplementary indirect attractive interaction between the particle lying on the LLLs in view of their virtual transitions on the ELLs and their return back [22]

$$\begin{aligned} H_{\text{suppl}} = -\frac{1}{2}\sum_{p,q,s} \phi_{e-e}(p,q;s) a_p^\dagger a_q^\dagger a_{q+s} a_{p-s} - \\ -\frac{1}{2}\sum_{p,q,s} \phi_{h-h}(p,q;s) b_p^\dagger b_q^\dagger b_{q+s} b_{p-s} - \sum_{p,q,s} \phi_{e-h}(p,q;s) a_p^\dagger b_q^\dagger b_{q+s} a_{p-s}. \end{aligned} \tag{2}$$

Here the creation and annihilation operators $a_p^\dagger, a_p$ for electrons and $b_q^\dagger, b_q$ for holes were introduced. The matrix elements of indirect interaction $\phi_{i-j}(p,q,z)$ are described by the common expressions [22]

$$\phi_{i-j}(p,q,s) = \sum_{n,m} \frac{\phi_{i-j}(p,q,z;n,m)}{n\hbar\omega_{ci} + m\hbar\omega_{cj}}. \tag{3}$$

In the case of electron-electron and hole-hole interaction, expression (3) has the form [22]

$$\begin{aligned} \phi_{i-i}(p,q,z;n,m) \cong \sum_{t,\kappa,\sigma} W_{t,k} W_{z-t,\sigma} \exp\left(i\kappa(p-q-t)l^2\right) \times \\ \times \exp\left(i\sigma(p-q-t-z)l^2\right)(t+i\kappa)^{n+m}(t-z+i\sigma)^{n+m}, \end{aligned} \tag{4}$$

but in the case of electron-hole interaction, it is

$$\begin{aligned} \phi_{e-h}(p,q,z;n,m) \cong \sum_{t,\kappa,\sigma} W_{t,k} W_{z-t,\sigma} \exp\left(i(\kappa+\sigma)(p+q)l^2\right) \times \\ \times (t+i\kappa)^n (t-i\kappa)^m (t-z+i\sigma)^n (t-z-i\sigma)^m, \end{aligned} \tag{5}$$

where

$$W_{s,\kappa} = \frac{2\pi e^2}{\varepsilon_0 S\sqrt{s^2+k^2}} \exp\left[-\frac{\left(s^2+k^2\right)l^2}{2}\right], \qquad W_{s,k} = W_{-s,-k} = W_{-s,k} = W_{s,-k}. \tag{6}$$

Hamiltonian (2) has a Hermitian conjugate form, if the requirements are fulfilled

$$\phi_{i-j}^*(p-s,q+s;-s) = \phi_{i-j}(p,q;s), \qquad i,j = e,h. \tag{7}$$

Hermiticity requirement (7) can be deduced, for example, in the case of electron-electron interaction as follows

$$\begin{aligned} \phi_{i-i}^*(p-z,q+z;-z;n,m) \cong \sum_{t,\kappa,\sigma} W_{t,\kappa} W_{-z-t,\sigma} \exp(-i\kappa(p-q-2z-t)l^2) \times \\ \times \exp(-i\sigma(p-q-t-z)l^2)(t-i\kappa)^{n+m}(t+z-i\sigma)^{n+m}. \end{aligned} \tag{8}$$

Introducing the new summation variables

$$t = t' - z, \qquad \kappa = -\sigma', \qquad \sigma = -\kappa' \tag{9}$$

and taking into account the properties (5) we will obtain exactly expression (4), what proves the affirmation. In the same way, we can write

$$\begin{aligned} \phi_{e-h}^*(p-z,q+z;-z;n,m) \cong \sum_{t,\kappa,\sigma} W_{t,\kappa} W_{t+z,\sigma} \exp(-i(\sigma+\kappa)(p+q)l^2) \times \\ \times (t-i\kappa)^n (t+i\kappa)^m (t+z-i\sigma)^n (t+z+i\sigma)^m, \end{aligned} \tag{10}$$

which after substitution (9) coincides with expression (5). There are two other properties of the coefficients $\phi_{i-j}(p,q;s)$, namely, their reality and parity, that is,

$$\phi^*_{i-j}(p,q;s)=\phi_{i-j}(p,q;s);\ \phi_{i-j}(-p,-q;-s)=\phi_{i-j}(p,q;s). \quad (11)$$

They can be proved as was demonstrated above using the substitution $\sigma=-\sigma'$ and $\kappa=-\kappa'$, when the reality is considered and the substitution $t=-t'$, $\sigma=-\sigma'$, and $\kappa=-\kappa'$ when the parity is discussed.

Side by side with the properties demonstrated above, there is another property related with the translational symmetry of the system in one in-plane direction, which does exist in the Landau gauge description. As a result, the coefficients $\phi_{i-j}(p,q;s)$ do not depend separately on the variables p and q but in their linear combination as follows

$$\begin{aligned}\phi_{i\text{-}i}(p,q;s)&=\tilde{\phi}_{i\text{-}i}(s,\kappa); && \kappa=p-q-s;\\ \phi_{e-h}(p,q;s)&=\tilde{\phi}_{e-h}(s,\sigma); && \sigma=p+q.\end{aligned} \quad (12)$$

They have the properties

$$\tilde{\phi}^*_{i-j}(-s,\sigma)=\tilde{\phi}_{i-j}(s,\sigma);\ \tilde{\phi}^*_{i-j}(s,\sigma)=\tilde{\phi}_{i-j}(s,\sigma);\ \tilde{\phi}_{i-j}(-s,-\sigma)=\tilde{\phi}_{i-j}(s,\sigma). \quad (13)$$

Their Fourier transforms are

$$\psi_{i-j}(s,\sigma)=\sum_{\kappa}\tilde{\phi}_{i-j}(s,\kappa)\exp(i\kappa\sigma l^2)\,. \quad (14)$$

Their symmetry properties follow directly from the previous ones

$$\begin{aligned}\psi^*_{i-j}(s,\sigma)&=\psi_{i-j}(-s,-\sigma); &&\text{hermiticity}\\ \psi^*_{i-j}(s,\sigma)&=\psi_{i-j}(s,-\sigma); &&\text{reality}\\ \psi_{i-j}(-s,-\sigma)&=\psi_{i-j}(s,\sigma). &&\text{parity}.\end{aligned} \quad (15)$$

They lead to the conclusion

$$\psi^*_{i-j}(s,\sigma)=\psi_{i-j}(s,\sigma)\,. \quad (16)$$

These properties will be used below during the transformation of Hamiltonian (2) written in terms of the single particle operators $a^\dagger_p, a_p, b^\dagger_p, b_p$ to the form expressed through the two-particle operators of the electron and hole densities $\hat{\rho}_e(\vec{Q})$ and $\hat{\rho}_h(\vec{Q})$ of the type

$$\hat{\rho}_e(\vec{Q})=\sum_t e^{iQ_y t l^2} a^\dagger_{t-\frac{Q_x}{2}} a_{t+\frac{Q_x}{2}};\ \hat{\rho}_h(\vec{Q})=\sum_t e^{iQ_y t l^2} b^\dagger_{t+\frac{Q_x}{2}} b_{t-\frac{Q_x}{2}}. \quad (17)$$

The relations between two sets of operators are

$$\begin{aligned}a^\dagger_{p-\frac{s}{2}} a_{p+\frac{s}{2}}&=\frac{1}{N}\sum_{\kappa}\hat{\rho}_e(s,\kappa)\exp(-i\kappa p l^2);\\ a^\dagger_p a_{p-s}&=\frac{1}{N}\sum_{\kappa}\hat{\rho}_e(-s,\kappa)\exp\left(-i\kappa p l^2+\frac{is\kappa}{2}l^2\right);\\ a^\dagger_q a_{q+s}&=\frac{1}{N}\sum_{\kappa}\hat{\rho}_e(s,\kappa)\exp\left(-i\kappa q l^2-\frac{is\kappa}{2}l^2\right),\end{aligned} \quad (18)$$

where $N=S/2\pi l^2$, S is the layer surface area and l is the magnetic length. Here the $\delta-$Kronecker symbol was used

$$\frac{1}{N}\sum_p \exp(ip(\sigma-\kappa)l^2)=\delta_{kr}(\sigma,\kappa)\,. \quad (19)$$

Taking into account that

$$\sum_{p,q,s}\phi_{e-e}(p,q;s)a_p^\dagger a_{p-s}a_q^\dagger a_{q+s}=\frac{1}{N}\sum_{s,\sigma}\psi_{e-e}(s,\sigma)\widehat{\rho}_e(-s,-\sigma)\widehat{\rho}_e(s,\sigma),$$

$$\sum_{s}\phi_{e-e}(p,p-s;s)=\sum_{s}\tilde{\phi}_{e-e}(s,0)=B_{i-i}, \tag{20}$$

$$\sum_{p}a_p^\dagger a_p=\widehat{N}_e;\quad \sum_{p}b_p^\dagger b_p=\widehat{N}_h;\qquad \widehat{N}=\widehat{N}_e+\widehat{N}_h$$

and the similar expressions for the hole-hole interaction, we can write

$$\frac{1}{2}\sum_{p,q,s}\phi_{e-e}(p,q;s)a_p^\dagger a_q^\dagger a_{q+s}a_{p-s}+\frac{1}{2}\sum_{p,q,s}\phi_{h-h}(p,q;s)b_p^\dagger b_q^\dagger b_{q+s}b_{p-s}=$$
$$=-\frac{1}{2}B_{i-i}\widehat{N}+\frac{1}{2N}\sum_{s,\sigma}\psi_{i-i}(s,\sigma)\left[\widehat{\rho}_e(-s,-\sigma)\widehat{\rho}_e(s,\sigma)+\widehat{\rho}_h(-s,-\sigma)\widehat{\rho}_h(s,\sigma)\right]. \tag{21}$$

The supplementary electron-hole interaction can be transformed as follows

$$\sum_{p,q,s}\phi_{e-h}(p,q;s)a_p^\dagger b_q^\dagger b_{q+s}a_{p-s}=\frac{1}{N}\sum_{s,\sigma}\psi_{e-h}(s,\sigma)\widehat{\rho}_e(-s,-\sigma)\widehat{\rho}_h(-s,-\sigma). \tag{22}$$

The Hamiltonian of supplementary indirect attractive interaction (2) has the form

$$H_{\text{suppl}}=\frac{1}{2}B_{i-i}\widehat{N}-\frac{1}{2N}\sum_{s,\sigma}\psi_{i-i}(s,\sigma)\left[\widehat{\rho}_e(-s,-\sigma)\widehat{\rho}_e(s,\sigma)+\widehat{\rho}_h(-s,-\sigma)\widehat{\rho}_h(s,\sigma)\right]-$$
$$-\frac{1}{N}\sum_{s,\sigma}\psi_{e-h}(s,\sigma)\widehat{\rho}_e(-s,-\sigma)\widehat{\rho}_h(-s,-\sigma). \tag{23}$$

Instead of density operators for electrons and holes, we can introduce their in-phase and in opposite-phase linear combinations

$$\hat{\rho}(\vec{Q})=\hat{\rho}_e(\vec{Q})-\hat{\rho}_h(-\vec{Q});\ \hat{D}(\vec{Q})=\hat{\rho}_e(\vec{Q})+\hat{\rho}_h(-\vec{Q});$$
$$\hat{\rho}_e(\vec{Q})=\frac{1}{2}\left[\hat{\rho}(\vec{Q})+\hat{D}(\vec{Q})\right];\ \hat{\rho}_h(\vec{Q})=\frac{1}{2}\left[\hat{D}(-\vec{Q})-\hat{\rho}(-\vec{Q})\right]. \tag{24}$$

They lead to the following relations

$$\hat{\rho}_e(-\vec{Q})\hat{\rho}_e(\vec{Q})+\hat{\rho}_h(-\vec{Q})\hat{\rho}_h(\vec{Q})=\frac{1}{2}\left[\hat{\rho}(-\vec{Q})\hat{\rho}(\vec{Q})+\hat{D}(-\vec{Q})\hat{D}(\vec{Q})\right];$$

$$\sum_{Q}\psi_{e-h}(Q)\left[\hat{\rho}(-\vec{Q})\hat{D}(\vec{Q})-\hat{D}(-\vec{Q})\hat{\rho}(\vec{Q})\right]=\sum_{Q}\psi_{e-h}(Q)\left[\hat{\rho}(-\vec{Q})\hat{D}(\vec{Q})-\hat{D}(\vec{Q})\hat{\rho}(-\vec{Q})\right]=0$$

and to the final expression

$$H_{\text{suppl}}=\frac{1}{2}B_{i-i}\widehat{N}-\frac{1}{4N}\sum_{Q}V(Q)\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})-\frac{1}{4N}\sum_{Q}U(Q)\hat{D}(\vec{Q})\hat{D}(-\vec{Q}), \tag{25}$$

where

$$U(Q)=\psi_{i-i}(Q)+\psi_{e-h}(Q);$$
$$V(Q)=\psi_{i-i}(Q)-\psi_{e-h}(Q). \tag{26}$$

The estimations show that

$$U(0)=2A_{i-i};\quad V(0)=0;\quad \frac{1}{N}\sum_{\vec{Q}}U(\vec{Q})=B_{i-i}+\Delta(0).$$

It means that one can suppose the dependences

$$U(\vec{Q})\cong U(0)e^{-\frac{Q^2l^2}{2}};\quad V(\vec{Q})\cong V(0)=0. \tag{27}$$

3. Bose-Einstein Condensation of magnetoexcitons in two alternative descriptions

BEC of 2D magnetoexcitons was considered in [20, 21] within the frame of Keldysh-Kozlov-Kopaev method [27], when the influence of the ELLs was neglected. The main results of this description will be mentioned below.

The creation $d^{\dagger}(\vec{P})$ and annihilation $d(\vec{P})$ operators of the 2D magnetoexciton have the form

$$d^{\dagger}(\vec{P}) = \frac{1}{\sqrt{N}}\sum_{t} e^{-iP_y t l^2} a^{\dagger}_{t+\frac{P_x}{2}} b^{\dagger}_{-t+\frac{P_x}{2}};$$
$$d(\vec{P}) = \frac{1}{\sqrt{N}}\sum_{t} e^{iP_y t l^2} b_{-t+\frac{P_x}{2}} a_{t+\frac{P_x}{2}}. \quad (28)$$

The energy of the two-dimensional magnetoexciton $E_{ex}(P)$ depends on the two-dimensional wave vector $\vec{P}$ and forms a band with the dependence

$$E_{ex}(\vec{P}) = -I_{ex}(\vec{P}) = -I_l + E(\vec{P});$$
$$I_{ex}(\vec{P}) = I_l e^{-\frac{P^2 l^2}{4}} I_0\left(\frac{P^2 l^2}{4}\right); \qquad I_l = \frac{e^2}{\varepsilon_0 l}\sqrt{\frac{\pi}{2}}; \ \sum_{\vec{Q}} W_{\vec{Q}} = I_l. \quad (29)$$

The ionization potential $I_{ex}(P)$ is expressed through the modified Bessel function $I_0(z)$, which has the limiting expressions

$$I_0(z)_{z\to 0} = 1 + \frac{z^2}{4} + ...; \ I_0(z)_{z\to\infty} = \frac{e^z}{\sqrt{2\pi z}}. \quad (30)$$

It means that the function $E(P)$ can be approximated as follows

$$E(\vec{P})_{P\to 0} = \frac{\hbar^2 P^2}{2M}; \quad M = M(0) = 2\sqrt{\frac{2}{\pi}}\frac{\hbar^2 \varepsilon_0}{e^2 l}; \quad E(P)_{P\to\infty} = I_l(1 - \frac{\sqrt{2/\pi}}{Pl}); \quad l^2 = \frac{\hbar c}{eH}. \quad (31)$$

To introduce the phenomenon of BEC of excitons, the gauge symmetry of the initial Hamiltonian was broken by means of the unitary transformation $\hat{D}(\sqrt{N_{ex}})$ following the Keldysh-Kozlov-Kopaev method [27]. We can shortly remember the main outlines of the Keldysh-Kozlov-Kopaev method [27], [33] as it was done in papers [20, 21]. The unitary transformation $\hat{D}(\sqrt{N_{ex}})$ was determined by formula (8) [20]. Here N_{ex} is the number of condensed excitons. It transforms the operators a_p, b_p to other ones α_p, β_p, as is shown in formulas (13) and (14) [20], and gives rise to the BCS-type wave function $|\psi_g(\bar{k})\rangle$ of the new coherent macroscopic state represented by expression (10) [20]. These results are summarized below

$$\hat{D}(\sqrt{N_{ex}}) = \exp[\sqrt{N_{ex}}(d^{\dagger}(\bar{k}) - d(\bar{k}))]; \ |\psi_g(\bar{k})\rangle = \hat{D}(\sqrt{N_{ex}})|0\rangle;$$
$$\alpha_p = \hat{D} a_p \hat{D}^{\dagger} = u a_p - \mathrm{v}(p - \frac{k_x}{2}) b^{\dagger}_{k_x - p}; \ \beta_p = \hat{D} b_p \hat{D}^{\dagger} = u b_p + \mathrm{v}(\frac{k_x}{2} - p) a^{\dagger}_{k_x - p}; \quad (32)$$
$$a_p = u\alpha_p + \mathrm{v}(p - \frac{k_x}{2})\beta^{\dagger}_{k_x - p}; \ b_p = u\beta_p - \mathrm{v}(\frac{k_x}{2} - p)\alpha^{\dagger}_{k_x - p}.$$

$$a_p|0\rangle = b_p|0\rangle = 0; \qquad \alpha_p|\psi_g(\bar{k})\rangle = \beta_p|\psi_g(\bar{k})\rangle = 0;$$

$$u = \cos g; \ \mathrm{v} = \sin g; \ \mathrm{v}(t) = \mathrm{v} e^{-ik_y t l^2}; \ g = \sqrt{2\pi l^2 n_{ex}}; \ n_{ex} = \frac{N_{ex}}{S} = \frac{\mathrm{v}^2}{2\pi l^2}; \ g = \mathrm{v}; \ \mathrm{v}=\mathrm{Sinv}. \quad (33)$$

The developed theory [20, 21] holds true in the limit $\mathrm{v}^2 \approx Sin^2\mathrm{v}$, what means the restriction $\mathrm{v}^2 < 1$. Within the frame of this approach, the collective elementary excitations can be studied constructing the Green's functions on the base of operators α_p, β_p and dealing with the transformed cumbersome Hamiltonian $\hat{\hat{\mathcal{H}}} = D(\sqrt{N_{ex}})\hat{H}D^{\dagger}(\sqrt{N_{ex}})$.

We propose another way, which is supplementary but completely equivalent to the previous one and is based on the idea suggested by Bogoliubov in his theory of quasiaverages [34]. Considering the case of a 3D ideal Bose gas with the Hamiltonian

$$H = \sum_{\bar{p}} \left(\frac{\hbar^2 p^2}{2m} - \mu \right) a_{\bar{p}}^{\dagger} a_{\bar{p}} , \tag{34}$$

where a_p^{+}, a_p are the Bose operators and μ is the chemical potential, Bogoliubov added the term

$$-\eta\sqrt{V}(a_0 e^{i\varphi} + a_0 e^{-i\varphi}) \tag{35}$$

breaking the gauge symmetry and proposed to consider the BEC on the state with $p = 0$ within the frame of the Hamiltonian

$$\hat{\mathcal{H}} = \sum_{p} \left(\frac{\hbar^2 p^2}{2m} - \mu \right) a_p^{\dagger} a_p - \eta\sqrt{V}(a_0^{\dagger} e^{i\varphi} + a_0 e^{-i\varphi}), \tag{36}$$

where

$$\eta = -\mu\sqrt{\frac{N_0}{V}} = -\mu\sqrt{n_0}; \qquad -\frac{\eta}{\mu} = \sqrt{n_0}. \tag{37}$$

We will name the Hamiltonian of type (36) the Hamiltonian of the theory of quasiaverages. It is written in terms of the operators a_p^{+}, a_p of the initial Hamiltonian (34).

Our intention is to apply this idea to the case of BEC of interacting 2D magnetoexcitons and to deduce explicitly the Hamiltonian of type (36) with the finite parameters μ and η but with the relation of type (37). We intend to formulate the new Hamiltonian with broken symmetry in terms of the operators a_p, b_p avoiding the obligatory crossing to the operators α_p, β_p (32) at least at some stages of the investigation where the representation in the a_p, b_p operators remains preferential.

It is obvious that the two representations are completely equivalent and complement each other. We will follow quasiaverage variant (36) instead of u, v variant (32, 33), because it opens some new possibilities, which have not been studied up till now, to the best of our knowledge. For example, the Hamiltonian of type (36) is simpler than the Hamiltonian $\hat{\hat{\mathcal{H}}} = D(\sqrt{N_{ex}})\hat{H}D^{\dagger}(\sqrt{N_{ex}})$ in the α_p, β_p representation, and the deduction of the equation of motion for operators (35) and for the many-particle Green's functions constructed on their base is also much simpler. We will take this advantage at some stages of investigation. On the contrary, for the calculations of the average values of different operators on the base of the ground coherent macroscopic state (27) or using the coherent excited states, as we have done in papers [20, 21], the most convenient way is to use the α_p, β_p representation. We will use, in a wide manner, the two representations. The new variant in the style of the theory of quasiaverages can be implemented rewriting the transformed Hamiltonian $D(\sqrt{N_{ex}})\hat{H}D^{\dagger}(\sqrt{N_{ex}})$ in the a_p, b_p representation as follows below. To demonstrate it, we will represent the unitary transformation

$$\hat{D}(\sqrt{N_{ex}}) = e^{\hat{X}} = \sum_{n=0}^{\infty}\frac{\hat{X}^n}{n!};\ D^{\dagger}(\sqrt{N_{ex}}) = e^{-\hat{X}}, \quad (38)$$

where

$$\hat{X} = \sqrt{N_{ex}}(e^{i\varphi}d^{\dagger}(K) - e^{-i\varphi}d(K));\ \hat{X}^{\dagger} = -\hat{X}. \quad (39)$$

The creation and annihilation operators $d^{+}(k),\ d(k)$ are written in the Landau gauge when the electrons and holes forming the magnetoexcitons are situated on their LLLs. This variant was considered firstly without taking into account of the ELLs, as one can see in [20]. The BEC of 2D magnetoexcitons was considered on the single-exciton state characterized by two-dimensional wave vector $\vec{k}$. Expanding in series the unitary operators $D(\sqrt{N_{ex}}), D^{\dagger}(\sqrt{N_{ex}})$, we find the transformed operator $\hat{\tilde{\mathcal{H}}}$ in the form

$$\hat{\tilde{\mathcal{H}}} = e^{\hat{X}}\hat{H}e^{-\hat{X}} = \hat{H} + \frac{1}{1!}\left[\hat{X},\hat{H}\right] + \frac{1}{2!}\left[\hat{X},\left[\hat{X},\hat{H}\right]\right] + \frac{1}{3!}\left[\hat{X},\left[\hat{X},\left[\hat{X},\hat{H}\right]\right]\right] + ... = \hat{\mathcal{H}} + \hat{\mathcal{H}}'. \quad (40)$$

Here the Hamiltonian $\hat{\mathcal{H}}$ contains the main contributions of the first two terms in the series expansion (40), whereas the operator $\hat{\mathcal{H}}'$ gathers the all remaining terms.

As one can see looking at formulas (39), the operator $\hat{X}$ is proportional to the square root of the exciton concentration $\sqrt{N_{ex}}$, which is proportional to the filling number v. One can see that the contributions arising from the first commutator $\left[\hat{X},\hat{H}\right]$ are proportional to v, the contributions arising from the second commutator $\left[\hat{X},\left[\hat{X},\hat{H}\right]\right]$ are proportional to v^2 and so on. Following the Bogoliubov's theory of quasiaverages, the linear terms of the type $\left(d^{+}(k)e^{i\varphi} + e^{-i\varphi}d(k)\right)\nu$ arising from the first commutator $\left[\hat{X},\hat{H}\right]$ were included into $\hat{\mathcal{H}}$.

The Hamiltonian $\hat{\mathcal{H}}$ with the broken gauge symmetry describing the BEC of 2D magnetoexcitons on the state with wave vector $k \neq 0$ being written in the style of the Bogoliubov's theory of quasiaverages has the form

$$\hat{\mathcal{H}} = \hat{H} + \sqrt{N_{ex}}(\bar{\mu} - E(\vec{K}))(e^{i\varphi}d^{\dagger}(\vec{K}) + e^{-i\varphi}d(\vec{K})). \quad (41)$$

For the case of an ideal 2D Bose gas, we rewrite the coefficient $-\eta\sqrt{V}$ in Hamiltonian (36) in the form $-\eta\sqrt{N}$ and, comparing it with deduced expression (41), we find

$$\eta = \left(E(k) - \bar{\mu}\right)\mathrm{v}. \quad (42)$$

Relation (42) coincides exactly with relation (20) of the Bogoliubov's theory of quasiaverages. In the case of ideal Bose gas, η and $\left(E(k) - \bar{\mu}\right)$ both tend to zero, whereas the filling number is real and different from zero. In the case of interacting exciton gas, both the parameter η and $\left(E(k) - \bar{\mu}\right)$ are different from zero.

The chemical potential μ was determined in the HFB approximation in [21, 22]. In the first of them, only the simplest case of first ELLs was discussed; in the second one, a more general case representing the influence of the all ELLs was described. We shall mention the last results. They were obtained making the (uv) transformation (32) from the initial operators a_p, b_p to new operators α_p, β_p in the starting Hamiltonian H (1). After its normal ordering within the frame of the operators $\alpha_p^{\dagger}, \alpha_p, \beta_p^{\dagger}, \beta_p$, the transformed Hamiltonian $DHD^{\dagger}$ will contain a constant part playing the role of ground state energy, a quadratic Hamiltonian H_2

containing the diagonal terms of the type $\alpha_p^\dagger, \alpha_p$ and $\beta_p^\dagger, \beta_p$ as well as the nondiagonal terms of the type $\alpha_p^\dagger \beta_{k_x-p}^\dagger$ and $\beta_{k_x-p}\alpha_p$, and a quartic Hamiltonian H', which is neglected in the HFB approximation.

The quadratic Hamiltonian H_2 was represented by formula (32) in [22], which is repeated below

$$H_2 = \sum_p [E(\mathbf{k}, v^2, \mu) + (B-2A)v^2(1-2v^2) + 2v^2(1-v^2)\Delta(k)](\alpha_p^+\alpha_p + \beta_p^+\beta_p) +$$
$$+\sum_p [uv(\frac{k_x}{2} - p)\beta_{k_x-p}\alpha_p + uv(p - \frac{k_x}{2})\alpha_p^+\beta_{k_x-p}^+]\{-\psi(\mathbf{k}, v^2, \mu) + 2v^2(B-2A+\Delta(k)) - \Delta(k)\}. \quad (43)$$

Here the notations of [20] were used

$$E(\mathbf{k}, v^2, \mu) = 2v^2u^2 I_{ex}(k) + I_l(v^4 - v^2u^2) - \frac{\mu}{2}(u^2 - v^2),$$
$$\psi(\mathbf{k}, v^2, \mu) = 2v^2 I_l + I_{ex}(k)(1-2v^2) + \mu. \quad (44)$$

The coefficients $\Delta(k)$, A_{i-i} and B_{i-i} were deduced in [22]. They are

$$A_{i-i} = \frac{I_l^2}{\pi\hbar\omega_c} S; \quad S \approx 0.481;$$

$$B_{i-i} = \frac{2I_l^2}{\pi\hbar\omega_c} T; \quad T \approx 0.216;$$

$$\Delta(0) = \frac{2I_l^2}{\pi\hbar\omega_c} 0.344.$$

Putting to zero the last bracket in equation (43), i.e., compensating the dangerous diagrams describing the spontaneous creation and annihilation of quasielectron-quasihole pairs in the new vacuum state (32), we determine the chemical potential μ of the system in the HFB approximation

$$\mu^{HFB} = -\tilde{I}_{ex}(k) + 2v^2(B-2A+\tilde{I}_{ex}(k) - I_l) = -\tilde{I}_{ex}(k) + 2v^2(B-2A+\Delta(k) - E(k)). \quad (45)$$

Here the renormalized ionization potential of magnetoexcitons $\tilde{I}_{ex}(k)$ containing the correction due to influence of all ELLs was introduced

$$\tilde{I}_{ex}(k) = I_{ex}(k) + \Delta(k); \qquad I_{ex}(k) = I_l - E(k); \qquad E_{ex}(k) = -I_{ex}(k). \quad (46)$$

Upon introduction of the value μ^{HFB} in the remainder part of the first line of (43), the Hamiltonian H_2 will take the form

$$H_2 = \sum_p \frac{\tilde{I}_{ex}(k)}{2}(\alpha_p^+\alpha_p + \beta_p^+\beta_p). \quad (47)$$

This Hamiltonian describes the single-particle elementary excitation extracting it from a single-exciton state with wave vector k of the condensed magnetoexcitons. To extract from the condensate one pair of new quasiparticles, the energy cost $\tilde{I}_{ex}(k)$ is equivalent to unbinding energy. For this reason, the excitation energy for one quasiparticle is $\tilde{I}_{ex}(k)/2$. Notice that the chemical potential μ^{HFB} in the point $v^2 = 0$ coincides with the position of the renormalized magnetoexciton energy band on the energy scale $\tilde{E}_{ex}(k) = -\tilde{I}_{ex}(k)$, while in the point $v^2 = 1$ it amounts to the value $-I_l + B - 2A$ and does not depend on k. The concentration corrections to

μ^{HFB} are determined by the term $2v^2(B-2A+\Delta(k)-E(k))$. The term $-E(k)$ appears within the frame of the LLLs and was obtained in [20, 21]. It determines the instability of the ground state within the HFBA when the corrections due to ELL are neglected. The term $B-2A$ appears in both phases, not only in the case of BEC of magnetoexciton but also in the case of EHL. The term $-2A$ is related with the average Hartree terms of the supplementary e–e, h–h, and e–h interactions; the term B, with the average exchange terms of the supplementary e–e and h–h interactions. The term $2v^2\Delta(k)$ is related to e–h interaction and Bogoliubov u-v transformation and is named the Bogoliubov self-energy term.

Below, we shall construct the equations of motion for the operators of creation $d^\dagger(P)$ and annihilation $d(P)$ of magnetoexciton and density fluctuation operators for electrons $\rho(Q)$ and holes $D(Q)$ on the base of Hamiltonian (1) in the quasiaverages theory approximation (QATA).

4. Equations of motion for the two-particle operators and for the corresponding Green's functions

The starting Hamiltonian in QATA has the form

$$\hat{\mathcal{H}} = \frac{1}{2}\sum_{\vec{Q}} W_{\vec{Q}}\left[\rho(\vec{Q})\rho(-\vec{Q}) - \hat{N}_e - \hat{N}_h\right] - \mu_e\hat{N}_e - \mu_h\hat{N}_h -$$
$$-\tilde{\eta}\sqrt{N}\left(e^{i\varphi}d^\dagger(k) + e^{-i\varphi}d(k)\right) + \frac{1}{2}B_{i-i}\widehat{N} - \tag{48}$$
$$-\frac{1}{4N}\sum_{Q} V(Q)\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q}) - \frac{1}{4N}\sum_{Q} U(Q)\hat{D}(\vec{Q})\hat{D}(-\vec{Q}).$$

The density fluctuation operators (24) with different wave vectors P and Q do not commute, which is related with the helicity or spirality accompanying the presence of a strong magnetic field [18]. They are expressed by the phase factors in the structure of operators (6) and by the vector-product of two 2D wave vectors P and Q and its projection on the direction of the magnetic field. These properties considerably influence the structure of the equations of motion for the operators and determine new aspects of the 2D electron-hole (e-h) physics.

The equation of motion for the creation and annihilation operators $d^+(k)$, $d(k)$ (28) and for the density fluctuation operators (24) are

$$i\hbar\frac{d}{dt}d(\vec{P}) = [d(\vec{P}),\hat{\mathcal{H}}] = (-\bar{\mu} + E(\vec{P}) - \Delta(\vec{P}))d(\vec{P}) - 2i\sum_{\vec{Q}}\tilde{W}(\vec{Q})Sin\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)\hat{\rho}(\vec{Q})d(\vec{P}-\vec{Q}) -$$
$$-\frac{1}{N}\sum_{\vec{Q}}U(\vec{Q})Cos\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)D(\vec{Q})d(\vec{P}-\vec{Q}) - \tilde{\eta}\sqrt{N}e^{i\varphi}\delta_{kr}(\vec{P},0) + \tilde{\eta}e^{i\varphi}\frac{D(\vec{P})}{\sqrt{N}};$$
$$i\hbar\frac{d}{dt}d^\dagger(-\vec{P}) = [d^\dagger(-\vec{P}),\hat{\mathcal{H}}] = (\bar{\mu} - E(-\vec{P}) + \Delta(-\vec{P}))d^\dagger(-\vec{P}) + \tag{49}$$
$$+2i\sum_{\vec{Q}}\tilde{W}(\vec{Q})Sin\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)d^\dagger(-\vec{P}-\vec{Q})\hat{\rho}(-\vec{Q}) +$$
$$+\frac{1}{N}\sum_{\vec{Q}}U(\vec{Q})Cos\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)d^\dagger(-\vec{P}-\vec{Q})D(-\vec{Q}) + \tilde{\eta}\sqrt{N}e^{-i\varphi}\delta_{kr}(\vec{P},0) - \tilde{\eta}e^{-i\varphi}\frac{D(\vec{P})}{\sqrt{N}};$$

$$i\hbar\frac{d}{dt}\hat{\rho}(\vec{P})=[\hat{\rho}(\vec{P}),\hat{\mathcal{H}}]=$$
$$=-i\sum_{\vec{Q}}\tilde{W}(\vec{Q})Sin\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)[\hat{\rho}(\vec{P}-\vec{Q})\hat{\rho}(\vec{Q})+\hat{\rho}(\vec{Q})\hat{\rho}(\vec{P}-\vec{Q})]+$$
$$+\frac{i}{2N}\sum_{\vec{Q}}U(\vec{Q})Sin\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)\left[D(\vec{P}-\vec{Q})D(\vec{Q})+D(\vec{Q})D(\vec{P}-\vec{Q})\right];$$
$$i\hbar\frac{d}{dt}\hat{D}(\vec{P})=[\hat{D}(\vec{P}),\hat{\mathcal{H}}]=$$
$$-i\sum_{\vec{Q}}\tilde{W}(\vec{Q})Sin\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)[\hat{\rho}(\vec{Q})\hat{D}(\vec{P}-\vec{Q})+\hat{D}(\vec{P}-\vec{Q})\hat{\rho}(\vec{Q})]+$$
$$+\frac{i}{2N}\sum_{\vec{Q}}U(\vec{Q})Sin\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)[\hat{D}(\vec{Q})\hat{\rho}(\vec{P}-\vec{Q})+\hat{\rho}(\vec{P}-\vec{Q})\hat{D}(\vec{Q})]+$$
$$+2\tilde{\eta}\sqrt{N}\left[e^{-i\varphi}d(\vec{P})-e^{i\varphi}d^{\dagger}(-\vec{P})\right].$$

Here

$$\tilde{\eta}=(\tilde{E}_{ex}(k)-\mu)\mathrm{v}=(E(k)-\Delta(k)-\bar{\mu})\mathrm{v};\ \ \tilde{E}_{ex}(k)=E_{ex}(k)-\Delta(k)=-I_l-\Delta(k)+E(k);$$
$$E_{ex}(k)=-I_l+E(k);\ \ E(K)=2\sum_{Q}W_Q Sin^2\left(\frac{[K\times Q]_z l^2}{2}\right);$$
$$\bar{\mu}=\mu+I_l;\qquad v=\mathrm{v}^2;\qquad N_{ex}=\mathrm{v}^2N;\qquad \widetilde{W}(Q)=W_Q-\frac{1}{2N}V(Q);$$
$$\Delta(k)=\sum_{s}\phi_{e-h}(p,-p-k_x,s)e^{-k_y s l^2}=\frac{1}{N}\sum_{Q}\psi_{e-h}(Q)\exp(i[k\times Q]_z l^2).$$

Following equations of motion (49), we will introduce four interconnected retarded Green's functions at $T=0$ [35, 36]

$$G_{11}(\vec{P},t)=\left\langle\left\langle d(\vec{P},t);\hat{X}^{\dagger}(\vec{P},0)\right\rangle\right\rangle;$$
$$G_{12}(\vec{P},t)=\left\langle\left\langle d^{\dagger}(-\vec{P},t);\hat{X}^{\dagger}(\vec{P},0)\right\rangle\right\rangle;$$
$$G_{13}(\vec{P},t)=\left\langle\left\langle \frac{\hat{\rho}(\vec{P},t)}{\sqrt{N}};\hat{X}^{\dagger}(\vec{P},0)\right\rangle\right\rangle; \tag{50}$$
$$G_{14}(\vec{P},t)=\left\langle\left\langle \frac{\hat{D}(\vec{P},t)}{\sqrt{N}};\hat{X}^{\dagger}(\vec{P},0)\right\rangle\right\rangle.$$

They are determined by the relations

$$G(t)=\left\langle\left\langle \hat{A}(t);\hat{B}(0)\right\rangle\right\rangle=-i\theta(t)\left\langle\left[A(t),B(0)\right]\right\rangle;$$
$$\hat{A}(t)=e^{\frac{iHt}{\hbar}}\hat{A}e^{-\frac{iHt}{\hbar}}; \tag{51}$$
$$\left[\hat{A},\hat{B}\right]=\hat{A}\hat{B}-\hat{B}\hat{A},$$

where $\hat{H}$ is Hamiltonian (48).

The average $\langle \ \rangle$ will be calculated at $T=0$ in the HFB approximation using the ground state wave function $\left|\psi_g(k)\right\rangle$ (32). The time derivative of the Green's function is calculated as follows

$$\begin{aligned} i\hbar\frac{d}{dt}G(t) &= i\hbar\frac{d}{dt}\langle\langle A(t);B(0)\rangle\rangle = \\ &= \hbar\delta(t)\left\langle\left[\hat{A}(0),\hat{B}(0)\right]\right\rangle + \left\langle\left\langle i\hbar\frac{d}{dt}A(t);B(0)\right\rangle\right\rangle = \\ &= \hbar\delta(t)C + \left\langle\left\langle\left[\hat{A}(t),\hat{H}\right];\hat{B}(0)\right\rangle\right\rangle. \end{aligned} \tag{52}$$

C will stand for the average values, which do not depend on *t*. They are not needed in an explicit form for the determination of the energy spectrum of the elementary excitations.

The Fourier transforms of Green's functions (50) will be denoted as

$$\begin{aligned} G_{11}(\vec{P},\omega) &= \left\langle\left\langle d(\vec{P})\middle|\hat{X}^{\dagger}(\vec{P})\right\rangle\right\rangle_{\omega}; \\ G_{12}(\vec{P},\omega) &= \left\langle\left\langle d^{\dagger}(-\vec{P})\middle|\hat{X}^{\dagger}(\vec{P})\right\rangle\right\rangle_{\omega}; \\ G_{13}(\vec{P},\omega) &= \left\langle\left\langle \frac{\hat{\rho}(\vec{P})}{\sqrt{N}}\middle|\hat{X}^{\dagger}(\vec{P})\right\rangle\right\rangle_{\omega}; \\ G_{14}(\vec{P},\omega) &= \left\langle\left\langle \frac{\hat{D}(\vec{P})}{\sqrt{N}}\middle|\hat{X}^{\dagger}(\vec{P})\right\rangle\right\rangle_{\omega}. \end{aligned} \tag{53}$$

The two representations are related as follows

$$G(\vec{P},\omega) = \int_{-\infty}^{\infty} e^{i\omega t}G(\vec{P},t)dt = \int_{0}^{\infty} e^{i\omega t-\delta t}G(\vec{P},t)dt,$$

where the infinitesimal value $\delta \to +0$ guarantees the convergence of the integral in the interval $(0,\infty)$ for the retarded Green's function $G(\vec{P},t)$.

The equation of motion in the frequency representation can be deduced on the base of equation (52)

$$\begin{aligned} \int_{-\infty}^{\infty} dte^{i\omega t}i\hbar\frac{dG(t)}{dt} &= i\hbar\int_{0}^{\infty} dte^{i\omega t-\delta t}\frac{dG(t)}{dt} = -i\hbar\int_{0}^{\infty} dtG(t)\frac{de^{i\omega t-\delta t}}{dt} = \\ &= (\hbar\omega+i\delta)G(\omega) = C + \int_{-\infty}^{\infty} dt\left\langle\left\langle\left[\hat{A}(t),\hat{H}\right];\hat{B}(0)\right\rangle\right\rangle e^{i\omega t} \end{aligned} \tag{54}$$

Green's functions (53) will be named one-operator Green's functions because, in the left hand side of the vertical line, they contain only one summary operator of the types $d(P)$, $d^{\dagger}(P)$, $\hat{\rho}(P)$ and $\hat{D}(P)$. At the same time, these Green's functions are two-particle Green's functions, because the summary operators are expressed through the products of two Fermi operators. In this respect, Green's functions (53) are equivalent to the two-particle Green's functions introduced by Keldysh and Kozlov in their fundamental paper [27], forming the base of the theory of high density excitons in the electron-hole description. However, in contrast to [27], we are using the summary operators which represent integrals on the wave vectors of relative motions.

The equations of motion for the Green's functions are

$$(\hbar\omega + i\delta + \bar{\mu} - E(P) + \Delta(P))G_1(P,\omega) =$$
$$= C - 2i\sum_Q \tilde{W}(Q) Sin\left(\frac{[P\times Q]_z l^2}{2}\right)\left\langle\left\langle \rho(Q)d(P-Q)\middle| X\right\rangle\right\rangle_\omega -$$
$$-\frac{1}{N}\sum_Q U(Q) Cos\left(\frac{[P\times Q]_z l^2}{2}\right)\left\langle\left\langle D(Q)d(P-Q)\middle| X\right\rangle\right\rangle_\omega + \tilde{\eta} G_4(P,\omega)e^{i\varphi};$$
$$(\hbar\omega + i\delta - \bar{\mu} + E(-P) - \Delta(-P))G_2(P,\omega) =$$
$$= C + 2i\sum_Q \tilde{W}(Q) Sin\left(\frac{[P\times Q]_z l^2}{2}\right)\left\langle\left\langle d^\dagger(-P-Q)\rho(-Q)\middle| X\right\rangle\right\rangle_\omega +$$
$$+\frac{1}{N}\left\langle\left\langle d^\dagger(-P-Q)D(-Q)\middle| X\right\rangle\right\rangle_\omega - \tilde{\eta} G_4(P,\omega)e^{-i\varphi};$$
$$(\hbar\omega + i\delta)G_3(P,\omega) =$$
$$= C - i\sum_Q \tilde{W}(Q) Sin\left(\frac{[P\times Q]_z l^2}{2}\right)\left\langle\left\langle \frac{\rho(P-Q)\rho(Q)}{\sqrt{N}} + \frac{\rho(Q)\rho(P-Q)}{\sqrt{N}}\middle| X\right\rangle\right\rangle_\omega +$$
$$+\frac{i}{2N}\sum_Q U(Q) Sin\left(\frac{[P\times Q]_z l^2}{2}\right)\left\langle\left\langle \frac{D(P-Q)D(Q)}{\sqrt{N}} + \frac{D(Q)D(P-Q)}{\sqrt{N}}\middle| X\right\rangle\right\rangle_\omega;$$
$$(\hbar\omega + i\delta)G_4(P,\omega) =$$
$$= C - i\sum_Q \tilde{W}(Q) Sin\left(\frac{[P\times Q]_z l^2}{2}\right)\left\langle\left\langle \frac{D(Q)\rho(P-Q)}{\sqrt{N}} + \frac{D(P-Q)\rho(Q)}{\sqrt{N}}\middle| X\right\rangle\right\rangle_\omega + \tag{55}$$
$$+\frac{i}{2N}\sum_Q U(Q) Sin\left(\frac{[P\times Q]_z l^2}{2}\right)\left\langle\left\langle \frac{D(Q)\rho(P-Q)}{\sqrt{N}} + \frac{\rho(P-Q)D(Q)}{\sqrt{N}}\middle| X\right\rangle\right\rangle_\omega +$$
$$+2\tilde{\eta}\left[e^{-i\varphi}G_1(P,\omega) - e^{i\varphi}G_2(P,\omega)\right].$$

5. Dyson equation and self-energy parts

Using the Zubarev's procedure [36] for the Green's function, we obtain a closed system of Dyson equation for the Green's functions in the form

$$\sum_{j=1}^{4} G_{1j}(\vec{P},\omega)\Sigma_{jk}(\vec{P},\omega) = C_{1k};\quad k = 1,2,3,4. \tag{56}$$

There are 16 different components of the self energy part of the 4×4 matrix $\Sigma_{jk}(\vec{P},\omega)$ as follows

$$\Sigma_{11}(\vec{P},\omega) = \hbar\omega + i\delta + \bar{\mu} - E(\vec{P}) + \Delta(\vec{P}) -$$
$$-\frac{\langle D(A)D(-A)\rangle}{N^2}\sum_{\vec{Q}\neq\vec{P}}\frac{U^2(\vec{Q})Cos^2\left(\frac{[\vec{P}\times\vec{Q}]l^2}{2}\right)}{\hbar\omega + i\delta + \bar{\mu} - E(\vec{P}-\vec{Q}) + \Delta(\vec{P}-\vec{Q})};$$
$$\Sigma_{21}(\vec{P},\omega) = 0;$$
$$\Sigma_{31}(\vec{P},\omega) = i\frac{\langle D(A)d(-A)\sqrt{N}\rangle}{N^2}\sum_{\vec{Q}\neq\vec{P}}\frac{U(\vec{Q})U(\vec{Q}-\vec{P})Cos\left(\frac{[\vec{P}\times\vec{Q}]l^2}{2}\right)Sin\left(\frac{[\vec{P}\times\vec{Q}]l^2}{2}\right)}{\hbar\omega + i\delta + \bar{\mu} - E(\vec{P}-\vec{Q}) + \Delta(\vec{P}-\vec{Q})};$$

$$\sum_{41}(\vec{P},\omega) = -\tilde{\eta}e^{i\varphi} + U(\vec{P})\frac{\langle d(0)\rangle}{\sqrt{N}} +$$

$$+2\frac{\langle D(A)d(-A)\sqrt{N}\rangle}{N}\sum_{\vec{Q}\neq\vec{P}}\frac{\tilde{W}(\vec{Q})(U(\vec{P})-U(\vec{Q}-\vec{P}))Sin^2\left(\frac{[\vec{P}\times\vec{Q}]l^2}{2}\right)}{\hbar\omega+i\delta+\bar{\mu}-E(\vec{P}-\vec{Q})+\Delta(\vec{P}-\vec{Q})} -$$

$$-\frac{\langle D(A)d(-A)\sqrt{N}\rangle}{N^2}\sum_{\vec{Q}\neq\vec{P}}\frac{U(\vec{Q})U(\vec{P})Cos^2\left(\frac{[\vec{P}\times\vec{Q}]l^2}{2}\right)}{\hbar\omega+i\delta+\bar{\mu}-E(\vec{P}-\vec{Q})+\Delta(\vec{P}-\vec{Q})};$$

$$\sum_{12}(\vec{P},\omega) = 0;$$

$$\sum_{22}(\vec{P},\omega) = \hbar\omega+i\delta-\bar{\mu}+E(\vec{P})-\Delta(-\vec{P}) -$$

$$-\frac{\langle D(A)D(-A)\rangle}{N^2}\sum_{\vec{Q}\neq-\vec{P}}\frac{U(\vec{Q})U(-\vec{Q})Cos^2\left(\frac{[\vec{P}\times\vec{Q}]l^2}{2}\right)}{\hbar\omega+i\delta-\bar{\mu}+E(-\vec{P}-\vec{Q})-\Delta(-\vec{P}-\vec{Q})};$$

$$\sum_{32}(\vec{P},\omega) = i\frac{\langle d^{\dagger}(A)D(-A)\sqrt{N}\rangle}{N^2}\sum_{\vec{Q}\neq-\vec{P}}\frac{U(\vec{Q})U(-\vec{Q}-\vec{P})Cos\left(\frac{[\vec{P}\times\vec{Q}]l^2}{2}\right)Sin\left(\frac{[\vec{P}\times\vec{Q}]l^2}{2}\right)}{\hbar\omega+i\delta-\bar{\mu}+E(-\vec{P}-\vec{Q})-\Delta(-\vec{P}-\vec{Q})};$$

$$\sum_{42}(\vec{P},\omega) = \tilde{\eta}e^{-i\varphi} - U(-\vec{P})\frac{\langle d^{\dagger}(0)\rangle}{\sqrt{N}} -$$

$$-2\frac{\langle d^{\dagger}(A)D(-A)\sqrt{N}\rangle}{N}\sum_{\vec{Q}\neq-\vec{P}}\frac{\tilde{W}(\vec{Q})(U(-\vec{Q}-\vec{P})-U(\vec{P}))Sin^2\left(\frac{[\vec{P}\times\vec{Q}]l^2}{2}\right)}{\hbar\omega+i\delta-\bar{\mu}+E(-\vec{P}-\vec{Q})-\Delta(-\vec{P}-\vec{Q})} -$$

$$-\frac{\langle d^{\dagger}(A)D(-A)\sqrt{N}\rangle}{N^2}\sum_{\vec{Q}\neq-\vec{P}}\frac{U(\vec{Q})U(-\vec{P})Cos^2\left(\frac{[\vec{P}\times\vec{Q}]l^2}{2}\right)}{\hbar\omega+i\delta-\bar{\mu}+E(-\vec{P}-\vec{Q})-\Delta(-\vec{P}-\vec{Q})};$$

$$\sum_{13}(\vec{P},\omega) = 0;\ \sum_{23}(\vec{P},\omega) = 0;$$

$$\sum_{33}(\vec{P},\omega) = \hbar\omega+i\delta-\frac{\langle D(A)D(-A)\rangle}{N^2(\hbar\omega+i\delta)}\sum_{\vec{Q}\neq\vec{P}}U(\vec{Q})(U(-\vec{Q})-U(\vec{Q}-\vec{P}))Sin^2\left(\frac{[\vec{P}\times\vec{Q}]l^2}{2}\right);$$

$$\sum_{43}(\vec{P},\omega) = 0;$$

$$\sum_{14}(\vec{P},\omega) = -2\tilde{\eta}e^{-i\varphi};$$

$$\sum_{24}(\vec{P},\omega) = 2\tilde{\eta}e^{i\varphi};$$

$$\sum_{34}(\vec{P},\omega) = 0;$$

$$\sum_{44}(\vec{P},\omega) = \hbar\omega+i\delta-\frac{2\langle D(A)D(-A)\rangle}{N(\hbar\omega+i\delta)}\sum_{\vec{Q}\neq-\vec{P}}\tilde{W}(\vec{Q})(U(\vec{Q}-\vec{P})-U(\vec{P}))Sin^2\left(\frac{[\vec{P}\times\vec{Q}]l^2}{2}\right) +$$

$$+\frac{\langle D(A)D(-A)\rangle}{N^2(\hbar\omega+i\delta)}\sum_{\vec{Q}\neq\vec{P}}U(\vec{Q})(U(\vec{P})-U(-\vec{Q}))Sin^2\left(\frac{[\vec{P}\times\vec{Q}]l^2}{2}\right). \quad (57)$$

The self-energy parts $\Sigma_{jk}(\vec{P},\omega)$ represented by formulas (57) contain the different average values of the two-operator products. They were calculated using the ground state wave function $\left|\psi_g(0)\right\rangle$ (32) taken with $k=0$ and have the expressions

$$\begin{gathered}\left\langle D(\vec{Q})D(-\vec{Q})\right\rangle = 4u^2v^2N;\\ \bar{\mu} = -\Delta(0) + 2v^2(B_{i-i} - 2A_{i-i} + \Delta(0));\\ \left\langle D(\vec{Q})d(-\vec{Q})\sqrt{N}\right\rangle = \left\langle d^\dagger(\vec{Q})D(-\vec{Q})\sqrt{N}\right\rangle = -2uv^3N;\\ \left\langle d(0)\right\rangle = \left\langle d^\dagger(0)\right\rangle = uv\sqrt{N}; \qquad \tilde{\eta} = -(\Delta(0)+\bar{\mu})v.\end{gathered} \tag{58}$$

All these averages are extensive values proportional to N or $\sqrt{N}$, they essentially depend on the small parameters of the types $u^2\text{v}^2$ or $u\text{v}^3$, or uv.

The cumbersome dispersion equation is expressed in a general form by the determinant equation

$$\det\left|\Sigma_{ij}(\vec{P},\omega)\right| = 0. \tag{59}$$

We can substitute the self-energy parts $\Sigma_{jk}(\vec{P},\omega)$ (57) in formula (59), and the determinant equation (59) disintegrates into two independent equations. One of them concerns only optical plasmons and has the simple form

$$\Sigma_{33}(\vec{P};\omega) = 0, \tag{60}$$

whereas the second equation contains the self-energy parts Σ_{11}, Σ_{22}, Σ_{44}, Σ_{14}, Σ_{41}, Σ_{24}, Σ_{42} and the quasi-average constant $\tilde{\eta}$

$$\Sigma_{11}(\vec{P};\omega)\Sigma_{22}(\vec{P};\omega)\Sigma_{44}(\vec{P};\omega) - \Sigma_{41}(\vec{P};\omega)\Sigma_{22}(\vec{P};\omega)\Sigma_{14}(\vec{P};\omega) - \Sigma_{42}(\vec{P};\omega)\Sigma_{11}(\vec{P};\omega)\Sigma_{24}(\vec{P};\omega) = 0\,. \tag{61}$$

The solutions of dispersion equation (61) will be discussed in two limiting cases. One of them is the point $v^2=0$, where the system behaves as an ideal Bose gas and the other case is $v^2\neq 0$.

All contributions to the self-energy parts contain the averages $\left\langle D(\vec{Q})D(-\vec{Q})\right\rangle$, $\left\langle D(\vec{Q})d(-\vec{Q})\sqrt{N}\right\rangle$, $\left\langle d(0)\right\rangle$, which do not vanish in the point $\vec{k}=0$. The 2D magnetoexciton system now is not at all a pure ideal gas. It was an ideal gas when the influence of ELLs was neglected. This unusual result was revealed for the first time by Lerner and Lozovik [15-17] and was confirmed by Paquet, Rice and Ueda [19]. In the case $v^2=0$, due to the vanishing of averages (58), the self-energy parts become

$$\begin{aligned}&\sigma_{11}(\vec{P},\omega) = \hbar\omega - E(P); && \bar{\mu}+\Delta(0) = 0;\\ &\sigma_{22}(\vec{P},\omega) = \hbar\omega + E(-P); && \tilde{\eta} = 0;\\ &\sigma_{33}(\vec{P},\omega) = \hbar\omega; && \Delta(\vec{P}) \approx \Delta(0);\\ &\sigma_{44}(\vec{P},\omega) = \hbar\omega; && v^2 = 0;\ \vec{k} = 0,\end{aligned} \tag{62}$$

the excitonic part of the dispersion relation and the acoustic plasmon frequency look like

$$\begin{gathered}\hbar\omega_{ex}(P) = \pm E(P);\\ \hbar\omega_A(P) = \hbar\omega_O(P) = 0.\end{gathered} \tag{63}$$

The acoustical and optical plasmon branches have the frequencies equal to zero. This case is presented in Fig. 1.

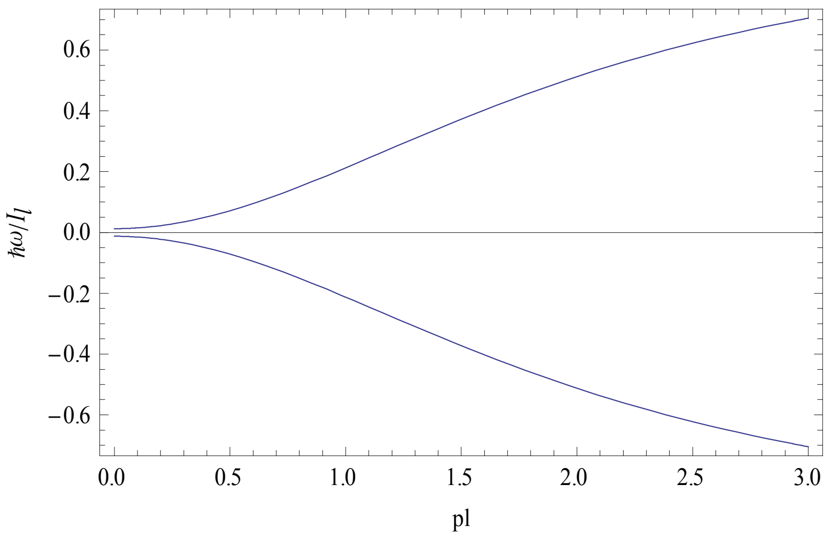

Fig. 1. The energy spectrum of elementary excitations of magnetoexcitons in the case when concentration corrections are not taken into account, the filling factor equals zero.

If we keep terms proportional to uv in formulas (58), then the self-energy parts included in (56) can be rewritten in following form

$$\begin{gathered}\sigma_{11}(\vec{P},\omega)=\hbar\omega+\bar{\mu}-E(\vec{P})+\Delta(\vec{P});\\ \sigma_{41}(\vec{P},\omega)=-\tilde{\eta}+U(\vec{P})\frac{\langle d(0)\rangle}{\sqrt{N}};\\ \sigma_{22}(\vec{P},\omega)=\hbar\omega-\bar{\mu}+E(\vec{P})-\Delta(-\vec{P});\\ \sigma_{42}(\vec{P},\omega)=\tilde{\eta}-U(-\vec{P})\frac{\langle d^{\dagger}(0)\rangle}{\sqrt{N}};\\ \sigma_{14}(\vec{P},\omega)=-2\tilde{\eta};\\ \sigma_{24}(\vec{P},\omega)=2\tilde{\eta};\\ \sigma_{44}(\vec{P},\omega)=\hbar\omega+i\delta;\\ \bar{\mu}=-\Delta(0)+2v^2(B_{i-i}-2A_{i-i}+\Delta(0));\\ \tilde{\eta}=-(\Delta(0)+\bar{\mu})v=-2v^3(B_{i-i}-2A_{i-i}+\Delta(0));\\ \Delta(\vec{P})\approx\Delta(0);\quad \vec{k}=0;\quad v^2\neq 0;\quad U(\vec{P})\cong U(0)e^{-\frac{P^2l^2}{2}},\quad U(0)=2A_{i-i}.\end{gathered}\tag{64}$$

Dispersion equation (61) in this case looks like

$$\hbar\omega=\pm\sqrt{\left(\bar{\mu}-E(\vec{P})+\Delta(0)\right)^2+4\tilde{\eta}\left(\tilde{\eta}-\frac{U(\vec{P})\langle d(0)\rangle}{\sqrt{N}}\right)}.\tag{65}$$

In the Ref. [22] the coefficient $(B_{i-i}-2A_{i-i}+\Delta(0))/I_l$ was determined to be 0.025 at the ratio $r=I_l/\hbar\omega=1/2$. It was used in the present calculations leading to the main parameters $\bar{\mu}$ and $\tilde{\eta}$ $(\bar{\mu}+\Delta(0))/I_l=2v^2\bullet 0.025=0.05v^2$, $\tilde{\eta}=-(\bar{\mu}+\Delta(0))v/I_l=-2v^3\bullet 0.025=-0.05v^3$, and $U(0)\langle d(0)\rangle/I_l\sqrt{N}=2A_{i-i}uv/I_l=0.15uv$. Introducing the dimensionless energies $\hbar\tilde{\omega}=\hbar\omega/I_l$ and $\tilde{E}(P)=E(P)/I_l$ one can transcribe solution (65) as follows

$$\hbar\omega = \pm\sqrt{\left(0.05v^2 - \tilde{E}(P)\right)^2 + 0.2v^2\left(0.05v^2 + 0.15uve^{-\frac{P^2l^2}{2}}\right)}. \quad (66)$$

Their solutions are presented in Fig. 2. The excitonic branch of elementary excitations is characterized by a roton-type behavior at the small and intermediary values of the wavevectors and by a monotonic increasing at higher values of the wavevectors. The acoustical and optical plasmon branches have frequencies equal to zero because the sums in expressions (57) containing the coefficients $U^2(\vec{Q})$, as well as $\tilde{W}(\vec{Q})U(\vec{Q})$, were not included in these calculations.

The neglected terms in the expressions for $\Sigma_{33}(\vec{P},\omega)$ and $\Sigma_{44}(\vec{P},\omega)$ can be calculated using the approximation (27) $U(\vec{Q}) \cong U(0)\exp[-Q^2l^2/2]$, whereas the terms proportional to $U^2(\vec{Q})$ in the expressions $\Sigma_{11}(\vec{P},\omega)$ and $\Sigma_{22}(\vec{P},\omega)$ must be summarized together with the denominators of the types $\hbar\omega + i\delta \pm \bar{\mu} \mp E(\pm\vec{P}-\vec{Q}) \pm \Delta(\pm\vec{P}-\vec{Q})$. They were represented as

$$\frac{\hbar\omega + i\delta \pm \bar{\mu} \mp E(\pm\vec{P}-\vec{Q}) \pm \Delta(\pm\vec{P}-\vec{Q}) - i\gamma}{(\hbar\omega + i\delta \pm \bar{\mu} \mp E(\pm\vec{P}-\vec{Q}) \pm \Delta(\pm\vec{P}-\vec{Q}))^2 + \gamma^2},$$

and the approach was proposed

$$\frac{Pf}{\hbar\omega \pm \bar{\mu} \mp E(\pm\vec{P}-\vec{Q}) \pm \Delta(\pm\vec{P}-\vec{Q})} \approx \frac{\hbar\omega \pm \bar{\mu} \mp E(\pm\vec{P}) \pm \Delta(\pm\vec{P})}{\gamma^2}.$$

In this rude approximation for denominators, using the known expressions for $W_{\vec{Q}}$ and $U(\vec{Q})$, the sums on $\vec{Q}$ were calculated; the results are represented in Figs. 3-5.

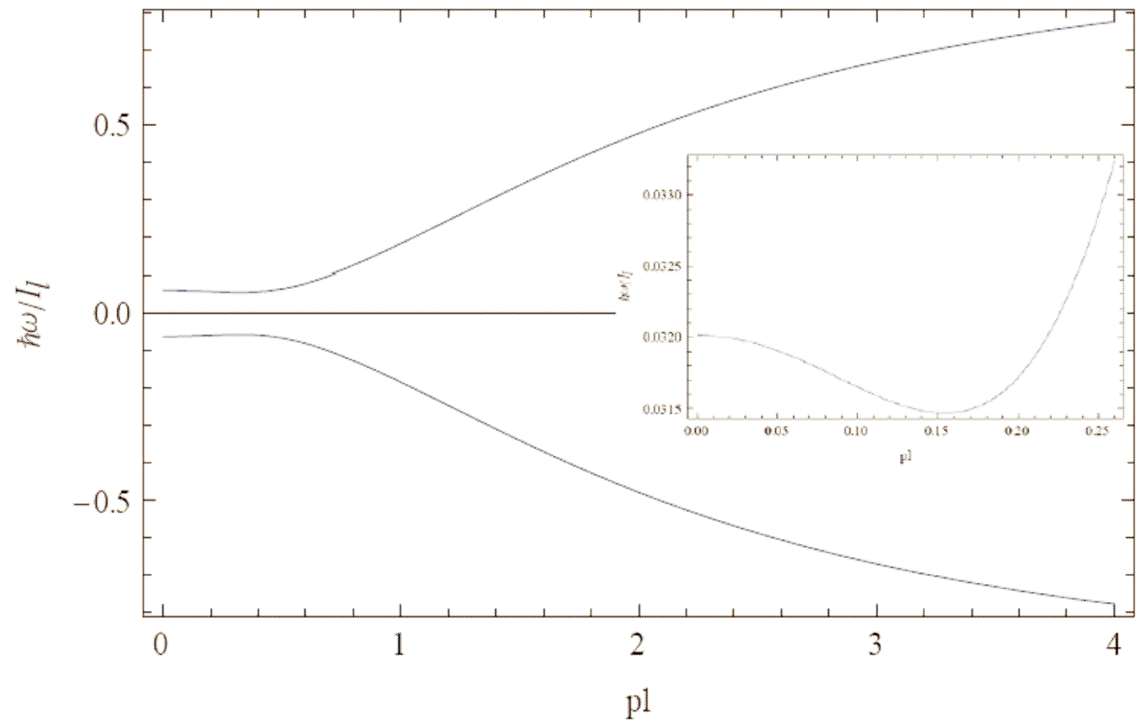

Fig. 2. Two exciton branches of the energy spectrum of collective elementary excitations of the Bose-Einstein condensed magnetoexcitons on the wave vector $\vec{k}=0$ calculated in HFBA using self-energy parts (64) and the filling factor $v^2 = 0.1$.

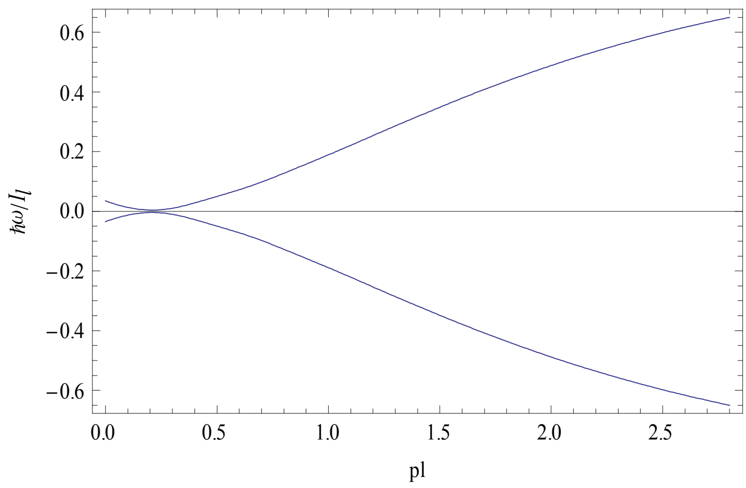

Fig. 3. Two exciton branches of the energy spectrum of collective elementary excitations of the Bose-Einstein condensed magnetoexcitons on the wave vector $\vec{k} = 0$ calculated in HFBA using self-energy parts (57) and the filling factor $v^2 = 0.1$.

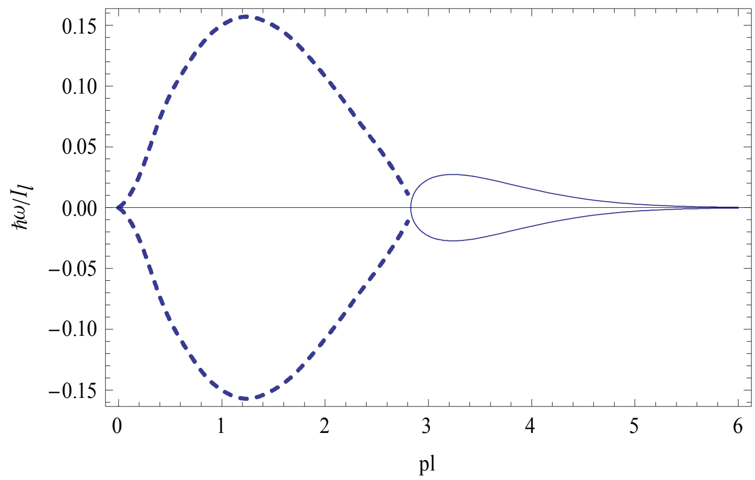

Fig. 4. The dispersion law of acoustical plasmon branch in the presence of the BEC of magnetoexcitons on the wave vector $\vec{k} = 0$ calculated in HFBA using self-energy parts (57) and filling factor $v^2 = 0.1$.

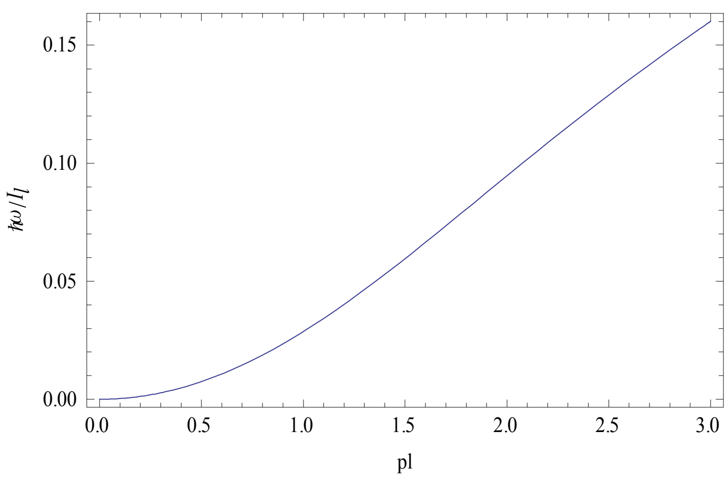

Fig. 5. The dispersion law of optical plasmon branch in the presence of the BEC of magnetoexcitons on the wave vector $\vec{k} = 0$ calculated in HFBA using self-energy parts (57) and the filling factor $v^2 = 0.1$.

Conclusions

The energy spectrum of the collective elementary excitations of a 2D e-h system situated in a strong perpendicular magnetic field in a state of BEC with wave vector $\vec{k}=0$ was investigated within the frame of Bogoliubov theory of quasiaverages. The starting Hamiltonian describing the e-h system contains not only the Coulomb interaction between the particles lying on the LLLs but also the supplementary interaction due to their virtual quantum transitions from the LLLs to the ELLs and return back. This supplementary interaction generates after the averaging on the ground state BCS-type wave function the direct Hartree-type terms with attractive character, the exchange Fock-type terms giving rise to repulsion as well as the similar terms arising after the Bogoliubov $u-v$ transformation. The interplay of these three parameters gives rise to the resulting nonzero interaction between the magnetoexcitons with wave vector $\vec{k}=0$ and to stability of their BEC as regards the collapse. It influences also the energy spectrum as well as the collective elementary excitations. It consists of four branches. Two of them are excitonic-type branches, one of them being the usual energy branch whereas the other is the quasienergy branch representing the mirror reflection of the energy branch. The other two branches are the optical and acoustical plasmon branches. The exciton energy branch has an energy gap due to the attractive interaction terms, which is needed to be got over during the excitation as well as a roton-type section in the range of intermediary values of the wave vectors. At higher values of wave vector, its dispersion law tends to saturation. The optical plasmon dispersion law is gapless with quadratic dependence in the range of small wave vectors and with saturation-type dependence in the remaining part of the spectrum. The acoustical plasmon branch reveals the absolute instability of the spectrum in the range of small and intermediary values of the wave vectors. In the remaining range of the wave vectors, the acoustical plasmon branch exhibits a very small real value of the energy spectrum tending to zero in the limiting case of high wave vectors.

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