A COMPARISON CONSTRAINECD OPTIMAL AND PARAMETRIZATION CONTROL ALGORITHMS FOR DYNAMIC PROCESSES

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Abstract: In the work are compares four different algorithms for the solution of control and state constrained optimal control problems. For dynamic processes is a given differential equations with the initial conditions. The results indicate that algorithms in general should be preferred in the order: simultaneous optimization and collocation, conjugate gradient, Goh, Teo, and Sakawa, Shindo.

Keywords: Computational methods, nonlinear programming, optimal control, control parameterization, conjugate gradient algorithm.

INTRODUCTION

Numerical algorithms computing open loop nominal controls have been proposed by numbers contributors. The key concepts of thesis algorithms are numerical integration of the state differential equations and a gradient algorithm in function space to up date the control function. The work compares four different algorithms for the solution of control and state constrained optimal control problems. The model predicative control schemes based on nonlinear processes mode have recently been proposed to cope with severe process nonlinearities [1, 2].

A different approach to the solution of the open loop optimal control problem is the control variable parameterization (CVP) strategy. The control is replaced by approximating functions, and the original optimal control problem is turned into the problem of deciding the optimal parameters are the approximating functions. This optimizations problem can be solved by a standard mathematical programming code, and state constraints are treated by Euler-Lagrange equations which provide the gradient information of the constraints with respect to the control parameters. A number of different control approximations have been suggested [3, 4]. By that approach, a good control approximation can usually be obtained with few parameters. The med to transform the control variables to ensure that the control bounds are met is a clear disadvantage.

IMPLEMENTED ALGORITHMS

The applied conjugate gradient algorithm computes the search direction by:

$$d^{i}(t) = -g_{u}^{\;i}(t) + \beta^{i-1} \cdot d^{i-1}(t); \;\; \beta^{i-1} = \frac{< g_{u}^{\;i}(t), g_{u}^{\;i}(t) - g_{u}^{\;i-1}(t) >}{< g_{u}^{\;i-1}(t), g_{u}^{\;i-1}(t) >}, \qquad (1)$$

where superscript i denotes iteration number $d(t)$ is the search direction and $g_u(t)$ is the gradient vector of the Hamiltonian with respect to the control vector at time t.

The inner product calculations in (1) for vector time functions $a(t)$ and $b(t)$ an the time interval $[t_1, t_2]$ are defined by

$$M[a(t), b(t)] = \int_{t_1}^{t_2} (a(t))^T b(t) dt$$

The classical 4th order Runge -Kutta method is applied to solve the differential state and cost equations.

Control variable up date iteration each is performed by:

$$\begin{aligned} u^{i+1}(t) &= u^{i}(t) + \alpha d^{i}(t), \\ u_{\min} &< u^{i+1}(t) < u_{\max}. \end{aligned} \qquad (2)$$

The controls are clipped at the bounds before calculating the value of the performance index in the line search phase, and the portions of the control gradients corresponding to intervals of saturated controls are omitted from the inner product calculations.

Controls are freed from the bounds by application of steepest descent when the gradient sign make this convenient.

Quadratic interpolation based on values of the performance index is applied in the line search. State variable inequality constraints and fixed final states are handled by the penalty function strategy proposed by Kelley (1962)

$$\hat{\chi} = \max\{h[x(t), t], Q\}^2,$$

where $h[x(t), t] \leq 0$ is the state variable inequality constraint, is added to the integrand of the performance index.

Given an initially small value of x, the problems are solved to meet the requested accuracy.

If the state trajectory of the solution is inside the infeasible region, x is increased by a - constant factor and the solution procedure continues from the current trajectories. This process is repeated if necessary.

One version applies constant control parameters in each finite element, and a limited number collocation points for the state approximating Lagrange polynomials within each element. The initial time and final time for each element are included as collocation points, but due to the given initial states and the state continuity at the knots between the elements, the residual equations for the first collocation point in each element are admitted. The constant controls are allowed to switch independently at the knots.

The residual equation in this implementation au given by:

$$r_i^s = \sum w_{ij}^s \cdot x_j^s - f\left(x_j^s, u^s\right) = \min,\ i = \overline{1, k+1}, s = Q, \quad (3)$$

and original performance index

$$J_a = \sum_{s=1}^{N} \sum_{i=0}^{k+1} I_i^s L\left(x_i^s, u_i^s\right) = \min\ , \quad (4)$$

where x_0^i – initial state, x_j^s – state at point j in element s, $x_0^s = x_{k+1}^{s-1}$ for $s=2, N$, u^s – control parameter vector in element s, S,N – number of elements, k – order of the Lagrange polynomial, $f(x_j^s, u^s)$ – the differential equation, x_{ij}^s – differentiation weight on state parameter vector nr. J in the residual equation for collocation point nr. i in element s, I_i^s – integration weight for collocation point I in element s, w_{ij}^s – differentiation weight on state parameter vector nr. j in the residual equation for collocation point nr. i, in element s.

The optimal control problems have been challenge a conjugate gradient function space algorithm, the algorithms due to Sakawa, Shindo and to Goh, Teo, and an implementation of a simultaneous collocation and optimization algorithm.

Piecewise constant control parameterization is applied by Goh and Teo.

The results indicate that the algorithms in general should be preferred in the order: simultaneous optimization and collocation, conjugate gradient, Goh and Teo and Sakawa and Shindo.

Sakawa and Shindo`s algorithm which was originally proposed to solve the container crane example problem, actually facile to solve that problem in a sates factory way.

The performance on the bang-bang problem was bad as well although the algorithm was excellent on the state variable inequality constrained problem, on indication was found that this algorithm should in general be preferred to the other three.

LINEAR SCHEDULING PROBLEMS

Scheduling problems arise from situations that requite the assignment of resources over a period to perform a set of activities. When the activities are deterministic these problems may be formulated as combinatorial optimization problems in the classical science. In this part it are dealing with a dynamic scheduling problems where the processing time $p_i(t)$ is defined by differential equation

$$\frac{dp_i(t)}{dt} = A_i, \quad i = \overline{1,n} \quad ,$$

with the following initial condition $p_i(t = T) = b_i$.

Hence the processing time $p_i(t) = A_i t + b$, the performance index c_i is defined as

$C_i = C_{i-1} + p_i(t), \; C_0 = T$.

Theorem1. Optical ordering for

$P_i(t) = A_i t + b_i$; $A_i > 0;\; b_i > 0;\; t_i > 0;\; \min(\max(C_i))$

is given by sequence

$\frac{A_1}{b_1} \geq \frac{A_2}{b_2} \geq ... \geq \frac{A_n}{b_n}$;

Proof.

1. Let $C_n = a_n(a_{n-1}C_{n-2} + b_{n-1}) + b_n$; for sequence 1, 2, …, n-1, n and
   $C'_n = a_{n-1}(a_n C_{n-2} + b_n) + b_{n-1}$; for sequence1, 2, …, n, n-1 $C_n \leq C'_n$;
   $a_n b_{n-1} + b_n \leq a_{n-1} b_n + b_{n-1}$; $A_n b_{n-1} \leq A_{n-1} b_n$; $A_n / b_n \leq A_{n-1} b_{n-1}$;
2. Let $C_n = a_j(a_i C_{j-2} + b_i) + b_j + p_{j+1} + ... + p_n$, for sequence $1,2,..., j,..., i,..., n$,
   and $C''_n = a_i(a_j C_{j-2} + b_j) + b_i + p_{j+1} + ... + p_n$, for sequence 1, … , j, …, m $C_n \leq C''_n$
   $a_j b_i + b_j \leq a_i b_j + b_i$; $A_j b_i \leq A_i b_j$; $A_j / b_i \leq A_i b_i$.

This means that in order to active the optimal sequence we have to order process for non-increasing values of A_l / b_l for $l \in \{1,2,..., h\}$.

Problem $p_i(t) = A_i + / \sum C_i$,

- C_i - denote a performance index for ordering of processes in the form $(1,2,...l,..., j,..., n-1, n)$.
- C''_i - denote a performance index for ordering of processes in the $(1,2,...j,..., i,..., n-1, n)$.

Theorem2. If the relation $A_i \leq A_2 \leq ... \leq A_{n-1} \leq A_n$

Are fulfilled, then the ordering of processes by means of the sequence $(1,2,..., n-1, n)$ is optimal.

Proof. The existence of a smaller value of the quality index $\sum C_i$ must have caused the existence of such a pair for which the following relation should be true

$A_{l+1} < A_l$ for $l \in \{1,2,..., n-1\}$

Despite the assumption $A_i \leq A_2 \leq ... \leq A_{n-1} \leq A$.

Problem $p_i(t) = A_t + b_i / \sum C_i$; $A_i b_i > 0, t \geq 0$.

The substance of the optimization method for nonlinear dynamic processes is to divide the set of index permutation into the variations of these indices. The base of the subsets which are method is an observation that finding k-element variation in a set comprising, n-elements, requires less than n^k computation as derived from the formula $n : |(n-k)| \leq n^k$

CONCLUSIONS

Dynamic parameters of the control systems are assumed to be unknown but constant. The three optimal problems considered challenged the four algorithms in different ways.

The conjugate gradient algorithm in function space performed reasonably well on all problems, although good tuning parameters in the penalty function approaches apparently had to be found by trial and error. Small tolerances satisfied before the first increase in the penalty parameters seemed to be choices on both penalty function problems. The optimal controller was shown to be globally stable in the sense that the control objectives achieved asymptotically. The optimization parameters of process of synchronization are carried out on the basis of mathematical information model by the structured Sakawa and Shindo`s algorithm. In the last algorithm the state variables are approximated by Lagrange polynomials while two different control parameterization are applied. Piecewise constant control parameterization is applied by Goh and Teo.

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