# THE RELIABILITY OF THE OBJECTS RECOGNITION AND LOCALIZATION IN THE OPTICAL CORRELATORS 

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#### Abstract

In the article were made the mathematical estimation and analyses of the objects' recognition and localization reliability in the optical correlation processors. There are presented the mathematical expressions for determination the probability of the correct and faulty objects recognition in the processors with single correlation maxima. It is given the estimation of the calculations results reliability in the processors with codified correlation maxima, taking into account that they consists the binary codes of $N$ digits and that at the scanning of the optical codes and their transformation to electrical signals the errors of different order are possible. There are presented the data regarding the objects recognition correctness in the correlation processors.


Keywords: correlator, detection, optical, processor, probability, recognition, reliability

## Introduction

One of the important problems, arising at the elaboration and utilization of the optical correlation processors represent the estimation of the reliability of calculations results, i.e. the probability of correct objects recognition and determination of their coordinates.

Optical correlation processors can be divided in two main groups - which are based on generation and analysis the correlation functions consisting of single and codified maxima. The examples of these kinds of the processors are described in article [1].

In this article were made the mathematical determination and analyses of the objects' recognition and localization probability in the optical correlation processors mentioned above. In section 1 there are presented the estimations of the probability of the correct and faulty objects recognition in the processors with single correlation maxima.

In section 2 it is given the estimation of the calculations results reliability of in the optical processors with codified correlation maxima, taking into account that they consists the binary codes of N digits from which there are K "units" and W "zeroes", and at the scanning of the optical codes and their transformation to electrical signals the errors of different order are possible.

In section 3 there are presented the results of the modeling of the calculations results reliability of in the optical correlation processors.

1. The estimation of the calculations results reliability of in the optical processors with a single correlation maxima

The general probability of the correct $\mathrm{P}_{\mathrm{s}}$ and faulty $\mathrm{P}_{\mathrm{fs}}$ objects recognition in the processors with a single correlation maxima (PSCM) can be described as the mathematical expectation of the probabilities $\mathrm{P}_{\mathrm{i}}$ of the correct objects recognition [2]:

$$
\begin{gather*}
\mathrm{P}_{\mathrm{s}}=\sum_{\mathrm{i}=1}^{\mathrm{M}} \mathrm{a}_{\mathrm{i}} \mathrm{P}_{\mathrm{i}}=\mathrm{a}_{1} \mathrm{P}_{1}+\mathrm{a}_{2} \mathrm{P}_{2}+\ldots . .+\mathrm{a}_{\mathrm{M}} \mathrm{P}_{\mathrm{M}}  \tag{1}\\
\mathrm{P}_{\mathrm{fs}}=1-\mathrm{P}_{\mathrm{s}}=\sum_{\mathrm{i}=1}^{\mathrm{M} a_{i} \mathrm{P}_{\mathrm{i}},} \tag{2}
\end{gather*}
$$

were $\alpha_{i}$ is the a priory probability of the i-th objects appearance; M is the number of the objects.

## 2. The estimation of the calculations results reliability in the optical processors with codified correlation maxima

In the optical correlation processors with codified correlation maxima (PCCM) there are generated the binary codes containing N digits, from which K - "units" and $\mathrm{W}=\mathrm{N}-\mathrm{K}$ - "zeroes". At the scanning of the optical codes and their transformation to electrical signals the errors are possible.

Let consider the errors of the two types - first and second. At the error of the first type, in a binary code an "unit" will be detected as a "zero" or "zero" - as an "unit". At the error of the second type at one digit of the code an "unit" will be detected as a "zero" and simultaneously in other digit a "zero" - as an "unit". These errors may be of different order.

In the optical processors using the device for the operative correlation field analysis (DOCFA) [1] the errors of the first type will be detected and don't influence on the recognition reliability. The wrong objects' detection can result only the errors of the second type.

The probability of correct i-th object recognition in the processors PCCM at occurrence of the errors of various order k can be described as follows:

$$
\begin{equation*}
\mathrm{P}_{\mathrm{ci}}=1-\mathrm{P}_{\mathrm{cfi}}=1-\sum_{\mathrm{k}=1}^{\mathrm{m}} \beta_{\mathrm{ik}} \mathrm{P}_{\mathrm{cfik}} \tag{3}
\end{equation*}
$$

where $\mathrm{P}_{\mathrm{cfi}}$ - probability of the faulty detection of i-th object; m-maximum order of an error; $\beta_{\mathrm{ik}}$ the probability of the second type errors occurrence; $\mathrm{P}_{\text {cfik }}$ - probability of faulty i-th object recognition at the error of the order k .

Let A be the event, which consists in the fact that in a binary code an "unit" will be detected as a "zero" in one of the digits; B - event, concluded in the fact that in the same binary code a "zero" will be detected as an "unit " in other digit; C - the event, then in the binary code at one digit an "unit" will be detected as a "zero" and simultaneously in other digit a "zero" - as an "unit".

It is obvious that the events A and B are independent. Under the theorem regarding the product of the probabilities of the independent events $A$ and $B$ we can determine the probability $\mathrm{P}(\mathrm{C})$ of the event $C$ as simultaneous occurrence of events $A$ and $B$. In this case $P(C)=P(A) P(B)$. Let's assume, that $\mathrm{P}(\mathrm{A})=\mathrm{P}(\mathrm{B})=\mathrm{P}_{\mathrm{fi}}=\left(1-\mathrm{P}_{\mathrm{i}}\right)$, where $\mathrm{P}_{\mathrm{fi}}, \mathrm{P}_{\mathrm{i}}$ are the probabilities of the faulty and correct i the objects recognition at the forming of the single correlation maxima.

Then the probability $\mathrm{P}(\mathrm{C})$ of occurrence of the event C is a probability of occurrence of an unitary error of the second type at the objects recognition, i.e. $\mathrm{P}(\mathrm{C})=\beta_{\mathrm{il}}$. The probability of occurrence of a k - order error will be equal to:

$$
\begin{equation*}
\beta_{\mathrm{ik}}=\left(1-\mathrm{P}_{\mathrm{i}}\right)^{2 \mathrm{k}} . \tag{4}
\end{equation*}
$$

Let's define the probability $\mathrm{P}_{\text {cfik }}$ of the faulty recognition of the i -th object as a result of occurrence of the errors of various order, taking into account the quantitative ratio in a code of "units" and "zeroes". Because on the wrong targets detection will result only errors of the second type, the probability of the faulty i-th target detection at occurrence of the errors of the second type of the order $k$ will be defined as the relation of the non detected number of errors $Q_{n k}$ to the total number of possible errors $\mathrm{Q}_{\mathrm{tk}}$ :

$$
\begin{equation*}
\mathrm{P}_{\mathrm{cfik}}=\mathrm{Q}_{\mathrm{nk}} / \mathrm{Q}_{\mathrm{tk}} \tag{5}
\end{equation*}
$$

Let's define the non detected number of errors $\mathrm{Q}_{\mathrm{nk}}$ of the second type of the order k . Let a binary code of the length $\mathrm{N}=8$ is describing as follows: $\mathrm{Z}=\mathrm{Z}_{8}, \mathrm{Z}_{7}, \mathrm{Z}_{6}, \mathrm{Z}_{5}, \mathrm{Z}_{4}, \mathrm{Z}_{3}, \mathrm{Z}_{2}, \mathrm{Z}_{1}$. Let $\mathrm{Z}_{\mathrm{i} .0}$ and $\mathrm{Z}_{\mathrm{j}, 1}$ be the faulty detection of the signals in the digits i and j respectively (i.e. transitions of the signals $1 \rightarrow 0$ and $0 \rightarrow 1$ ).

Let's consider the number of "units" in the code is $\mathrm{K}=5$, and number of "zeroes" $\mathrm{W}=3$ and, for example $\mathrm{Z}=10110101$. Then in this code the following combinations of the unitary errors of the second type are possible:

$$
\begin{aligned}
& \mathrm{Z}_{8.0} \mathrm{Z}_{7.1} ; \mathrm{Z}_{8.0} \mathrm{Z}_{4.1} \mathrm{Z}_{8.0} \mathrm{Z}_{2.1} ; \mathrm{Z}_{6.0} \mathrm{Z}_{7.1} ; \mathrm{Z}_{6.0} \mathrm{Z}_{4.1} ; \\
& \mathrm{Z}_{6.0} \mathrm{Z}_{2.1} ; \mathrm{Z}_{5.0} \mathrm{Z}_{7.1} ; \mathrm{Z}_{5.0} \mathrm{Z}_{4.1} ; \mathrm{Z}_{5.0} \mathrm{Z}_{2.1} \mathrm{Z}_{3.0} \mathrm{Z}_{7.1} ; \\
& \mathrm{Z}_{3.0} \mathrm{Z}_{4.1} ; \mathrm{Z}_{3.0} \mathrm{Z}_{2.1} ; \mathrm{Z}_{1.0} \mathrm{Z}_{7.1} ; \mathrm{Z}_{1.0} \mathrm{Z}_{4.1} ; \mathrm{Z}_{1.0} \mathrm{Z}_{2.1}
\end{aligned}
$$

The number of such errors is equal to $\mathrm{Q}_{\mathrm{n} 1}=\mathrm{C}^{1}{ }_{\mathrm{K}} \mathrm{C}^{1}{ }_{\mathrm{W}}=15$, where $\mathrm{C}^{1}{ }_{\mathrm{K}}, \mathrm{C}^{1}{ }_{\mathrm{J}}$ are the numbers of combinations from $K$ and $J$ on one. In the similarly way it is possible to state, that $Q_{n 2}=C^{2}{ }_{k} C^{2} w$; $\mathrm{Q}_{\mathrm{n} 3}=\mathrm{C}^{3}{ }_{\mathrm{k}} \mathrm{C}^{3}{ }_{\mathrm{w}} ; \mathrm{Q}_{\mathrm{n} 4}=\mathrm{C}^{4}{ }_{\mathrm{k}} \mathrm{C}^{4}{ }_{\mathrm{w}}$ etc.

So, the number of the second type errors of the order k which will lead to the wrong target detection, can be defined as follows:

$$
\begin{equation*}
\mathrm{Q}_{\mathrm{nk}}=\mathrm{C}^{\mathrm{k}}{ }_{\mathrm{K}} \mathrm{C}_{\mathrm{W}}^{\mathrm{k}} \tag{6}
\end{equation*}
$$

Taking into account the fact that the second type error of the order k is a consequence of the faulty detection of the signals simultaneously in $n=2 k$ digits, the total number of the errors can be defined as follows:

$$
\begin{equation*}
\mathrm{Q}_{\mathrm{tk}}=\mathrm{C}^{\mathrm{n}}{ }_{\mathrm{N}}=\mathrm{C}^{2 \mathrm{k}}{ }_{\mathrm{N}} \tag{7}
\end{equation*}
$$

On the basis of the expressions (6) and (7), the probability of the faulty objects' recognition at occurrence of the errors of the order k will be defined as:

$$
\begin{equation*}
P_{\text {cfik }}=Q_{n k} / Q_{t k}=C^{k}{ }_{k} C^{k}{ }_{W} / C^{2 k}{ }_{N} \tag{8}
\end{equation*}
$$

The general probability of the faulty objects' recognition at occurrence of the errors of the second type of various orders will be defined as:

$$
\begin{equation*}
\mathrm{P}_{\text {cfi }}=\sum_{\mathrm{k}=1}^{\mathrm{m}} \beta_{\mathrm{ik}} \mathrm{P}_{\text {cfik }}=\sum_{\mathrm{k}=1}^{\mathrm{m}}\left\{\left(1-\mathrm{P}_{\mathrm{i}}\right)^{2 \mathrm{k}} \mathrm{C}^{\mathrm{k}} \mathrm{~K}^{\mathrm{k}}{ }_{\mathrm{w}} / \mathrm{C}^{2 \mathrm{k}}{ }_{\mathrm{N}}\right\} \tag{9}
\end{equation*}
$$

The probability of the correct objects recognition at occurrence of the errors of various orders will be:

$$
\begin{equation*}
P_{c i}=1-P_{c f i}=1-\sum_{k=1}^{m}\left\{\left(1-P_{i}\right)^{2 k} C_{K}^{k} C^{k}{ }_{w} / C^{2 k}{ }_{N}\right\} \tag{10}
\end{equation*}
$$

Then the expressions for general probability of the correct and faulty objects recognition in the processors PCCM will be described as:

$$
\begin{align*}
& \mathrm{P}_{\mathrm{c}}=\sum_{\mathrm{i}=1}^{\mathrm{M}} \alpha_{\mathrm{i}} \mathrm{P}_{\mathrm{ci}}=\sum_{\mathrm{i}=1}^{\mathrm{M}}\left\{\alpha_{\mathrm{i}}\left[1-\sum_{\mathrm{k}=1}^{\mathrm{m}}\left\{\left(1-\mathrm{P}_{\mathrm{i}}\right)^{2 \mathrm{k}} \mathrm{C}_{\mathrm{K}}^{\mathrm{k}} \mathrm{C}_{\mathrm{w}}^{\mathrm{k}} / \mathrm{C}^{2 \mathrm{k}}{ }_{\mathrm{N}}\right\}\right]\right\}  \tag{11}\\
& \mathrm{P}_{\mathrm{fc}}=1-\mathrm{P}_{\mathrm{c}}=1-\sum_{\mathrm{i}=1}^{\mathrm{M}}\left\{\alpha_{\mathrm{i}}\left[1-\sum_{\mathrm{k}=1}^{\mathrm{m}}\left\{\left(1-\mathrm{P}_{\mathrm{i}}\right)^{2 \mathrm{k}} \mathrm{C}_{\mathrm{K}}^{\mathrm{k}} \mathrm{C}^{\mathrm{k}}{ }_{\mathrm{w}} / \mathrm{C}^{2 \mathrm{k}}{ }_{\mathrm{N}}\right\}\right]\right\} \tag{12}
\end{align*}
$$

3. The modeling of the calculations results reliability in the optical correlation processors

On the basis of expressions (1), (2), (11) and (12) there have been calculated the probabilities of correct and faulty objects recognition in the processors with a single and codified correlation maxima at the following parameters: number of objects $\mathrm{M}=10$, priory probability of the objects appearance $\alpha_{i}=1 / \mathrm{M}$, probability of the correct object recognition $\mathrm{P}_{\mathrm{i}}=0.75-0.95$, number of "units" and "zeros" in the code $\mathrm{K}=\mathrm{W}=4$, second type error maximum order $\mathrm{m}=4$.

The data presented in figure 1 show that the probability $P_{c}$ of the correct objects recognition in the processors with codified correlation maxima is higher than in the processors with a single correlation maxima and increase from the $P_{c}=0.962$ until 0.998 at change of the $P_{i}$ value from $P_{i}$ $=0.75$ to 0.95 . The probabilities of the faulty objects recognition in the processors PCCM are smaller than in the processors PSCM on 6.6 times at $P_{i}=0.75$ and on 25 times at $P_{i}=0.95$ (Fig.2).


Figure 1. The objects recognition probabilities: correct $\mathrm{P}_{\mathrm{s}}, \mathrm{P}_{\mathrm{c}}$ and faulty $\mathrm{P}_{\mathrm{fs}}, \mathrm{P}_{\mathrm{fc}}$ - in the processors PSCM and PCCM


Figure 2. Relation $\mathrm{P}_{\mathrm{fs}} / \mathrm{P}_{\mathrm{fc}}$ of the faulty objects recognition probabilities

## 4. Conclusion

An analytical estimation of targets detection reliability in the correlators has been performed. The analysis show that the probability of the correct targets detection in the correlators with codified correlation maxima is higher than in the correlators with single correlation maxima and increase until 0.998 .

The probability of faulty decisions in the correlators with codified correlation responses is much lower ( 25 times) than in the correlators with single responses.

## 5. References

1. Perju V., Casasent D. "Optical correlation processors for high-speed objects recognition and localization". In: Proc. of ICTEI-2012 - International Conference on Telecommunications, Electronics and Informatics. Chisinau, 2012.
2. Perju V. L., Perju V. V., Tsiberneac S. K., Saranciuc D. I. "Reliability of calculation results in optical-electronic holographic computer systems". In: Optical Pattern Recognition IX. - David P. Casasent, Tien-Hsin Chao, Eds. / Proc. SPIE Vol. 3386, pp.364-373(1998).
3. Perju V., Casasent D., Mardare I., Chirca O. „Experimental estimation of the recognition reliability in the optical pattern recognition systems". In Optical Pattern Recognition XVIII. David P. Casasent, Tien-Hsin Chao, Editors. Proc. SPIE 6574 (2007).
