# SYNTHESIS OF CARTESIAN COORDINATES METROLOGICAL IMPEDANCE SIMULATORS 

V. Nastas<br>Department of Telecommunications, Technical University of Moldova, 168, Stefan cel Mare ave., MD-2004, Chisinau, Republic of Moldova<br>E-mail: vitalienastas@gmail.com<br>(Received 2 October 2008)


#### Abstract

This paper contains some problems of synthesis and analysis of the metrological simulators of impedance, suitable for using as the measures in impedance meters. There are presented two types of simulators for reproducing the impedances represented in Cartesian coordinates: with current control and with voltage control. The reproduced impedance can possess any character and have the separate regulation of the components. For synthesis of the simulator structures, the formal-structural method is applied. Studies of the stability conditions of the proposed simulators are carried out and the conditions of their optimal application in the resonance meters of impedance components are defined. The advantages of the proposed devices are the possibility of reproducing of impedance with any character without switching in its circuit, absence of the adjustable reactive elements, high exactitude and simplicity of practical realization. The presented devices are suitable for application both in the simple and cheap automatic impedance meters and in the high-accuracy meters of impedance components.


## 1. Introduction

For high accuracy measurement of the impedance components, the method of simultaneous comparison with measure in present is used. This method can be practically implemented in the bridge, compensating or resonance measuring circuits. Last studies in the field of the resonance method of measurement [1] made it possible to develop the variety of this method, which possesses unique possibilities. This relates to the method of simulated resonance, which was called so because in comparison with the classical resonance method, the reference element is simulated impedance [2-4]. There are known applications of this method in polar coordinates [3], for measuring the components of impedance [5] and admittance [6]. However, irrespective of the variant of their application, its distinctive special feature is application of the metrological simulator (or, converter) of impedance (MSI) as measure of the impedance components.

In comparison with traditional measures of impedance, for which there are used precision elements (resistors, capacitors, inductance coils) and also the adjustable boxes of these elements, MSI possess some of special features, among which can be mentioned the following:

- Low error rate and high stability of reproduced impedances;
- Possibility of reproduction of impedance with any character and the separate control of its components;
- The known and warranted value of systematic error;
- Digital control;
- Absence of switching elements for changing the character of the reproduced impedance;
- Simplicity of construction, small overall sizes and low cost.

At present, some authors take a shot at the development of the same devices [7, 8]. But the proposed devices are of special purpose and cannot be used as measure in the universal meters of the impedance components.

In the result of our research there is developed a class of MSI suitable for application in various impedance meters. Among them, we can mention the polar-coordinate [9, 10] and the Cartesian- coordinate [11, 12] MSI, MSI for reproduction of floating impedance [13], MSI with ladder structure. Among all types of MSI, the Cartesian-coordinates MSI with separate regulation of the components, suitable for application in the Cartesian-coordinates impedance meters should be mentioned.

Investigations of the accuracy of MSI [14] showed that their systematic error can be determined and under specific conditions it can have very low values (up to the values $\delta \sim$ $0.001 \%$ ). There are also determined the ways of increasing the MSI accuracy, depending on the conditions of their specific application. But, for examination of some problems of its synthesis and analysis, the conception of generalized impedance and admittance must be determined.

## 2. Generalized impedance and admittance

For effective analysis of the proposed devices, the conception of generalized passive value is used. It relates to the both known passive electrical values: impedance and admittance.

The classical impedance, as it is known, may be defined in the Cartesian and in the polar coordinates as

$$
\begin{equation*}
\mathrm{Z}=\mathrm{R}+\mathrm{j} \mathrm{X}=Z_{m} \exp (\mathrm{j} \varphi), \tag{1}
\end{equation*}
$$

where $\mathrm{R}, \mathrm{X}, Z_{m}, \varphi$ are the classically defined respective components. The domain of variation of these components is

$$
\begin{equation*}
\mathrm{R}=\{0 \div \infty\} ; \mathrm{X}=\{-\infty \div+\infty\} ; Z_{m}=\{0 \div \infty\} ; \varphi=\left\{0 \div \pm 90^{\circ}\right\} \tag{2}
\end{equation*}
$$

Analogously, for the classical admittance the same relations may be defined

$$
\begin{gather*}
\mathbf{Y}=\mathrm{G}+\mathrm{jB}=Y_{m} \exp (\mathrm{j} \psi),  \tag{3}\\
\mathrm{G}=\{0 \div \infty\} ; \mathrm{B}=\{-\infty \div+\infty\} ; Y_{m}=\{0 \div \infty\} ; \psi=\left\{0 \div \pm 90^{\circ}\right\} . \tag{4}
\end{gather*}
$$

As it is known, values (1)-(4) are represented graphically on the complex plan $\{ \pm \mathbf{1} ; \pm \mathbf{j}\}$ (Fig. 1a). The domain of definition of classical $\mathbf{Z}, \mathbf{Y}$ is the right semi plan limited by the axis $\pm \mathbf{j}$. Due to the duality of impedance and admittance, the mathematical apparatus used in conversions with its presence make it possible to analyze only the processes for the one of the, for example, for the impedance. The analysis for the other, e.g., for admittance, looks similarly.

Values (1)-(4) are well known in the classical technique for measurement of the impedance and the methods of its measurement are well developed.

The concepts of generalized impedance and admittance are wider and include, as a particular case, the classical concepts of these values. The definitions for them are the same as (1) and (3); however, they are different from the domain of definition of the components. For the generalized $\mathbf{Z}, \mathbf{Y}$

$$
\begin{align*}
& \mathrm{R}=\{-\infty \div+\infty\} ; \mathrm{X}=\{-\infty \div+\infty\} ; Z_{m}=\{0 \div \infty\} ; \varphi=\left\{0 \div 360^{\circ}\right\},  \tag{5}\\
& \mathrm{G}=\{-\infty \div+\infty\} ; \mathrm{B}=\{-\infty \div+\infty\} ; Y_{m}=\{0 \div \infty\} ; \psi=\left\{0 \div 360^{\circ}\right\} . \tag{6}
\end{align*}
$$

As it follows from (5) and (6), the domain of definition of generalized $\mathbf{Z}, \mathbf{Y}$ is the all complex plan $\{ \pm \mathbf{1} ; \pm \mathbf{j}\}$ (Fig. 1b). In the general case, an impedance $\mathbf{Z}$ (or an admittance $\mathbf{Y}$ ) contains a combination of positive or negative active component $\pm \mathrm{R}( \pm \mathrm{G}$, for admittance) with positive or negative reactive component $\pm \mathrm{j} \mathrm{X}( \pm \mathrm{jB}$, for admittance $)$.


Fig. 1. Representation of the classical impedance (a) and of the generalized impedance (b).
As it follows from above-stated, the conception of generalized passive value includes, in addition to the classical ones, a class of the values situated on the left of the axis $\pm \mathbf{j}$ on the complex plan. We must note that this definition is introduced only from the functional point of view, without affecting questions of energy supplying of these devices. In their physical essence, these values, strictly speaking, are not passive, but characterize the properties of objects with energy sources. This category includes the objects that possess properties of negative resistance (tunnel diodes, thyristors, and other electronic devices) and also a class of circuits with combined negative and positive feedbacks, used for the purpose of reproduction of the simulated impedances and admittances (impedance converters) [15]. The values reproduced by the impedance converters follow the same laws as the classical passive values, if the condition of guaranteeing their stability is satisfied. Thus, for the analysis of measuring circuits with their application, the methods which are used for analysis of the circuits with classical complex values can be used.

## 3. Synthesis of the MSI structure

As it is known [15], two basic types of impedance simulators are possible: the current controlled (I-MSI) and the voltage controlled (U-MSI). The both types of MSI are of a practical value, since I-MSI save the stability down to the condition of no-load operation; U-MSI, down to the condition of a short-circuit on its terminals. These properties make them suitable for application as impedance measures in the series (for I-MSI) and in the parallel (for U MSI) resonance measuring circuits, respectively [1].

For synthesis of the impedance simulator circuits, the method of formal synthesis based on the conversion algorithm of magnitudes in the synthesized device is used. As it will be shown later, this method allows, on the basis of the necessary conversion algorithm of the values, synthesizing its block diagram and, later, - the practical circuit of the device.

### 3.1. Synthesis of the current - controlled impedance simulator (I-MSI)

I-MSI is an impedance converter, which maintains the stability up to the regime of noload operation on its terminals [15]. As initial entering value for synthesis the I-MSI structure, the current $\mathbf{I}_{\mathbf{i}}$, flowing through the reproduced impedance is used. The impedance $\mathbf{Z}_{\mathbf{i}}$, reproduced by the simulator, should be represented in Cartesian coordinates in the form

$$
\begin{equation*}
\mathbf{Z}_{i}=\mathrm{R}_{\mathrm{i}}+\mathrm{j} \mathrm{X}_{\mathrm{i}}, \tag{7}
\end{equation*}
$$

where $R_{i}$ is the active component of reproduced impedance, $X_{i}$ is its reactive component. The components $R_{i}$ and $X_{i}$ must possess the property of independent control.

For synthesis of the block diagram of the simulator circuit, the diagram of information conversion algorithm presented in Fig. 2 is used.


Fig. 2. The algorithm of information conversion for I-MSI.
The entering current $\mathbf{I}_{\mathbf{i}}$ is converted into the voltage $\mathbf{U}_{\mathbf{1}}$, used for forming a voltage drops on the active $\left(\mathbf{U}_{\mathbf{R}}\right)$ and reactive $\left(\mathbf{U}_{\mathbf{X}}\right)$ components of the reproduced impedance $\mathbf{Z}_{\mathbf{i}}$. For forming the voltage $\mathbf{U}_{\mathbf{X}}$, the rotation of the phase of voltage $\mathbf{U}_{\mathbf{1}}$ by an angle of $90^{\circ}$, with consequent regulation of its magnitude by factor $N_{X}$ are used. The voltage $U_{R}$ is formed only by regulation the magnitude of $\mathbf{U}_{1}$ by the factor $N_{R}$. The voltages $\mathbf{U}_{R}$ and $\mathbf{U}_{\mathbf{X}}$ are summarized and form the voltage $\mathbf{U}_{\mathbf{i}}$, that, in conjunction with the entering current $\mathbf{I}_{\mathbf{i}}$, forms the reproduced impedance $\mathbf{Z}_{\mathbf{i}}$.

The above presented algorithm of the information conversion is implemented in the block - diagram of the simulator presented in Fig. 3.


Fig. 3. The block-diagram of I-MSI. IUC - current- to-voltage converter, DA - differential amplifier, PA1, PA2 - programmable amplifier, $\mathbf{9 0}^{\circ}$ - phase shifter (by $90^{\circ}$ ), SA - summing amplifier.

For conversion of the current $\mathbf{I}_{\mathbf{i}}$ into the voltage $\mathbf{U}_{\mathbf{1}}$ the current-voltage converter IUC is applied. The voltage $\mathbf{U}_{1}$ on its output [16] is

$$
\begin{equation*}
\mathbf{U}_{\mathbf{1}}=\mathbf{U}_{\mathbf{i}}-\mathbf{I}_{\mathbf{i}} \cdot \mathrm{R}, \tag{8}
\end{equation*}
$$

where R is the conversion factor of the converter IUC.
In order to obtain an algorithmically correct dependence between the current $\mathbf{I}_{\mathbf{i}}$ and the voltage $\mathbf{U}_{1}$, by elimination the effect of unnecessary common feedback of IUC [17], the differential amplifier DA is used. The voltage $\mathbf{U}_{1}{ }^{1}$ on its output is

$$
\begin{equation*}
\mathbf{U}_{1}{ }^{1}=\mathbf{U}_{\mathbf{i}}-\left(\mathbf{U}_{\mathbf{i}}-\mathbf{I}_{\mathbf{i}} \cdot R\right)=\mathbf{I}_{\mathbf{i}} \cdot R . \tag{9}
\end{equation*}
$$

The phasor and the programmable amplifiers PA1 and PA2 are used for introduction of the phase shift of $90^{\circ}$ and for regulation of the voltages $\mathbf{U}_{\mathbf{R}}$ and $\mathbf{U}_{\mathbf{X}}$. The voltages $\mathbf{U}_{\mathbf{R}}$ and $\mathbf{U}_{\mathbf{X}}$ formed with these elements amount, respectively, to

$$
\begin{gather*}
\mathbf{U}_{\mathrm{R}}=\mathrm{N}_{\mathrm{R}} \cdot \mathbf{U}_{\mathbf{1}}{ }^{1}=\mathrm{N}_{\mathrm{R}} \cdot \mathrm{R} \cdot \mathbf{I}_{\mathbf{i}},  \tag{10}\\
\mathbf{U}_{\mathbf{X}}=\mathrm{N}_{\mathrm{X}} \cdot \mathbf{U}_{\mathbf{1}}{ }^{1} \cdot \mathbf{j} \sin 90^{\circ}=\mathbf{j} \mathrm{N}_{\mathrm{X}} \cdot \mathrm{R} \cdot \mathbf{I}_{\mathbf{i}} . \tag{11}
\end{gather*}
$$

The summer amplifier $\mathbf{S A}$ summarizes the voltages $\mathbf{U}_{\mathbf{R}}$ and $\mathbf{U}_{\mathbf{X}}$ and forms the voltage $\mathbf{U}_{\mathbf{i}}$ applied to the input of the simulator

$$
\begin{equation*}
\mathbf{U}_{\mathbf{i}}=\mathbf{U}_{\mathbf{R}}+\mathbf{U}_{\mathbf{X}}=\mathrm{N}_{\mathrm{R}} \cdot \mathrm{R} \cdot \mathbf{I}_{\mathbf{i}}+\mathbf{j} \mathrm{N}_{\mathrm{X}} \cdot \mathrm{R} \cdot \mathbf{I}_{\mathbf{i}}=\mathrm{R}\left(\mathrm{~N}_{\mathrm{R}}+\mathbf{j} \mathrm{N}_{\mathrm{X}}\right) \mathbf{I}_{\mathbf{i}} . \tag{12}
\end{equation*}
$$

The impedance $\mathbf{Z}_{\mathbf{i}}$ reproduced by the simulator at its terminals is determined as

$$
\begin{equation*}
\mathbf{Z}_{\mathbf{i}}=\mathbf{U}_{\mathbf{i}} / \mathbf{I}_{\mathbf{i}}=\mathrm{R}\left(\mathrm{~N}_{\mathrm{R}}+\mathbf{j} \mathrm{N}_{\mathrm{X}}\right) . \tag{13}
\end{equation*}
$$

As follows from (13), the reproduced impedance $\mathbf{Z}_{\mathbf{i}}$ is represented in Cartesian coordinates and ensures the separate regulation of active and reactive components by changing the gain factors $\mathrm{N}_{\mathrm{R}}$ and $\mathrm{N}_{X}$ of the programmable amplifiers PA1 and PA2, respectively.

It also follows from (13) that if the band of variation of $\mathrm{N}_{\mathrm{R}}$ is located in the field of positive values and that of $\mathrm{N}_{\mathrm{X}}$ in the band of positive or negative values, the reproduced impedance can have the character of positive resistance in combination with inductive or capacitive component. The case when $\mathrm{N}_{\mathrm{R}}$ and $\mathrm{N}_{\mathrm{X}}$ have the range of change $\left(-\mathrm{N}_{0} \div 0 \div+\mathrm{N}_{0}\right)$ is more interesting. The area of variation of $\mathbf{Z}_{\mathbf{i}}$ in this case covers all complex plane; that is, $\mathbf{Z}_{\mathbf{i}}$ can have the character of a different combination of the positive or negative resistance with the capacitive or inductive impedance component (Fig. 4).


Fig. 4. Various character of the simulated impedance.
The circuit diagram of the analyzed impedance simulator is presented in Fig. 5.
The components of the simulator are implemented on the basis of operational amplifiers (OA) in the traditional connections; the precision resistors and capacitor are used as passive elements. The resistive DAC can be applied for assurance regulation of the programmable amplifier gain factor in the field of positive and negative values.


Fig. 5. The circuit diagram of I - MSI.

### 3.2. Synthesis of the voltage-controlled impedance simulator (U-MSI)

The difference between U-MSI and I-MSI consists in the fact that as input value for U MSI, the voltage drop on the reproduced impedance $\mathbf{Z}_{\mathbf{i}}$ is used. In addition to this, for the simulators of this type it is convenient to examine the admittance $\mathbf{Y}_{\mathbf{i}}$ instead of the impedance $\mathbf{Z}_{\mathrm{i}}$. The procedure of synthesis of the U-MSI structure is analogous to the above examined procedure of synthesis of I-MSI, taking into account replacement of the corresponding values. The diagram of information conversion algorithm in this simulator is presented in Fig. 6a; the synthesized block diagram of U-MSI, in Fig. 6b.


Fig. 6. The algorithm of information conversion in U-MSI (a) and the block-diagram of U-MSI (b).
The input voltage $\mathbf{U}_{\mathbf{i}}$ is transferred by two separate channels for the formation of initial values for active and reactive components of the reproduced impedance. Conversion of values in the channels is analogous to conversion in I-MSI. These voltages undergo regulation in the programmable amplifiers PA1 and PA2 and also, to the phase rotation by an angle of $90^{\circ}$ (for reactive component)

$$
\begin{gather*}
\mathbf{U}_{\mathbf{R}}=\mathbf{N}_{\mathbf{R}} \cdot \mathbf{U}_{\mathbf{i}},  \tag{14}\\
\mathbf{U}_{\mathbf{X}}=\mathbf{N}_{\mathbf{X}} \cdot \mathbf{U}_{\mathbf{i}} \cdot \mathrm{j} \sin 90^{\circ}=\mathrm{j} \mathbf{N}_{\mathbf{X}} \cdot \mathbf{U}_{\mathbf{i}} . \tag{15}
\end{gather*}
$$

The voltages $\mathbf{U}_{\mathbf{R}}$ and $\mathbf{U}_{\mathbf{X}}$ command a voltage-current converter DUIC with differential entrance for formation of the input current $\mathbf{I}_{\mathbf{i}}$

$$
\begin{equation*}
\mathbf{I}_{\mathbf{i}}=\left(\mathbf{U}_{\mathbf{R}}-\mathbf{U}_{\mathbf{X}}\right) \mathrm{G}=\mathrm{G}\left(\mathrm{~N}_{\mathrm{R}}-\mathbf{j} \mathrm{N}_{\mathrm{X}}\right) \mathbf{U}_{\mathbf{i}}, \tag{16}
\end{equation*}
$$

where G is the conversion factor of DUIC.
The obtained current $\mathbf{I}_{\mathbf{i}}$ is introduced into the input circuit of the simulator and, together with the input voltage $\mathbf{U}_{\mathbf{i}}$, reproduces on the input terminals the simulated admittance $\mathbf{Y}_{\mathbf{i}}$

$$
\begin{equation*}
\mathbf{Y}_{\mathbf{i}}=\mathbf{I}_{\mathbf{i}} / \mathbf{U}_{\mathbf{i}}=\mathrm{G}\left(\mathbf{N}_{\mathrm{R}}-\mathbf{j} \mathbf{N}_{\mathrm{X}}\right) \tag{17}
\end{equation*}
$$

As follows from (17), the simulated admittance $\mathbf{Y}_{\mathbf{i}}$, as the simulated impedance of I-MSI $\mathbf{Z}_{\mathbf{i}}$, is represented in Cartesian coordinates and has a separate regulation of its active and reactive components. Regulation is assured by variation of the gain factors $\mathrm{N}_{\mathrm{R}}$ and $\mathrm{N}_{\mathrm{X}}$ of the programmable amplifiers PA1 and PA2 in the value range ( $-\mathrm{N}_{0} \div 0 \div+\mathrm{N}_{0}$ ), that ensure the domain of variation of $\mathbf{Y}_{\mathbf{i}}$ similarly as for $\mathbf{Z}_{\mathbf{i}}$ in I-SIM.

The circuit diagram of synthesized U-MSI is shown in Fig. 7. It is analogous to I-MSI, yet the blocks on the basis of operational amplifiers are used. On the OA $\mathrm{A}_{1}$ the voltage repeater for increasing the input impedance is realized, on the basis of $\mathrm{A}_{2}, \mathrm{~A}_{4}$ - the programmable amplifiers PA1 and PA2 and on the basis of $A_{3}$ - the phase shifter by $90^{\circ}$ Variation of the values $\mathrm{N}_{\mathrm{R}}$ and $\mathrm{N}_{\mathrm{X}}$ in the domain ( $-\mathrm{N}_{0} \div 0 \div+\mathrm{N}_{0}$ ) is carried out by means of the variable resistors $R_{1}$ and $R_{7}$ respectively. If it is necessary, they can be replaced with DAC. The differential voltage-to-current converter on the basis of OA A ${ }_{5}$ produces the entering current $\mathbf{I}_{\mathbf{i}}$ flying through the reproduced admittance $\mathbf{Y}_{\mathbf{i}}$.


Fig. 7. The circuit diagram of $\mathrm{U}-\mathrm{MSI}$.

## 4. Stability of the Cartesian-coordinates impedance simulators

The problem of MSI stability is rather important and requires a complex analysis. In [17] the analysis of stability for the polar coordinates MSI is carried out. The similar requirements for guaranteeing the stability are made to the Cartesian coordinates MSI.

The standard MSI circuit consists of several links based on OA and contains local and common feedback loops, and also contains a phase shifter and external impedance connected to OA terminals. Thus, as it is known [16], OA stability is affected by the following factors:

- Nonideality of OA performances at high frequencies;
- OA properties at the direct current;
- Character of impedances included in the feed - backs and the phase angle introduced by the phase shifter;
- Character and magnitude of the external impedance connected to the input of MSI.

As follows from the theory of OA [16], it is necessary to consider three types of stability:

- Stability on the direct current,
- Stability on the high frequencies,
- The functional stability.

Stability at the direct current may be interrupted because of appearance of trigger effect [16] at excess of the depth of positive feedback above the negative one. To ensure the absolute stability at the direct current, it is necessary to ensure a negative character of the common feedback at variation of parameters of the circuit and of external resistance in the all operating range. Obviously, for the considered circuit, this condition will be respected when the summary transfer factor at a direct current through the common feedback loop $\mathrm{K}_{0 \text { summ }}$ will have a negative value. It is achieved by combination of inverting and noninverting connection of all circuit units at the direct current.

Stability at the high frequencies is ensured by correct frequency correction of the OA characteristic in all circuit units, considering the MSI structure as a multisection amplifier with common feedback [16].

The functional stability of MSI represents the more complex problem. The following factors render influence on the condition of functional stability

- Type of MSI;
- Values of the module and phase of the simulated impedance;
- Character and value of the external impedance connected to the terminals of MSI.

Thus, the conditions of functional stability should be determined for each type of MSI separately, depending on the band of variation of the simulated impedance components and on the value and character of the external impedance connected to its terminals.

For estimation the conditions of the functional stability, we shall take advantage to Ny quist criterion in application to circuits containing the operational amplifiers [16]. As it is known, the circuits with feedbacks maintain the stability if the critical point $(-1,+\mathrm{j} 0)$ is located to left of the hodograph of the transfer characteristic on the loop of the feedback at frequency change from $\mathrm{f}=0$ up to $\mathrm{f} \rightarrow \infty$ (Fig. 8).

Thus, for estimation of the stability conditions for the circuits according to the Nyquist criterion, it is necessary to determine its loop gain factor $\boldsymbol{\beta} \mathbf{A}$ and to examinee it in neighborhood of the critical point $(-1,+\mathrm{j} 0)$ in coordinates $\operatorname{Re}(\beta \mathbf{A})$, $\operatorname{Im}(\beta \mathbf{A})$. The condition of functional stability for the MSI circuit looks like

$$
\begin{equation*}
\boldsymbol{\operatorname { R e }}\left[\mathrm{H}_{\mathrm{io}}\right]>-1, \tag{18}
\end{equation*}
$$

where $\mathrm{H}_{\mathrm{io}}$ is the loop gain factor for MSI circuit.

b)

Fig. 8. Hodograph of loop gain factor for stable (a) and unstable (b) circuits.
Figure 9a presents the model of I-MSI for determining the stability conditions. Through the equivalent differential amplifier with the gain factor $\mathbf{A}$, the portion of the circuit of I-MSI, limited by the dotted line (Fig. 3) is designated.
a)


b)

Fig. 9. The models of I-MSI (a) and U-MSI (b) for stability analysis.
For determining the expression of loop gain factor $\mathbf{H}_{24}$ from point 1 to point 2 we will use the method of the conductance matrices [14]. The general matrix of I-MSI circuit has the form

$$
Y=\begin{array}{|l|l:l:l|}
\hline & \mathbf{1} & \mathbf{2} & \mathbf{3}  \tag{19}\\
\hline \mathbf{1} & \boldsymbol{Y}_{e}+\boldsymbol{Y} & \mathbf{0} & -\boldsymbol{Y} \\
\hline \mathbf{2} & \mathbf{0} & \boldsymbol{Y}_{1} & \mathbf{0} \\
\hdashline \mathbf{3} & -\mathbf{1} & +\mathbf{1} & \mathbf{0} \\
\hline
\end{array}
$$

where $\mathbf{Y}$ stands for the same corresponding values of $\mathbf{Z}$.
The loop gain factor for I-MSI is

$$
\begin{equation*}
\mathbf{H}_{24}=\mathbf{U}_{\mathbf{0}} / \mathbf{U}_{\mathbf{i}}=\left(\boldsymbol{\Delta}_{23} / \boldsymbol{\Delta}_{22}-1\right) \mathbf{A}, \tag{20}
\end{equation*}
$$

where $\Delta_{23}$ are $\Delta_{22}$ are the cofactors of the matrix (19). Their values are

$$
\Delta_{23}=(-1)\left|\begin{array}{cc}
Y_{e}+\boldsymbol{Y} & 0  \tag{21}\\
-1 & +1
\end{array}\right|=-\left(Y_{e}+Y\right), \quad \Delta_{22}=(+1)\left|\begin{array}{cc}
Y_{e}+\boldsymbol{Y} & -\boldsymbol{Y} \\
-1 & 0
\end{array}\right|=-\boldsymbol{Y} .
$$

After substitution of (21) into (20), $\mathbf{H}_{24}$ takes the form

$$
\begin{equation*}
\mathbf{H}_{24}=\mathbf{A}\left[\left(\mathbf{Y}_{\mathbf{e}}+\mathbf{Y}\right) / \mathbf{Y}-1\right]=\mathbf{A} \cdot \mathbf{Z} / \mathbf{Z}_{\mathbf{e}} \tag{22}
\end{equation*}
$$

A stands for the summary gain factor of the I-MSI circuit (Fig. 3). It is

$$
\begin{equation*}
\mathbf{A}=\mathrm{K}_{\mathrm{d}} \cdot\left(\mathrm{~N}_{\mathrm{R}}+\mathbf{j} \mathrm{N}_{\mathrm{X}}\right) . \tag{23}
\end{equation*}
$$

Taking into account that in the real case $\mathbf{Z} \equiv R, \mathbf{Z}_{\mathbf{e}}=R_{e}+\mathbf{j} X_{e}$ and $K_{d}=1$, (22) takes the form

$$
\begin{equation*}
\mathbf{H}_{24}=\mathrm{R}\left(\mathrm{~N}_{\mathrm{R}}+\mathbf{j} \mathrm{N}_{\mathrm{X}}\right) /\left(\mathrm{R}_{\mathrm{e}}+\mathbf{j} \mathrm{X}_{\mathrm{e}}\right) . \tag{24}
\end{equation*}
$$

Let us apply Nyquist criterion (18) to (24), and let us assume that the range of values for $\mathrm{N}_{\mathrm{R}}$ comprises $\left(-\mathrm{N}_{\mathrm{R} 0} \div 0+\mathrm{N}_{\mathrm{R} 0}\right)$ and for $\mathrm{N}_{\mathrm{X}}:\left(-\mathrm{N}_{\mathrm{X} 0} \div 0 \div+\mathrm{N}_{\mathrm{X} 0}\right)$. Then, stability condition (18) for I-MSI in the most critical case $\left(N_{R}=-\mathrm{N}_{\mathrm{R} 0}, \mathrm{~N}_{\mathrm{X}}=-\mathrm{N}_{\mathrm{X} 0}\right)$ takes the form

$$
\begin{equation*}
\mathrm{R}\left(\mathrm{~N}_{\mathrm{R} 0} \mathrm{R}_{\mathrm{e}}+\mathrm{N}_{\mathrm{X} 0} \mathrm{X}_{\mathrm{e}}\right)<\mathrm{R}_{\mathrm{e}}^{2}+\mathrm{X}_{\mathrm{e}}^{2} . \tag{25}
\end{equation*}
$$

But, $\mathrm{R}_{\mathrm{e}}{ }^{2}+\mathrm{X}_{\mathrm{e}}{ }^{2} \equiv\left|\mathbf{Z}_{\mathrm{e}}\right|^{2}, \mathrm{RN}_{\mathrm{R} 0} \approx \mathrm{R}_{\mathrm{X} \text { max }}, \mathrm{RN}_{\mathrm{X} 0} \approx \mathrm{X}_{\mathrm{X} \text { max }}$, where $\mathrm{R}_{\mathrm{X} \text { max }}$ and $\mathrm{X}_{\mathrm{X} \text { max }}$ are the maximal values of active and reactive components of the measured impedance, $\left|\mathbf{Z}_{\mathrm{e}}\right|$ is the modulus of the external impedance. The folowing condition of the absolute stability for I-MSI rersults from (25)

$$
\begin{equation*}
\left|\mathbf{Z}_{\mathrm{e}}\right|^{2}>\mathrm{R}_{\mathrm{e}} \mathrm{R}_{\mathrm{X} \max }+\mathrm{X}_{\mathrm{e}} \mathrm{X}_{\mathrm{X} \text { max }} . \tag{26}
\end{equation*}
$$

It follows from condition (26) that the circuit of I-MSI preserves the absolute stability in all domain of variation of the components $\mathrm{R}_{\mathrm{X}}$ and $\mathrm{X}_{\mathrm{X}}$, if the following conditions are satisfied

$$
\begin{equation*}
\left|\mathbf{Z}_{e}\right|>R_{X_{\text {max }}},\left|\mathbf{Z}_{\mathbf{e}}\right|>X_{X_{\text {max }}} . \tag{27}
\end{equation*}
$$

As follows from [1], conditions (27) are satisfied easily in the series resonance measuring circuit. Thus, condition (27) determines the field of application of I-MSI.

For the model of U-MSI presented in Fig. 9b, the analysis of stability can be executed analogously. Its loop gain factor is

$$
\begin{equation*}
\mathbf{H}_{16}=-\mathbf{A} \mathbf{Z}_{e} / R_{1}=-\left(N_{R}+\mathbf{j} N_{X}\right)\left(R_{e}+\mathbf{j} X_{e}\right) / R_{1} . \tag{28}
\end{equation*}
$$

Applying condition (18) to (28) and admitting the domain of variation of $N_{R}$ and $N_{X}$ analogous to I-MSI, the stability condition for U-MSI for the extreme critical values of simulated impedance components takes the form

$$
\begin{equation*}
\mathrm{N}_{\mathrm{R} 0} \mathrm{R}_{\mathrm{e}}+\mathrm{N}_{\mathrm{X} 0} \mathrm{X}_{\mathrm{e}}<\mathrm{R}_{1} . \tag{29}
\end{equation*}
$$

It follows from expression (29) that the conditions of stability for U-MSI are fulfilled automatically if it is used in the parallel resonant circuit with voltage generator having the zero internal resistance as source of measuring signal [6].

The problems of the error analysis for impedance simulators on the basis of operational amplifiers were investigated in [14]. As it follows from [14], the greatest influence on the error of the given value of the reproduced impedance is exerted by the factors of nonideality of the operational amplifier and, in particular, the limited value of the gain factor and its frequency dependence. For minimization of this component of the error, the operating frequency of measuring signal must not strongly exceed the frequency of pole of the operational amplifier characteristic [16]. The acceptable error $\delta \leq 0.1 \%$ was obtained at the operating frequency $\mathrm{F}=100 \mathrm{~Hz}$.

## 5. Conclusions

Application of the Cartesian-coordinates impedance simulators as reference elements in the Cartesian - coordinate impedance meters ensures the high accuracy and simple algorithm of measurements. The separate control of reproduced impedance components is an indispensable condition for this.

By algorithmical synthesis, having accepted as a basis the necessary conversion algorithm of information, the current controlled and the voltage controlled impedance simulators are designed.

The simulators have the separate control of the impedance components and ensure reproduction of the impedance with any character, without use of the variable reactive elements and commutations in the circuit.

As follows from the stability analysis, the current controlled impedance simulator maintain absolute stability in the series resonant circuits with current source of measuring signal; the voltage controlled simulator, in the parallel resonant circuits with voltage source of measuring signal.

## References

[1] V. Nastas and M. Scanteianu, Masurarea impedantei prin metoda de rezonanta, Meridian ingineresc, Chisinau, "Tehnica-Info", 2, 70, (2001).
[2] V. Nastas and M. Scanteianu, "Impedance measurement by method of simulated resonance", Proceedings of the $8^{\text {th }}$ Int. Conf. OPTIM 2002, Brasov, 3, 683, (2002).
[3] V. Nastas, Method of impedance components measurements. Patent MD 2086, Chisinau, (2003).
[4] V. Nastas, Method of impedance components measurements. Patent MD 2509, Chisinau, (2004).
[5] V. Nastas, Device for measuring the impedance components. Patent MD 2248, Chisinau, (2003).
[6] V. Nastas, Device for measuring the admitance components. Patent MD 2463, Chisinau, (2004).
[7] A. Nordeng, Impedance synthesizer. European Patent 0656542 A2, European Patent Office, (1995).
[8] J.C.P. Liu, C.K. Tse, F.N.K. Poon, M.H. Pong, and Y.M. Lai, General Impedance Synthesizer Using Minimal Configuration of Switching Converters, Proceedings ECCTD 2005, Cork Ireland, 1, 173, (2005).
[9] V. Nastas and Scanteianu M, Impedance converter. Patent MD 2130, Chisinau, (2003).
[10] V. Nastas and A. Cazac, Impedance converter. Patent MD 2462, Chisinau, (2004).
[11] V. Nastas and A. Nastas, Impedance converter. Patent MD 3154, Chisinau, (2006).
[12] V. Nastas, Admitance converter. Patent MD 3111, Chisinau, (2006).
[13] V. Nastas, Impedance converter. Patent MD 3173, Chisinau, (2006).
[14] V. Nastas, Simulator de impedanta metrologic, Meridian ingineresc, "Tehnica-Info", Chisinau, 3, 49, (2003).
[15] F. Bening, Negative viderstande in elektronischen schaltungen, Veb Verlag, Berlin, 288, 1971.
[16] J. Dostal, Operational amplifiers, Elsevier SPC, New York, 512, 1981.
[17] V. Nastas, Polar-coordinates impedance simulators and polar-coordinates impedance meter. Proceedings of the Int. Conf. OPTIM - 2004, Brasov, 4, 89, (2004).

