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Influence functions, integral formulas, and explicit solutions for thermoelastic spherical wedges

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Abstract In this study, new exact Green's functions and a new exact Green-type integral formula for a boundary value problem (BVP) in thermoelasticity for some spherical wedges with mixed homogeneous mechanical boundary conditions are derived. The thermoelastic displacements are subjected to a heat source applied in the inner points of the spherical wedges and to a mixed non-homogeneous boundary heat conditions. When the thermoelastic Green's function is derived, the thermoelastic displacements are generated by an inner unit point heat source, described by Dirac's δ-function. All results are obtained in elementary functions that are formulated in a special theorem. Exact solutions in elementary functions for two particular BVPs of thermoelasticity for spherical wedges also are included. In these particular BVPs, the thermoelastic displacements are subjected to a constant temperature (in the first particular BVP) or to a constant heat source (in the second particular BVP). In both BVPs, the constant temperature or the constant heat source is given on the segment of the radius of the quarter-space. On the boundary half-planes of the quarter-space zero temperature and zero heat flux are prescribed.

1 Introduction

The main objective of this paper is to prove a theorem (Sect. 2) on the derivation of thermoelastostatic Green functions (Sect. 2.2) and a Green-type integral formula (Sect. 2.3) for a homogeneous isotropic 3D spherical wedge $V(0 \leq r < \infty; 0 \leq \varphi \leq \alpha; 0 \leq \beta \leq \pi)$; $\alpha = \pi/n$; $n = 2, 3, 4 \ldots$, which is bounded by the half-planes $\Gamma_{\varphi 0}(0 \leq r < \infty;\ \varphi = 0; 0 \leq \beta \leq \pi)$ and $\Gamma_{\varphi\alpha}(0 \leq r < \infty; \varphi = \alpha; 0 \leq \beta \leq \pi)$, where spherical coordinates r, φ, β are used. When deriving the Green-type integral formula, on the boundary half-planes $\Gamma_{\varphi 0}, \Gamma_{\varphi\alpha}$ the homogeneous mixed mechanical boundary conditions are given (zero normal displacements and zero tangential stresses on $\Gamma_{\varphi 0}$ and zero normal stresses and zero tangential displacements on $\Gamma_{\varphi\alpha}$). The searched thermoelastic field of displacements is created by the inner heat source $F(M)$, $M(r, \varphi, \beta) \in V$, by non-homogeneous Dirichlet's boundary conditions for the temperature T, and by non-homogeneous Neumann's

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