



LOAD-BEARING CAPACITY OF DAMAGED REINFORCED CONCRETE SPAN STRUCTURES STRENGTHENED WITH PRESTRESSED METAL CASINGS

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Abstract. This article deals with the results of system experimental research of the load-bearing capacity, stress-strain behaviour and crack resistance of damaged 2,000x200x100 mm reinforced concrete beams with prestressed external metal casings that have been subjected to high-level static and low-cycle alternate loading. Specifically, the test-statistical curves illustrates strengths of the inclined cross-sections (collapsing transverse force), sagging deflections, widths of the normal and inclined cracks before collapse of the beams, projection lengths of the dangerous inclined cracks on the longitudinal axis and the average distances between normal cracks along the element lengths versus the design and external factors. Comparative analysis was made to check the impact of the main design and external factors on the indicated load-bearing capacity parameters of the common reinforced concrete beams subjected to a single-time stepwise increasing (series 1) and high-level low-cycle alternate loading (series 2) as well as to the strengthened with metal casings beam elements that have been damaged through normal and cross inclined cracks in the course of previous tests (series 3) subjected to a repeated action of similar low-cycle alternate loading. All tested beam samples were manufactured and tested in accordance with the experimental design theory and D-optimum plan of Box B4.

Keywords: *damaged reinforced concrete beam, prestressed metal casings, low-cycle alternate loading.*

Introduction

Every day the engineers face new challenges that are connected with higher seismicity in the areas where buildings and facilities are located, their physical wear and damage, changes of their functional purpose, higher loading, etc. For this reason they have,

from time to time, to solve complicated technical problems associated with replacement or reinforcement of the damaged structures. Replacement of the damaged reinforced concrete structures requires considerable labour resources, financial costs and long-time suspension of production. Along with that, analysis of the damaged critically important reinforced concrete span structures of a facility or a building has shown, on the whole, that in the course of their operation it is observed a considerable damage of the protective concrete layer, corrosion of the reinforcement, appearance and excessive opening of cracks, achievement of the non-permissible deflections, etc.

Further experience has shown that operation and the life cycle of the damaged structures can be extended due to their reinforcement and renewal during total renovation and without suspension of the technological process.

Reinforced concrete structures can be strengthened with the aid of reinforced concrete or soluble mixtures which, besides increase of their weight and other known deficiencies, have such drawbacks: a reinforcement system can commonly work with the damaged element only after the materials have acquired the necessary strength in the reinforced cross-sections; and absence of full control of the joint performance of the damaged structure and the reinforcement system.

Recently, it is made use of reinforcement structures with glued composite materials. However, high cost of such materials, absence of the direct control of the reinforcement quality, sensitivity to temperature effects, a possibility of support area scaling and imperfection of the existing methods of their calculation hold back a chance for the wider application of this advanced, on the whole, method to reinforce the damaged structures.

Within operation the majority of span reinforced concrete structures are subject to cyclic or low-cycle repeated and alternate loads which arise within the operational level limit and, sometimes, even exceed it. Load reversal, change of its level and indefinite number of repetitions in the course of operation often lead to the consequences that qualitatively differ from those obtained at the stage when the permanent, not alternate loading of maximum intensity was calculated, which the most effective standards provide for. There is no alternative for reinforcement of the damaged through normal and inclined cross and excessively wide cracks in reinforced concrete beam structures which, in the course of their above performance, have reached the limit or ultimate limit stage resulted from the increasing cyclic high-level loading without suspension of the technological production process with the aid of prestressed metal casings. However, design of such reinforcement is held back by absence of the normative methodology and clear cut guidelines in the authors' methodologies that adequately reflect the real stress-strain condition both of the damaged structure and the elements of its reinforcement. That is why the topicality of this work is evident.

Review of recent sources of publications

It was experimentally established [1] that the low-cycle, alternate and non-alternate high-level loads not only reduces the load-bearing capacity of the test reinforced concrete beams to 20%, and their crack resistance, considerably increases width of normal and, particularly, inclined cracks and sagging deflections to 35% but also changes the nature of their collapse as compared with a single time static proportionally increasing loading. With the multiple cyclic loading under unfavourable conditions the absolute limit of concrete and reinforcement endurance decreased to 50% of their design resistance which leads to

ever increasing cracking resistance and reduction of the load-bearing capacity of such structures, their deformability increases and the collapse process is accelerated.

It is evident that in order to reinforce concrete structures that have reached the limit or ultimate condition it is necessary to use metal as one of the most well studied materials according to its stable strength and physical and chemical characteristics which can be easily controlled. Doing so, presence of plastic properties in steel facilitates re-distribution of inner forces between the reinforcement elements and the damaged structure.

Let us consider the existing methods to reinforce the damaged concrete structures with the use of steel arrangements. The known method deals with strengthening of the reinforced concrete structure [2] by way of attaching additional reinforcement to the naked working rods of the tensioned zone, its previous tension "on concrete" and further coating with concrete. However, this method provides for, first, compressing of the tensioned zone, which leads to resistance of only the tensioned zone that is subject to bending, i.e., the damaged structure is not fully reinforced which results in partial and is not always acceptable (at higher static, fluctuating or seismic impacts) increase of the load-bearing capacity of the structure being reinforced.

Among many other methods it is worthwhile to mention a method [3] that provides for a fixture of the reinforcing element within the compressed zone of the reinforced concrete beam followed by its two-stage compression before and after prestressing of the strengthening element of the tensioned zone.

Still, this method allows to strengthen the normal cross-sections of the whole (containing no cracks) reinforced concrete beam against the non-alternate transverse loading or the bending moment in the main vertical force plane, and does not lead to strengthening of inclined cross-sections and support areas, as a whole, against the action of the same non-alternate loading. In case of the alternate transverse loading (e.g., during an earthquake) this method may, on the contrary, result in a premature collapse caused by both normal previously tensioned upper reinforcement elements and normal cross-sections and by cross inclined cross-sections.

Prof. V.V. Pynadjan [4] was one of the first in the Soviet Union who studied a possibility to apply various arrangements to reinforce the span reinforced concrete structures. Specifically, he established that reinforcement of the damaged and loaded T-shape reinforced concrete beam with the reinforced soluble "casing" improved its load-bearing capacity by 42% while reinforcement of a similar beam with the aid of prestressed longitudinal steel rods improved it by 8%, its simultaneous reinforcement with the aid of two above described arrangements resulted in improvement by 71% and reinforcement of the beam with prestressed U-shaped clamps in the support areas only – by 10%. So as to join the old concrete with a new one the author [4] recommends installation of additional metal links in order to prevent their mutual displacement. The author asserted that reinforcement of the tensioned zone of the structure with prestressed longitudinal reinforcement in kind of camber rods presents one of the most effective arrangements.

This conclusion has been confirmed by testing [5] reinforced concrete beams strengthened with prestressed metal clamps that indicated simultaneous increase of their load-bearing capacity and stiffness in parallel with the increase of the transverse cross-section of the clamps. The author tested [5] the reinforced two-span beams and the result showed that the prestressed clamps turned the beams into eccentrically compressed elements and that there is a dependence of re-distribution of the inner forces in the beam

versus the value of the compressing forces produced by clamps. These tests also confirmed expediency of the vertical transverse rods prestressing.

Detailed qualification of the most spread causes and conditions of the building structures that emphasize the necessity of their reinforcement as well as the methods of such reinforcement is described in [6]. It contains a full set of diverse arrangements for strengthening reinforced concrete structures and became an indispensable handbook of many experts in reconstruction. However, as it appeared, the attached atlas of diagrams and drawings [6] still does not contain the arrangements aimed at reinforcement of the dangerous structures against the action of cyclic alternate increasing loading.

It is known the design of the arrangement [7] for strengthening a reinforced concrete beam with a metal sheet glued to its lower surface and with linked to the sheet prestressed horizontal elements of two flexible ties on the side surfaces; the stress in the ties is created by tightening (stressing) the vertical elements anchored in the upper compressed zone. However, with the aid of the above described construction it is possible to reinforce the lower tensioned zone only and the support areas of the beam against permanent or non-alternate cyclic transverse loading only.

An original device for strengthening building structures subject to bending [8] presents a stiff wavelike reinforcing element, stressed vertical clamps, stressed reinforcement and support rollers. The reinforcing element is fixed at its ends and in the contact points to the being bent reinforced structure on the side of the compressed zone. Longitudinally stressed reinforcement is fixed in the ends of the structure being reinforced and the support rollers are located between the stressed reinforcement and the tensioned surface of the structure. The height of the wavelike element crest reinforcement is made so that it decreases from the middle part towards the ends of the structure being reinforced and encompasses the stressed reinforcement between the support rollers. The clamp is made as a shackle that has threads at its ends for a bar that is fixed with nuts. When the nuts are tightened, the clamps and the rods are tensioned. This construction is clamped from its middle towards the ends. Doing so, the upper compressed zone is partially unloaded while the lower reinforcement appears a moment of the sign opposite to the moment produced by external loading. Therefore, this device makes it possible to strengthen the upper compressed and the lower tensioned zones as well as, partially, the support areas. As a result, the load-bearing capacity increases, however, in case of the transverse loading of one sign only.

It is of interest to reinforce the compressed concrete and bent elements with the aid of steel sheets fixed to their sides with bolt connections [9]. In this work the authors established a link between sliding of the bolt connection and the load-bearing capacity of the beam. Taking into consideration the cyclic nature of loading, the authors [10, 11, 12] stress both a positive quality of this device performance and a plastic character of the strengthened structure collapse.

The performed experimental studies [13] of a not damaged reinforced concrete beam strengthened by gluing a steel strip onto it have shown an increment of its load-bearing capacity to 300%. The authors [13] proceed with their study of the strip end fixture so as to fully use its strength and preclude crushing of the beam.

The authors [14] tested four beams: the first one – without strengthening; the second – strengthened on the bottom in the tensioned zone with glued steel strip; the third – strengthened in the compressed and tensioned zones with similar steel strips without

gluing them to the beam; and the fourth – strengthened with the same strips glued on the top and on the bottom of the beam with the aid of epoxy glue. All strengthened beams had the same anchoring of the steel plates on their ends. The higher load-bearing capacity within the range of 130...160% was achieved, the deformability of the beams strengthened with epoxy glue was reduced same as the plastic nature of their collapse. And the beam strengthened with steel plates without their gluing was working as the tensioned beam and had higher deformation property.

Experimental study [15] of three reinforced concrete beams strengthened with steel plates installed in the tensioned zone and fixed with the aid of end anchors and epoxy glue have shown the higher load-bearing capacity that was reaching 48%. In paper [16] it was indicated that application of the similar strengthening arrangement (though with the other anchor design) resulted in the higher load-bearing capacity by 10% only as compared with the beam without anchors.

The authors of [17] who performed experimental study of the new strengthening arrangement – an external rod-and-roller strut bracing have achieved to 4.5 times greater load-bearing capacity and considerably reduced the deformability.

At the moment of strengthening building structures are basically subjected to loading value, according to [18], defines reliability of the strengthened reinforce concrete element. As a result of the study of the stressed state of reinforced concrete beams by extension of tensioned rod reinforcement under loading, the authors [19] proposed two new methodologies to evaluate reliability of the strengthened reinforced concrete elements.

Definition of the earlier unresolved parts of the general problem

The national design norms and the known authors' methodologies do not contain clear guidelines for calculating joint performance of the damaged and brought into a state of limit or dangerous condition reinforced concrete structures with the elements of the prestressed metal elements.

According to the author's methodologies design of the metal elements that strengthen reinforced concrete structures is accomplished in compliance with the existing provisions of the effective design norms which calculation formulas use installation and transverse reinforcement, the longitudinal, transverse and inclined strengthening elements without sufficient account of their interaction with the damaged structures and their real technical condition, levelling, necessity of straightening and shaping their initial form in the majority of instances.

Analysis of the available publications has proved that the existing methods to strengthen span reinforced concrete structures possess the following common features:

- fixture of the strengthening elements on the damaged beam;
- prestressing of the beam in the longitudinal and vertical transverse direction.

The arrangements that are used for strengthening span structures have the following common features:

- longitudinal strengthening elements;
- transverse vertical strengthening elements;
- transverse vertical strengthening elements that are connected to the longitudinal strengthening elements.

The drawbacks of the above described methods and arrangements for strengthening reinforced concrete beam structures are:

- strengthening normal (more frequently) and inclined (not so frequently) cross-sections of the whole reinforced concrete beam against the action of the non-alternate transverse loading in the main plane only. Under impact of the alternate loading the majority of the existing methods and arrangements may lead to a premature destruction of the studied elements;

- impossibility to renew or improve, if necessary, the load-bearing capacity of the dangerous or brought to the limit state beam structures subjected to the high-level alternate cyclic or fluctuating loading or seismic impacts as well as their impossibility to redistribute inner forces in the whole beams.

Comparison of the test data pertaining to the load-bearing capacity both of common and strengthened with metal casings reinforced concrete beams damaged through normal and cross inclined cracks that are subjected to the high-level low-cycle alternate loading with the results of the calculations made according to the available authors' methodologies has shown their, on the whole, unsatisfactory convergence both for common and for strengthened test beams. At that, the main portion of the publications deals with determination of the load-bearing capacity of the normal cross-sections of common beams and the beams strengthened with metal casings on the basis of the first and second groups of boundary conditions while the strength of their inclined cross-sections and of the support area remain hardly studied at all.

Goal of the work and tasks of the study

This work is aimed at studying, both experimentally and theoretically, the load-bearing capacity, crack resistance and deformability of the damaged through normal and cross inclined cracks reinforced concrete beams strengthened with prestressed metal casings against the action of the high-level low-cycle alternate loading with a view to improve the existing and of further development of a new physical and mathematical model of the load-bearing capacity of the normal and inclined cross-sections of the span reinforced concrete structures, damaged during military activity and strengthened with the aid of metal casings, that are subjected to the indicated loading by similarity with the earlier developed similar models.

Tasks of the study:

- to develop an ingenious method to renew and strengthen the damaged through normal and cross inclined cracks reinforced concrete beams and the arrangement for its achievement, as well as to receive an appropriate patent for the invention, if possible;

- to study the stress-strain behaviour, load-bearing capacity, opening width of the normal and inclined cracks, spans, nature of destruction of the common whole and strengthened with prestressed metal casings reinforced concrete beams that have been essentially damaged and brought almost to destruction in the course of previous studies, when they were subjected to static and high-level low-cycle alternate loading, with the use of the experimental design theory;

- to study the impact of the main design factors on the load-bearing capacity of the support areas of the studied common and strengthened with prestressed metal casings damaged reinforced concrete beams, their crack resistance and deformability with the aid of experimental and statistical dependencies obtained in the course of the received data;

- to make a comparative analysis of the impact of the main design factors upon said parameters of the common whole and strengthened with prestressed metal casings damaged reinforced concrete beams with due account of static and low-cycle alternate loading.

Methodology of the experiments, materials and equipment

Experimental research system [1, 20, 21 and 22] of the support areas load-bearing capacity of the complex stressed reinforced concrete span structures are conducted in Odesa State Academy of Construction and Architecture.

In order to achieve this goal the Academy has set a task and developed a method of renewal and strengthening the damaged reinforced concrete beams and arrangements for renewal and strengthening a reinforced concrete beam which is in the ultimate limit state because of its division through normal and cross inclined cracks into separate blocks that are interconnected with longitudinal and transverse reinforcement that are subjected to high-level alternate cyclic or fluctuating loading, as well as strengthening of a common span reinforced concrete structure which, though being technically satisfactory, may be subject to said loading or seismic impacts [23]. The set task was solved with the aid of two inventions joined by a single idea:

a) a method for renewal and strengthening of a damaged reinforced concrete beam by fixing the strengthening elements onto it and subsequent prestressing them followed by three-sided clamping of this beam;

b) an arrangement for renewal and strengthening of a damaged beam that contains the interconnected longitudinal and transverse vertical and horizontal elements clamped together by threaded couplings.

The claimed method to renew and strengthen damaged reinforced concrete beams and the arrangement for its accomplishment provide the following technical result:

1. Complete renewal of the damaged beam.

2. Enhancement of the load-bearing capacity of the reinforced concrete beam with strengthened prestressed metal casings.

3. Considerable economy of material and technical resources due to saving, renewal and strengthening, if required, of the existing beam structures to withstand loading that have changed, including increasing, alternate, fluctuating or other dynamic loading.

4. Installation works connected with the arrangement and a method to use it do not necessitate termination of the main technological process.

Proceeding from the above, there has been accomplished one more additional series of tests of the single-span experimental beams damaged in the previous tests and strengthened with prestressed metal casings subjected to high-level low-cycle alternate loading within the frame of the research projects financed by the central government budget (state registration Nos.0107U000809 and 0108U000559) with the use of the experimental design theory and efficient PC software COMPLEX of Prof. V.A. Voznesenskyi.

It is known from literary sources that the main performance parameters of reinforced concrete structures are governed by the Gaussian law and that it is possible to process the results with the least square method. As the studied factors can influence the output function in a non-linear manner, it is expedient to approximate it with the second order polynomial. That is why the test samples were prepared and twice tested with the use of four-factor three-level D-optimum Box plan B-4 [24] which ensures the same accuracy of

the output parameter prediction within the area that is described with the radius that equals the conventional "1" with respect of the "zero" point.

The following factors were chosen as design factors that were changing at three levels:

X_1 – relative shear span (distance from the support to the concentrated force line), $a/h_0=1, 2, 3$ at $h_0=d=175$ mm;

X_2 – concrete grade C, MPa, C16/20, C30/35 and C40/50;

X_3 – transverse reinforcement coefficient of the external metal casings $\rho_{fw}(A240C)=0.0046; 0.0105; 0.0263; c_{sw}(Bpl)=0.0016; 0.0029; 0.0044;$

X_4 – level of the external transverse loading in the reinforced concrete beams of the first (ASD) and the second series (A) $\eta_{1,2}=0.5; 0.65; 0.80$; the prestress levels in the metal casing elements $\eta_{of}=0.25; 0.50; 0.75$.

Coefficients of the upper and low longitudinal reinforcement $\rho_{ls}=\rho_{ls}=0.176$ for all beam types. Coefficients of the upper and low additional strengthening of the damaged reinforced concrete beams in kind of prestressed elements of the metal casings made of angles $25 \times 25 \times 4$ $\rho_{ef}=\rho_{ef}=0.0191$, at $l_0=9h_0$; $h_0=175$ mm and $b=100$ mm for common reinforced concrete beams.

Each study of the field test of the first (ASD) and the second (A) series provided for two twin beams of the third series (D) had four support areas. The test sample beams of the third series (D) were the same as second series beams (A) damaged and divided into separate blocks through normal and inclined cross cracks that were brought to the limit state during previous tests when they were subjected to high-level alternate loading and which were renewed and strengthened with the use of the patented by the authors method and arrangement in the course of the third series (D) [23]. Hence, altogether in the first two series there were prepared and tested $50+50=100$ reinforced concrete beams for the action of, accordingly, static increasing and high-level low-cycle alternate loading until the advent of the limit state that arises in the test elements at $\eta \approx 0.95 F_{ult}$ loading level, i.e., on the verge of collapse. In the third series of experiments (D) all renewed and strengthened damaged test sample beams of the second series (A) were strengthened with prestressed metal casings [23] and repeatedly subjected to the low-cycle alternate loading until their practical collapse.

The tested reinforced concrete beam samples were strengthened in accordance with the experiment plan B4 with two flat welded frames. In order to make the present elements it was used heavy concrete of the above indicated grades prepared with granite chips of 5-10 mm fractions and 1.5 fineness modulus quartz sand. Portland cement grade 500 without additives was used as a binder. In order to reduce the water/cement ratio and to make it more convenient to place concrete mix and shorten the cure period, all tests were conducted with Relaxol-Super M (ISO 9001 № 04.156.26) complex additive in quantity of 1% of the cement weight as recalculated to dry substance.

To renew and strengthen the reinforced concrete beams damaged with cracks that appeared due to loading, it was used the patent for invention No.119294 [23] and special arrangements for its accomplishment according to the claimed method were manufactured, and a certified special power unit was made for testing. In the course of cycles, loading was applied according to the four-point pattern with the aid of a DG-50 hydraulic jack and two distribution traverse beams that applied two concentrated forces in stages: by (0.04 ... 0.06) F_{ult} until appearance of the first normal and inclined cracks, and then – by (0.08 ... 0.12) F_{ult}

until collapse in the first series (ASD) or until the preset level η of the alternate loading in its first cycle, afterwards – until collapse at the final stage in the second (A) and third (D) series. The delay exposure at a stage comprised up to 15 minutes including all measurements at the beginning and at the end of each loading stage.

Before making the test beams, chains of KF5P1-5-200 strain gauges were glued on the tensioned reinforcement of one of the flat frames (base 5 mm) in compliance with the technology recommended by the manufacturer (LLC "Veda", Kyiv).

Deformation of the test concrete samples were measured with the aid of wire and foil strain gauges with the 40 and 50 mm base according to the generally adopted methodology; the strain gauges were glued on one lateral and the top polished sides.

Deformations of the longitudinal and transverse elements of the prestressed metal casings were also measured with the aid of similar strain gauges. The transfer from the deformations in the reinforcement and metal elements measured in the study were accomplished in accordance with the Hooke's law, and in the concrete – on the basis of the cross-section elasticity modulus. The control of the concrete deformations in the compressed zone and in the tensioned reinforcement as well as of the prestress level in the metal casings was accomplished with the aid of the dial type indicators, the vertical displacement was measured with Aistov's flexometers.

Statement of basic materials and results

Testing of the studied reinforced concrete beams of the first (ASD) and the second (A) series has proved that their deformation, cracking and collapse took place in accordance with the structural theory rules and was predictable. At first, normal cracks have appeared in the maximum bending moment zones. As the transverse loading was increasing, the normal cracks were developing towards the beam depth, their width was increasing and new normal cracks were appearing. Further increase of the static stepwise loading has led to further development of the normal and inclined cracks with the prevailing opening of the inclined cracks up to the collapse of the first series beams (ASD) along the dangerous inclined cracks.

The tested beam samples of the first (ASD) and the second (A) series were designed almost of equal strength in the normal and inclined cross-sections, however, they were made so that their collapse was still taking place along the inclined cracks under action of the collapsing transverse forces and the associated bending moments at the final stage of beam performance.

The earlier formed normal and inclined cracks of the second series (A) samples were fully closing due to the action of the compressing springs when the sign of beam loading changed at half-cycle point in the zone which was tensioned during the previous half-cycle and now became compressed. On the contrary, in the earlier compressed zone of "pure bend" during the next half-cycle new normal cracks appear which join the previous normal cracks thereby forming visible through cracks at rest. When the sign of loading is being changed at half-cycle point new inclined cracks appear in the beam support areas, these new cracks cross the earlier formed cracks at an angle close to 90°, which is well explained by the direction of the main tensioning stress.

As the alternate loading levels increase, the height of normal and length of the inclined dangerous cracks were increasing worse, concrete cohesion with the longitudinal and transverse reinforcement was becoming worse, the concrete was becoming loose and

its strength decreased, the cracks were becoming wider and the tested beam samples deformed accompanied with relative displacement of separate concrete blocks. Collapse of the second series (A) beams at high levels of $\eta \geq 0.8F_{ult}$ had, as a rule, a crushing character and took place, mainly, in the support areas.

Renewal and strengthening of the second series (A) damaged beams with the aid of the method and arrangement claimed in [23] allowed, first, of their straightening and bringing the beams back to their initial shape, afterwards it became possible to considerably renew and strengthen them. As the results of the third series (D) tests the load-bearing capacity of the beams was considerably improved thereby confirming expediency of the practical application of the invention.

The obtained experimental data, removal of the insignificant coefficients and re-calculation of the remaining coefficients with the aid of COMPEX software program made it possible to get adequate experimental and statistical dependencies of the main parameters of the studied series tests samples which are very useful from the information viewpoint and exhibit good convergence with the test data.

Strength (load-bearing capacity) of the tested elements can be represented by the following experimental dependencies:

$$\hat{Y}(V_{u1}^{exp}) = 98 - 41X_1 + 12X_2 + 6X_3 + 16X_1^2 - 7X_2^2 - 5X_3^2 - 7X_1X_2, \text{ kH}, \quad \mathfrak{U} = 5.2\%; \quad (1)$$

$$\begin{aligned} \hat{Y}(V_{u2}^{exp}) &= 80 - 33X_1 + 13X_2 + 6X_3 - 2X_4 + 21X_1^2 - 12X_2^2 - 5X_3^2 - 7X_1X_2, \text{ kH}, \\ \mathfrak{U} &= 5.8\%; \end{aligned} \quad (2)$$

$$\hat{Y}(V_{uf}^{exp}) = 148 - 32X_1 + 17X_2 + 2X_1^2 - 8X_2^2 - 10X_1X_2, \text{ kH}, \quad \mathfrak{U} = 5.3\%; \quad (3)$$

where, V_{u1}^{exp} , V_{u2}^{exp} - a collapsing transverse force under statistical stepwise increasing loading and low-cycle alternate loading of the common reinforced concrete beams [1], accordingly;

V_{uf}^{exp} - a collapsing transverse force in the damaged with forced normal cracks reinforced concrete beams of the second series (A) that have been strengthened with prestressed metal casings according to [23] that have been subjected to high-level low-cycle alternate loading with the same values of the design factors.

The presented adequate experimental and statistical dependencies (1) ... (3), which Prof. V.A. Voznesenskyi called mathematical models [24], have essential advantage over correlation and other dependencies because they enable to comprehensively evaluate the impact of each of the above indicated factors upon the initial output parameters, not collectively but in their interaction with each other, as well as the value of such impact both on the common reinforced concrete beams subjected to static stepwise increasing and high-level low-cycle alternate loading but also on the damaged and divided into several blocks with forced crack reinforced concrete beams strengthened with prestressed metal casings subjected to similar type loading. Geometrical interpretation of the load-bearing capacity of the tested sample beams can be partially presented in Figure 1.

The greatest impact on the load-bearing capacity of the test element support areas is produced by such design factor as the relative shear span (Figure 1, a). Generally, the dependence identified by O.S. Zalesov, Yu.A. Klymov [25] and V.M. Karpiuk [20, 21 and 22] and other researches, namely, a decrease of strength in the inclined cross-sections of the reinforced concrete beam structures versus an increase of the shear span that follows the

non-linear law. The concrete grade is the next design factor. At this, the higher the grade, from C16/20 to C40/50, the higher the load-bearing capacity – this dependence also follows the non-linear law (Figure 1, b).

The similar picture is observed with respect of the increase of the transverse reinforcement coefficients ρ_{sw} , ρ_{fw} (Figure 1, c).

The impact of the increase of the transverse loading η from 0.5 to 0.8 turned to be negative at the alternate loading for the common beams of the second series (A) only (Figure 1, d).

Analysis of the dependencies (1) ... (3) indicates that they are of the same type by quality and that the impacts of the design factors as well as the factor of low-cycle loading are also of similar quality. So, the load-bearing capacity of the inclined cross-sections in the tested beam samples, which is expressed via the collapsing transverse force V_u , increases relatively to their average values (free terms b_0), 98, 80 i 148 kN, accordingly:

- as the relative shear span a/h_0 decreases from 3 to 1 in the first (ASD), second (A) and third (D) series, it increases by 84, 83 and 43%, accordingly;

- as the concrete grade is higher from C16/20 to C40/50, it increases by 24, 33 and 23%, accordingly;

- as the quantity of the transverse steel reinforcement ρ_{sw} increases from 0.0016 to 0.0044 (expressions (1), (2)), it increases by 12 and 15%, accordingly;

- as the quantity of the prestressed transverse clamps of the metal casings ρ_{fw} increases from 0.0046 to 0.0263, the V_{uf}^{exp} value remains the same – evidently it happens due to the joint performance effect of all prestressed elements of the metal casings and partially renewed previously damaged reinforced concrete beam because of the three-side clamping of that beam;

- as the low-cycle alternate loading η decreases from 0.8 to 0.5 in case of the reinforced concrete beams of the second (A) series – it increases by 5%;

- as the value of the relative shear span a/h_0 decreases simultaneously with the increase of the concrete grade C by 7, 9 and 7%, accordingly. Presence of the quadratic effects at factors X_1^2 , X_2^2 and X_3^2 that have the signs opposite to the direct impact of factors indicates that, beyond the interval of the studied factor limits ($a/h_0 > 3$, $C > 40/50$ МПа, $\rho_{sw} > 0,0044$ i $\rho_{fw} > 0,0263$), further increase of a/h_0 will not lead to the essential reduction of the load-bearing capacity (collapsing transverse force) (Figure 1, a) further increase of the concrete grade C (Figure 1, b) and further increase of the transverse steel reinforcement quantity ρ_{sw} and of the prestressed external clamps ρ_{fw} (Figure 1, c) is inexpedient because the increase process V_u has, at that, a fading character.

Analysis of the obtained experimental data has proved that the low-cycle repeated loading reduces the ability of the reinforced concrete beam support areas by 8% on the average according to [22] while the low-cycle alternate loading reduced it by 18% as compared with the load-bearing capacity of similar beams tested under static stepwise increasing loading. This conclusion is confirmed by the works of Prof. Ye.M. Babych, Prof. G. Kh. Masyuk and their followers.

As the field experiments have shown, the load-bearing capacity of the reinforced concrete beams strengthened with prestressed metal casings [23] after being damaged in the previous tests was increased by 51% as compared with common reinforced concrete beams of the first series (ASD), which tells about high effectiveness of such strengthening.

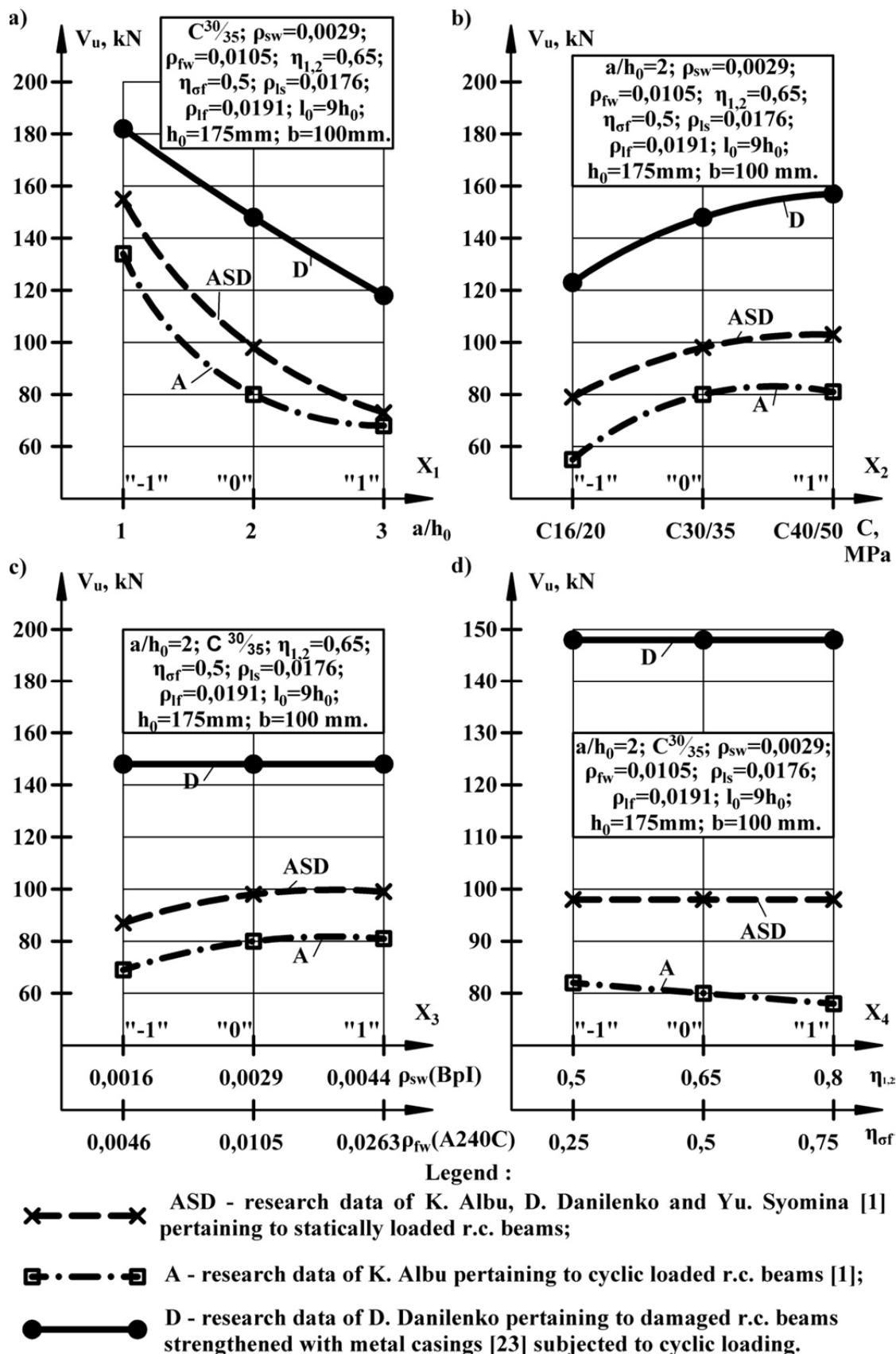


Figure 1. Impact of the relative shear span (a), concrete grade (b) and quantity of the transverse reinforcement (c) and levels of the transverse loading and previous stress in the metal casings (d) on the load-bearing capacity of the test element support areas before their collapse.

The number of repeated and alternate loading cycles was dictated by the criterion referring to the stabilization of deformations in concrete introduced by Prof. Ye.M. Babych, and amounted to at least 10 in case when the test beam samples did not collapse at the lower number of cycles.

The absolute majority of the tested beams collapsed along the inclined cross-sections in both or in one (more frequently) shear spans. The collapse criterion of the test samples occurred upon reaching limit deformations in concrete or reinforcement with the evident signs of the arising plastic deformations, excessive width (to 1mm and more) of the inclined (more often) or normal (not so frequent) cracks, considerable increase of sagging ($f \geq \frac{1}{100} l_0$) and absence of the increment or a certain decrease (to 15%) of the power unit pumping station pressure gauge readings.

It is experimentally proved that the structural disturbance of the concrete is the main reason of the load-bearing capacity reduction in the tested sample beams subjected to low-cycle repeated and alternate loading, especially in the support areas, concrete crushing and partial loss of its cohesion with the reinforcement.

The greatest increment of the residual deformations in concrete and in transverse reinforcement is recorded during the first two-three cycles and, as a rule, they stabilize before the fifth-sixth cycles at $\eta=0 \dots 0.65$ loading levels. In some samples, where the concrete grade was minimum and the transverse reinforcement quantity at loading levels $\eta=0 \dots 0.8$, the deformations did not stabilize and the beams collapsed at 6 ... 9 cycles because of reaching fatigue strength or possible reduction of their strength parameters due to statistic error in the determining the collapsing single-time stepwise increasing loading.

Non-overreinforced ($\rho_{sw} \leq 0.003$ (Bpl), $\rho_{fw} \leq 0.018$ (A500C)) reinforced concrete test beams collapsed, as a rule, according to the B/M pattern when subjected to a single-time stepwise increasing static low-cycle, repeated and alternate loadings [1, 21, 22], i.e., along the inclined cross-sections with the prevailing action of the bending moment because the longitudinal working reinforcement yielded in the mouth of a dangerous inclined crack and the transverse reinforcement that crosses it. As the transverse reinforcement $\rho_{sw} \geq 0.0044$ quantity was increasing, the similar tested elements with average ($a/h_0=2$) and high $a/h_0=3$ shear spans were collapsing according to C/V pattern, i.e., along the inclined crack with the prevailing action of the transverse force due to the yield of the transverse reinforcement and shear or crushing of concrete because of compressing force above the apex of the dangerous inclined crack. With small shear spans ($a/h_0 \leq 1$) the tested reinforced concrete beams collapsed, as a rule, according to D//cm pattern along the inclined compressed strip between two inclined cracks due to crushing of concrete in that strip along the route of the main compressing stresses or along its shear under the influence of the maximum tangential stresses.

Collapse of the strengthened beams of the third (D) series began from a gradual loss of the prestress in the compressed elements of the metal casings, opening of the earlier formed cracks, mutual displacement of separate blocks and excessive deformation of the metal casings, and ended with a loss of the support area or areas shape.

Deflections of the test beams before their collapse can be characterized by the following experimental and statistical dependencies

$$\hat{Y}(f_{u1}^{\exp}) = 6 + 1.5X_1 + 0.65X_2 + 0.7X_3 + 0.35X_4 - 0.5X_1^2 + 0.2X_1X_3, \text{ MM}, \quad U = 5.5\%; \quad (4)$$

$$\hat{Y}(f_{u2}^{\exp}) = 7.2 + 2.1X_1 + 0.8X_2 + 0.8X_3 + 0.4X_4 - 0.5X_1^2, \text{ MM}, \quad U = 5.3\%; \quad (5)$$

$$\begin{aligned} \hat{Y}(f_{uf}^{\exp}) = & 13.2 + 6.6X_1 - 0.7X_2 + 0.85X_3 - 0.8X_4 + 1.5X_1^2 + 0.4X_2^2 + 0.6X_1X_3 - 0.8X_1X_4 \\ & + 0.9X_2X_4 - 0.6X_3X_4, \text{ MM}, \end{aligned} \quad U = 5.1\%; \quad (6)$$

that indicate that the deflections of the first test series beams (ASD) comprised, on the average, $(1/263)$ of the design span l_0 , of the second series (A) – $(1/280) l_0$, and of the third (D) series – $(1/119) l_0$. At that, the effect of the low-cycle alternate loading was increasing the deflections of the tested beam samples as compared with a single time statistical stepwise increasing loading by 20%.

Application of the metal prestressed metal casings for strengthening the damaged beams subjected to low-cycle alternate loading increased not only their load-bearing capacity by 1.85 times but the deflections to the same extent as well. Graphical representation of the deflections prior to collapse of the beams and their dependence on the studied factors is shown in Fig. 2.

Width of the normal and inclined cracks in the tested beam samples.

Width of the normal cracks in the “pure bend” zone can be represented by dependencies (7), (8) and in Figure 3:

$$\begin{aligned} \hat{Y}(W_{cr1,2}^{\exp}) = & 0.14 + 0.02X_1 + 0.03X_2 + 0.01X_3 + 0.05X_4 + 0.01X_1^2 - 0.03X_2^2 + 0.02X_4^2 \\ & + 0.01X_1X_3 + 0.01X_1X_4 + 0.02X_2X_4 + 0.01X_3X_4, \text{ MM}, \end{aligned} \quad U = 6.2\%, \quad (7)$$

$$\begin{aligned} \hat{Y}(W_{crf}^{\exp}) = & 0.06 + 0.04X_1 + 0.02X_2 + 0.01X_3 - 0.01X_4 + 0.02X_1X_2 + 0.02X_1X_3 - 0.01X_1X_4 \\ & + 0.01X_2X_3, \text{ MM}, \end{aligned} \quad U = 21.6\%. \quad (8)$$

As follows from (7) and (8), presence of the prestressed metal casings restrains development of the normal cracks in the middle of the beams and reduces their width by 2.3 times, on the average.

Widths of the inclined cracks in the support areas of the tested elements can be represented by experimental and statistical dependencies and are shown in Figure 4:

$$\begin{aligned} \hat{Y}(W_{cr1}^{\exp}) = & 0.35 - 0.06X_1 - 0.03X_2 - 0.01X_3 - 0.14X_4 - 0.01X_1X_3 - 0.03X_2X_4 - \\ & - 0.02X_3X_4, \text{ MM}, \end{aligned} \quad U = 10.4\%; \quad (9)$$

$$\begin{aligned} \hat{Y}(W_{cr2}^{\exp}) = & 0.63 + 0.05X_1 + 0.05X_2 - 0.06X_3 + 0.24X_4 - 0.02X_2^2 + 0.02X_3^2 + 0.02X_4^2 + \\ & + 0.11X_1X_2 - 0.03X_1X_3 - 0.06X_3X_4, \text{ MM}, \end{aligned} \quad U = 11.5\%; \quad (10)$$

$$\begin{aligned} \hat{Y}(W_{crf}^{\exp}) = & 0.50 + 0.20X_1 - 0.01X_2 - 0.05X_3 - 0.09X_4 + 0.03X_1^2 - 0.03X_1X_2 - \\ & - 0.04X_1X_4, \text{ MM}, \end{aligned} \quad U = 8.4\%. \quad (11)$$

Analysis of expressions (7), (8), (9), (10) and (11) indicates that the width of the inclined cracks in common beams exceeds 2.5 times the width of the normal cracks in the same beams when are subjected to single time static loading.

The action of high-level low-cycle alternate loading increases the width of the inclined cracks as much as by 4.5 times.

Application of the metal prestressed metal casings enabled not only to increase the load-bearing capacity of the damaged and separated into blocks test beams by 1.85 times but also reduce the width of the inclined cracks by 21%, which is still another advantage of the used method to strengthen the damaged span structures.

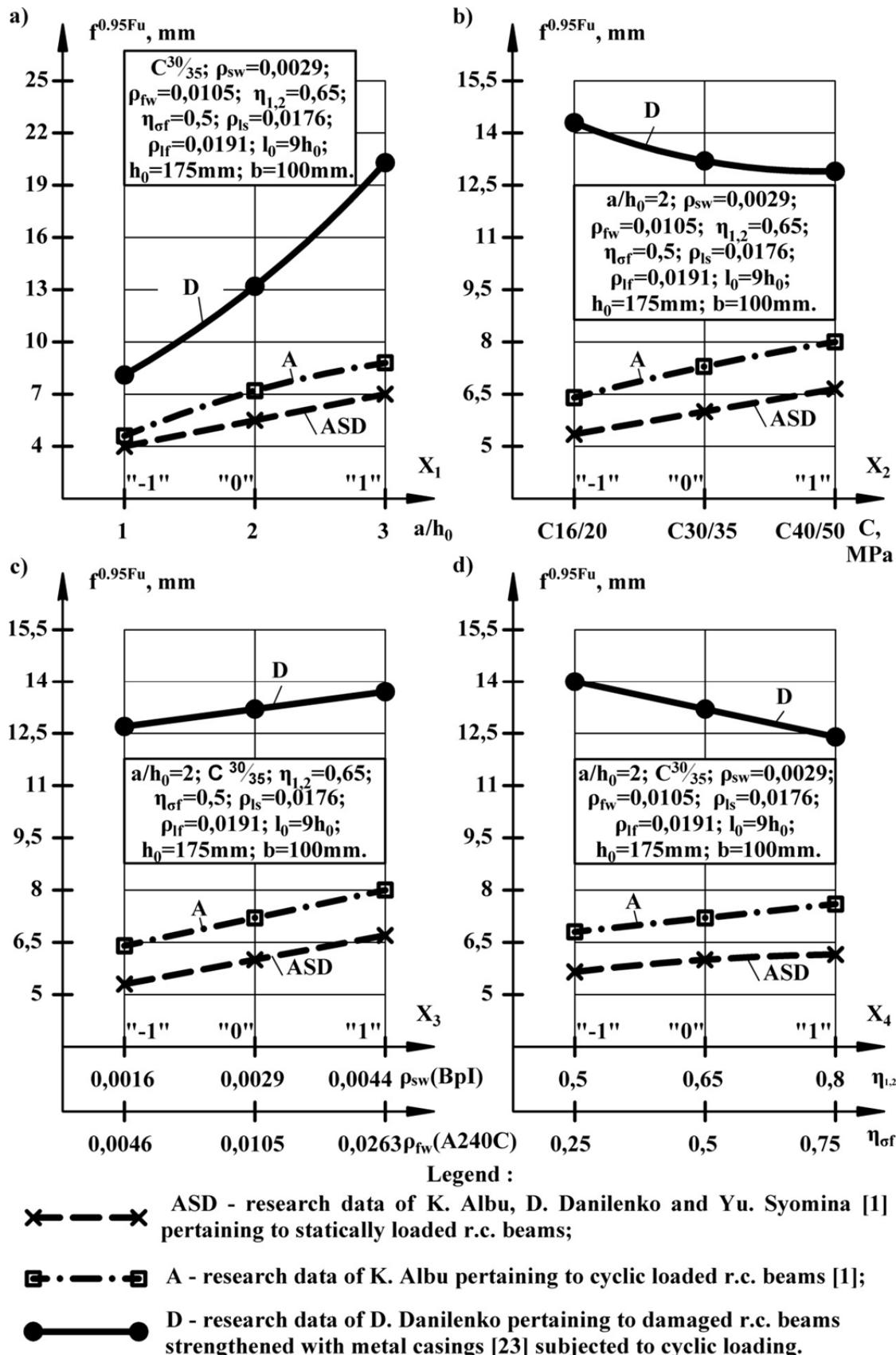


Figure 2. Deflections before collapse of the test beams versus the relative shear span value (a), concrete grade (b), transverse reinforcement quantity (c) and levels of transverse loading and prestress of the metal plate elements (d).

Length of the dangerous inclined crack projection on the horizontal axis of the beam is represented by expressions (12), (13) and is shown in Figure 5:

$$\hat{Y}(l'_{cr1}^{exp}) = 201 + 53X_1 - 19X_3 - 4X_1X_2 - 23X_1X_3, \text{ MM}, \quad U = 8.5\%; \quad (12)$$

$$\hat{Y}(l'_{cr2,f}^{exp}) = 173 + 40X_1 - 16X_3 + 19X_4 + 16X_1X_2 - 7X_1X_3 + 14X_1X_4 - 11X_3X_4, \text{ MM}, \\ U = 8.9\%; \quad (13)$$

As follows from (12) and (13), the average length of the critical inclined crack projection, when the beam is subjected to low-cycle alternate loading, it is reduced as compared with the static single time loading by 14% at the expense of the fatigue rupture of the concrete in the compressed zone and destruction of the protection layer of concrete of the supports.

Distance between the normal cracks in the “pure bend” zone is characterized by dependencies (14), (15) and is shown in Figure 6:

$$\hat{Y}(l_{cr1}^{\perp\perp exp}) = 68 - 6X_1 + 2X_2 + 3X_1X_3, \text{ MM}, \quad U = 5.1\%; \quad (3.16)$$

$$\hat{Y}(l_{cr2,f}^{\perp\perp exp}) = 70.9 - 2.15X_1 + 1.93X_2 - 1.75X_3, \text{ MM} \quad U = 4.7\%; \quad (3.17)$$

Conclusions

1. The experiments have proved a possibility and expediency of practical application of the ingenious method developed by the authors which is a distinctive renewal and strengthening the damaged reinforced concrete beam structures damaged through normal and cross inclined cracks which divide the beam into separate blocks due to three-side clamping, and the arrangement for its accomplishment claimed by the patent of invention [23] subjected to action of the increasing static, cyclic, alternate, seismic, fluctuating and other dynamic loading, including, among other things, the reinforced concrete beams damaged during military activity, such beams did not collapse completely and preserve their initial shape.

2. The accomplished system approach to the experimental and theoretical research of the stress-strain behaviour of the common, renewed and strengthened reinforced concrete span structures damaged through forced cracks with the aid of metal prestressed metal casings enabled to make a reliable quantitative and qualitative evaluation of the design factors and external factors impact on their load-bearing capacity, stiffness, crack resistance and other performance parameters, either individually or in their interaction with each other, considerably specify the physical model of present structures performance when subjected to static and low-cycle alternate loading. In particular, it was established that the load-bearing capacity of the studied elements according to the 1st group of boundary conditions is increasing in a non-linear manner:

- as the relative shear span a/h_0 decreases from 3 to 1 in the indicated series ASD, A, D - by 43 ... 84%;
- as the concrete grade is higher from C16/20 to C40/50 - by 23 ... 33%;
- as the quantity of the transverse steel reinforcement ρ_{sw} increases from 0.0016 to 0.0044 – by 12 ... 15%;
- as the level of low-cycle alternate loading η increases from 0.8 to 0.5 – by 5%;

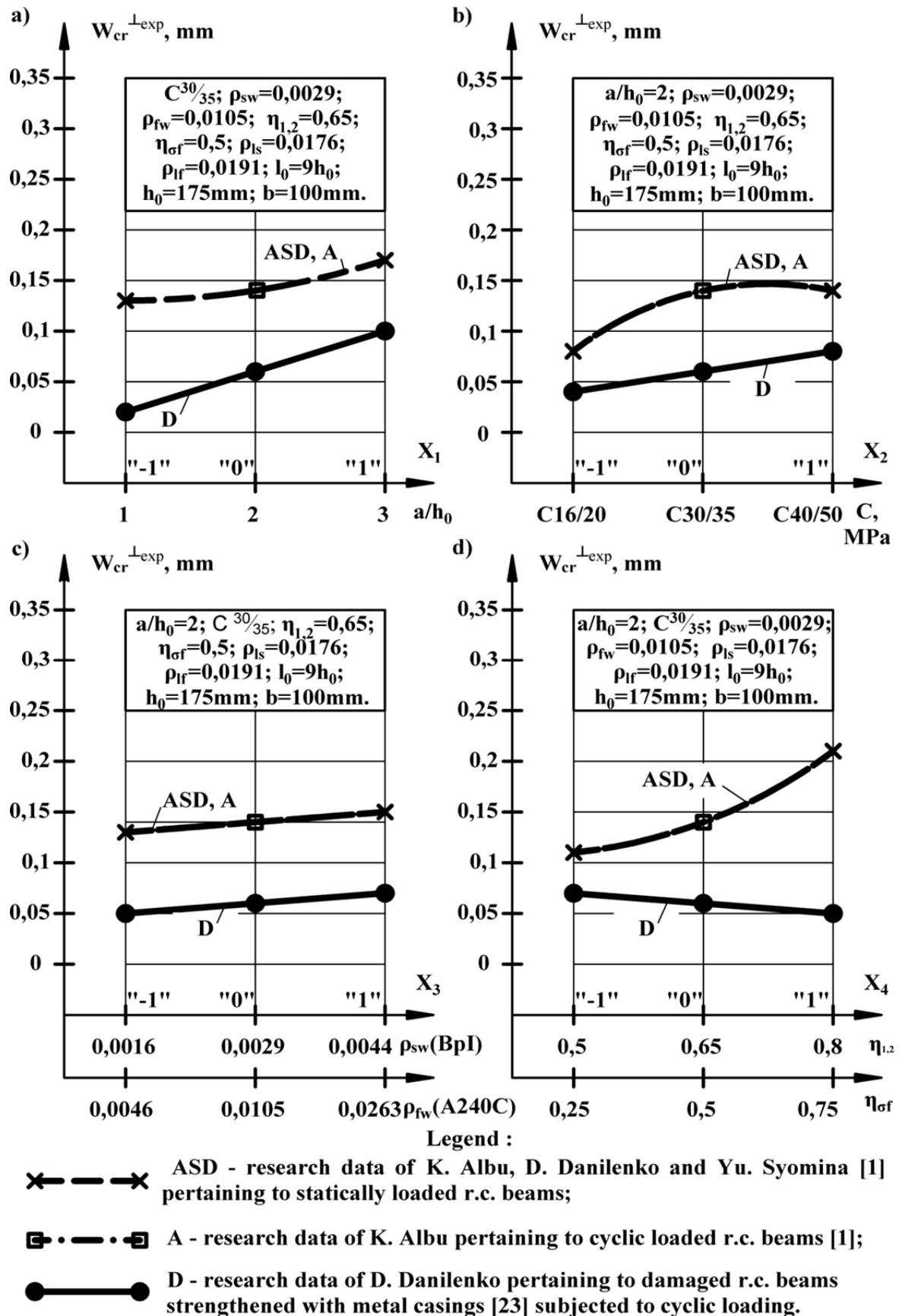


Figure 3. Impact of the relative shear span (a), concrete grade (b), quantity of the transverse reinforcement (c), levels of the transverse loading and prestress of the metal casing elements (d) on the normal crack width in the tested beams prior to their collapse.

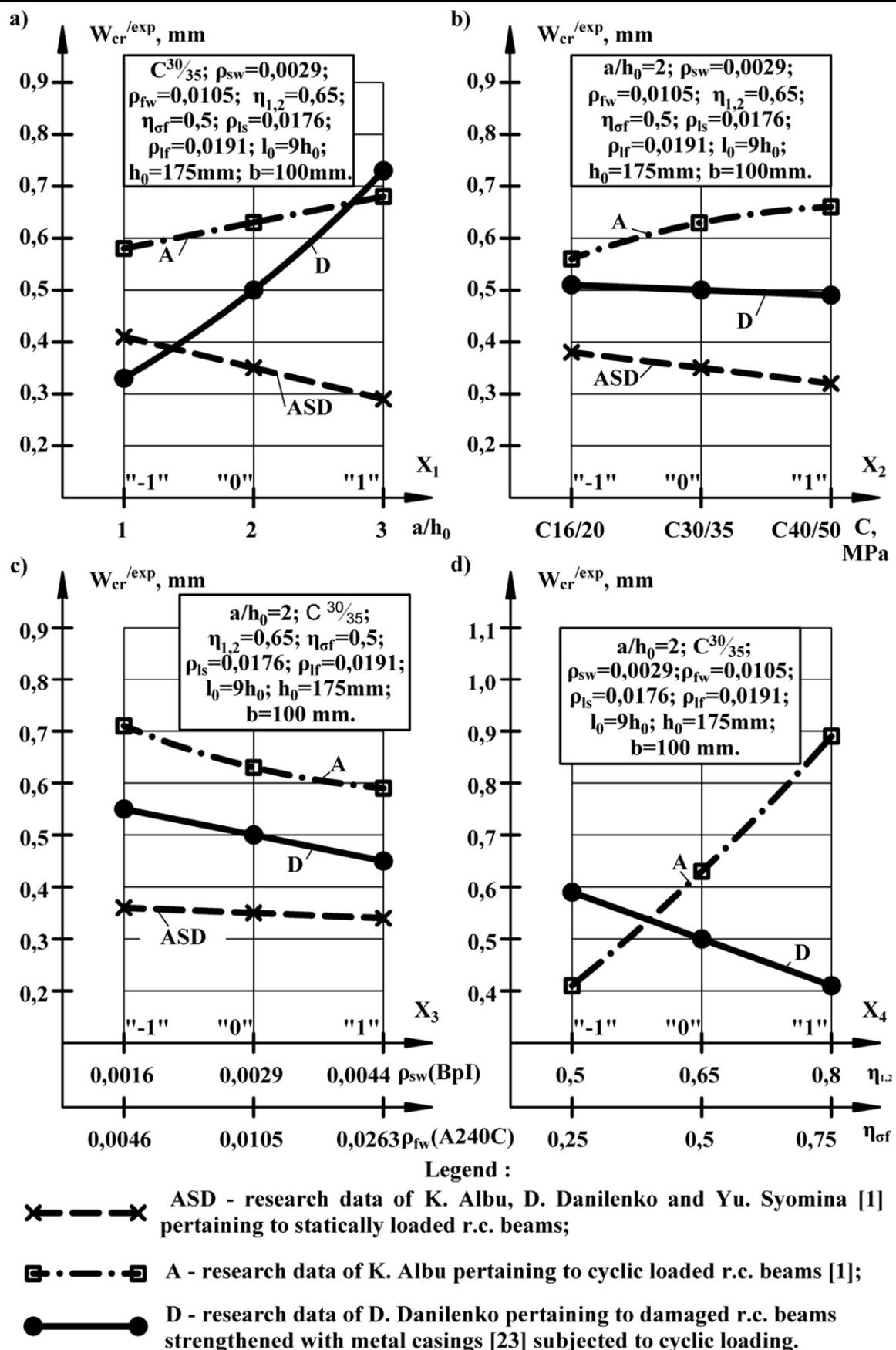


Figure 4. Dependence of the inclined crack width prior to collapse of the test beams on the value of the relative shear span (a), concrete grade (b), quantity of the transverse reinforcement (c), levels of the transverse loading and prestress of the metal casing elements (d).

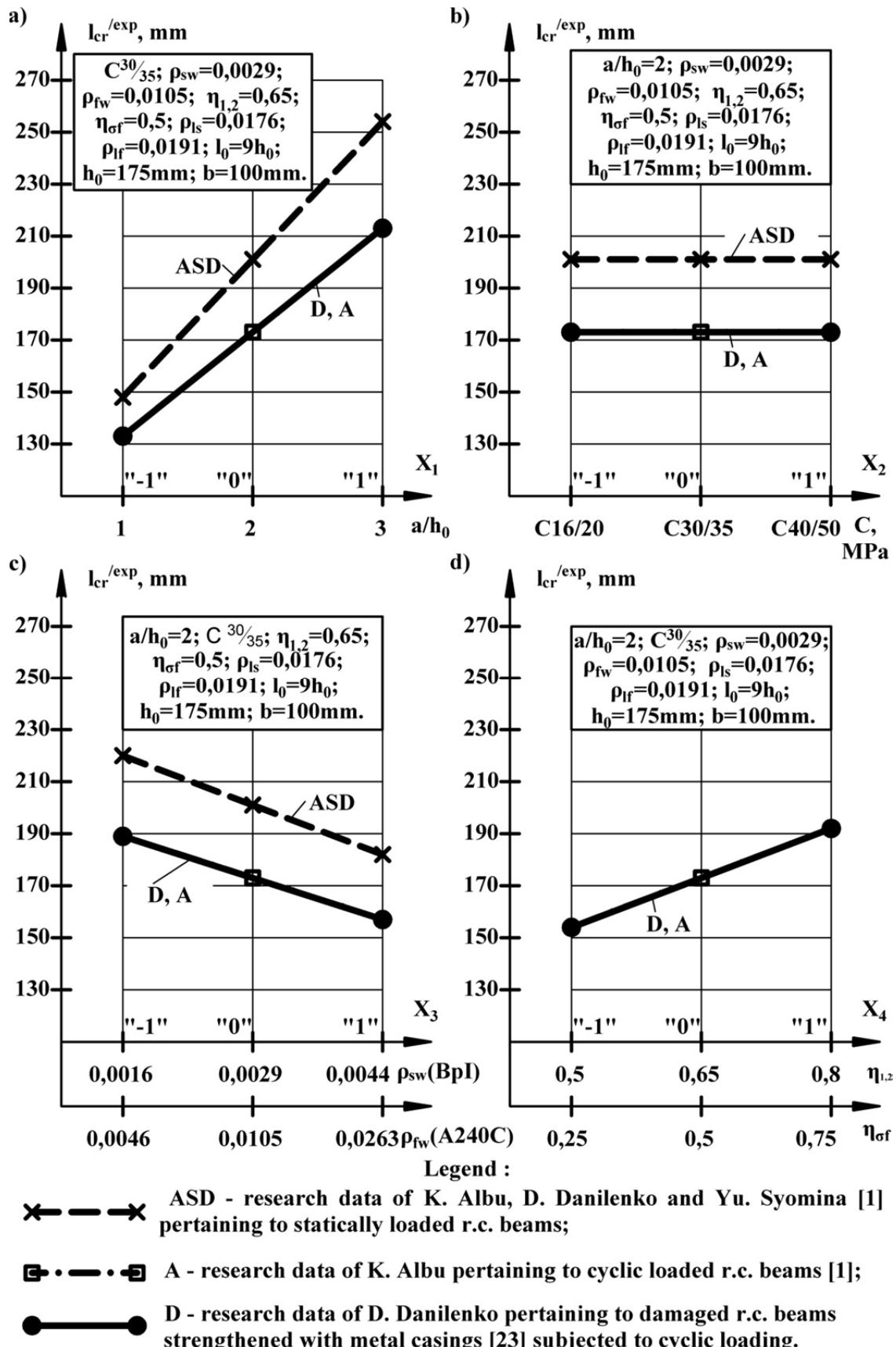


Figure 5. Impact of the relative shear span (a), concrete grade (b), quantity of the transverse reinforcement (c), levels of the transverse loading and prestress of the metal casing elements (d) on the length of inclined crack projection in the tested elements prior to their collapse of normal crack width in the tested beams prior to their collapse.

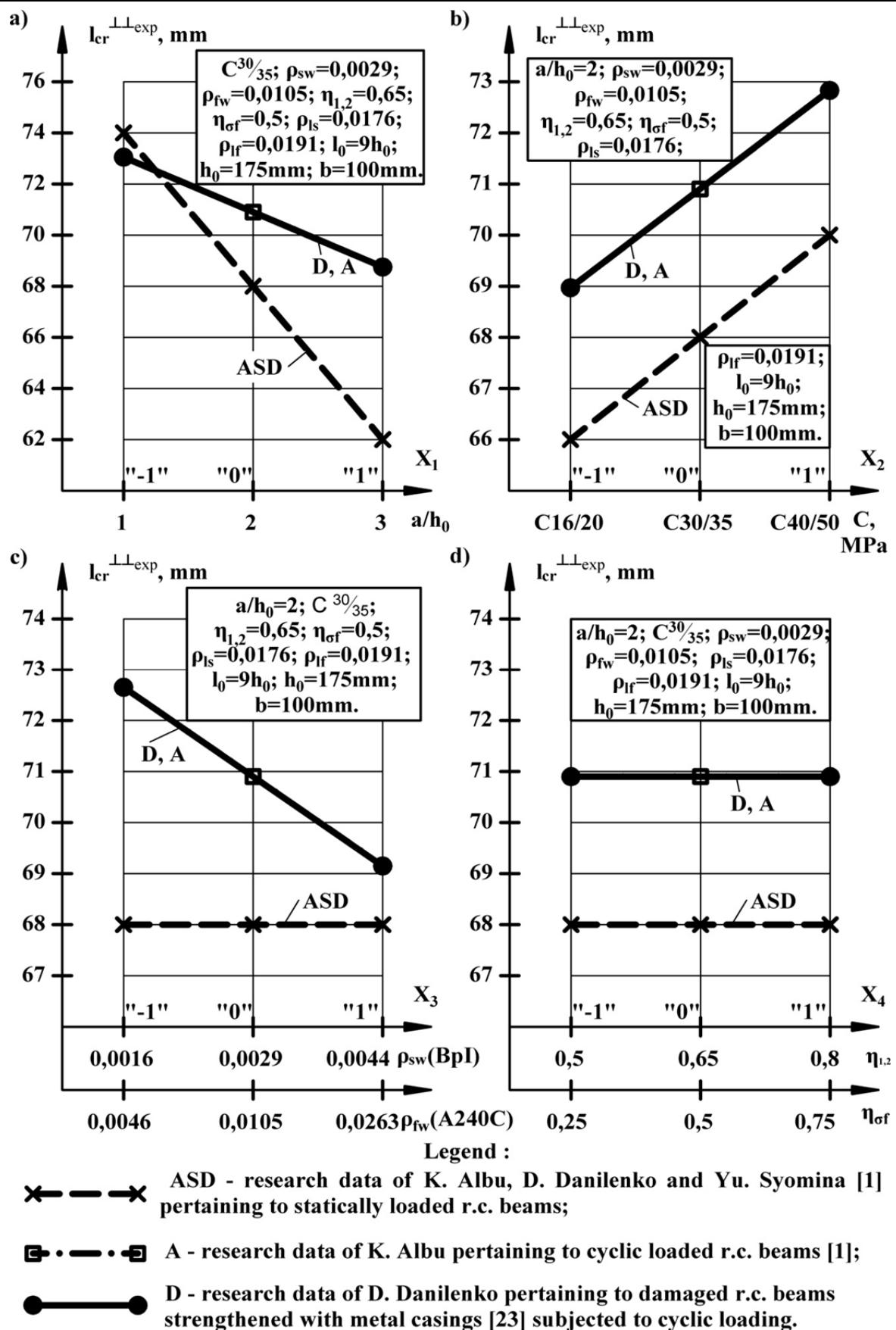


Figure 6. dependence of the distance between normal cracks prior to collapse on the value of the relative shear span (a), concrete grade (b), quantity of the transverse reinforcement (c), levels of the transverse loading and prestress of the metal casing elements (d).

- as the value of the relative shear span a/h_0 decreases simultaneously with the increase of the concrete grade C by 7 ...9.

3. The low-cycle repeated loading reduces the load-bearing capacity of the common reinforced concrete beam support areas by 8% on the average according to [22], and the low-cycle alternate loading – by 18%.

4. The load-bearing capacity of the damaged reinforced concrete beams strengthened with prestressed metal casings [23] subjected to alternate loading increased by 51% on the average as compared with the common reinforced concrete beams tested for a single time static loading, it indicates high effectiveness of such strengthening.

5. The low-cycle alternate loading increases deflections of the tested beam samples as compared with a single time static stepwise increased loading by 20%.

Application of the metal prestressed metal casings for strengthening damaged beams increases both their load-bearing capacity and the deflections by 1.85 times subjected to low-cycle alternate loading as compared with these parameters in the common whole beams subjected to single-time loading.

6. The normal crack width in the middle part of the tested beams of all series does not exceed the permissible values and varies within 0.02 ... 0.22 mm.

The average value of the inclined crack width in the tested elements subjected to operational loading levels does not exceed the permissible values and varies, prior to collapse, within 0.35 ... 0.63 mm. At that, application of the metal prestressed strengthening reduces the inclined crack width by 21% on the average, and the width of the normal cracks – by 2.3 times.

7. The average length of the horizontal projection of a critical inclined crack subjected to low-cycle alternate loading is reduced as compared with the static single-time loading by 14% due to the fatigue rupture of the compressed concrete zone and destruction of its protection layer in the support areas.

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