Synthesis of the Active Low - Pass Filters (LPF) based on RC - Elements With Distributed Parameters

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Abstract — **A method of synthesis of active filters based on RC-elements with distributed parameter is described. Is described the procedures, is made synthesis of a low-pass Buttervorth filter of the second degree. It is calculated amplitude-frequency characteristic error synthesis filter to the amplitude-frequency characteristic of second-degree Butterworth filter and the error of the amplitude-frequency characteristic of the model filter to the amplitude-frequency characteristic of second-degree Butterworth filter.**

Index Terms — **Active Low - Pass Filters, amplitude-frequency characteristic**, **Buttervorth filter, RC - Elements With Distributed Parameters, second-degree Butterworth filter.**

I. INTRODUCTION

The indispensable step of active filter design presents the procedure of their synthesis. The notion of synthesis is understood the finding of the principal electrical scheme of the filter and the parameters of its components, which would have a set of desired properties, indicated before the synthesis procedure. This can be illustrated by the diagram of fig. 1, which can be read as: given a set of properties of filter. Is necessary to find filter (electric principle scheme and parameters of the scheme components), which would the same properties.
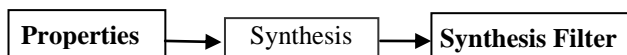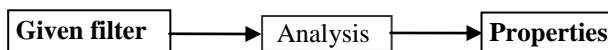

Figure 1 The procedure of filter synthesis

Figure 2 The procedure of filter analysis

In the procedure of the filter analysis is given a diagram of principle scheme and values of they components, but need to find the properties and characteristics in the frequency or time of the given filter. It should be mentioned that in the synthesis of active filter with pre-set properties so there is a feature that the lack of solutions or possible solutions to obtain more technical data features. In other words, if the technical solution exists, then it can lead to the scheme by several characteristics of the data or data about. In this case from all technical resolutions found necessary to select that technical solution that will ensure obtaining a circuit that would meet the practical point of view of circuit type and minimum number of passive and active elements. Decisive role in assessing of the quality of selected technical solution is the possibility of achieving practical simple, requirements of mass and dimensions , economic

and ergonomic requirements. It should be mentioned that the synthesis and design concepts are not equivalent. Usually the result of an active filter synthesis can be achieved a number of variants of technical solutions. The procedure for selecting the best options from a viewpoint or another, called optimization, refers to the design.

II. LPF BASED on $\overline{RC}$ - ELEMENTS

In literature is widely described procedure for synthesis of active RC filters based on discrete elements [1, 2 and others]. But nowhere describes the procedure for synthesis of active filters based on RC elements with distributed parameters ($\overline{RC}$ - elements). In this paper we describe the method of synthesis of active filters on $\overline{RC}$ - elements. As a basic circuit diagram is used from fig. 3.
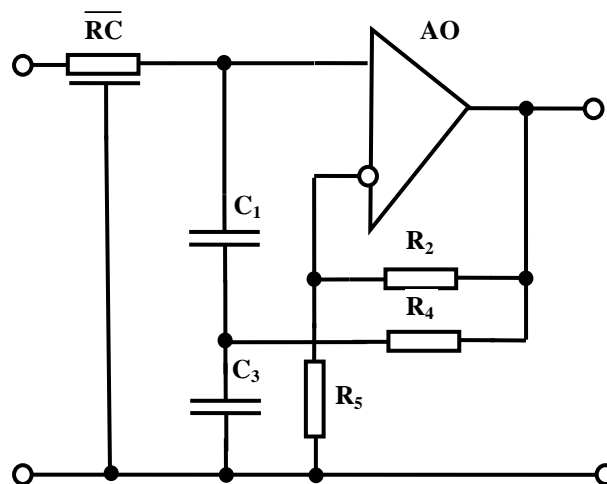

Figure 3. LPF scheme based on $\overline{RC}$ -element

The transfer function of this scheme is expressed by the expression:

$$T(\Theta) = \frac{1 + (1+n)\alpha\Theta^2}{[1+(1+n)\alpha\Theta^2]ch\Theta + \beta\Theta(\alpha\Theta^2 - \gamma)sh\Theta} \quad (1)$$

where:

$$\Theta = \sqrt{p\overline{RC}} \quad , \quad n = \frac{C_3}{C_1}, \quad \beta = \frac{C_1}{\overline{C}}, \quad \gamma = \frac{R_4}{R_5},$$

$$\alpha * \beta = \frac{C_1^2 R_2}{\overline{C}^2 \overline{R}}, \quad \rho = jw, \quad \tau = \overline{RC}.$$

Transfer function (1) is transcendental and the variable $\rho = jw$ is contained under the signs of hyperbolic functions **sh, ch** and under the sign $\sqrt{\ }$. Therefore in such a form it can not be used. In paper [3] has been shown that transfer functions of active filter, based on $\overline{RC}$-elements can be expressed by transfer approximated functions so as second and third degree, depending on the number of terms used in approximating hyperbolic functions and imposed performance conditions. To achieve the second degree LPF transfer function (1), approximated by the equation fractional-rational polynomial that is characteristic of second-degree described by the following expression:

$$T_2(p) = \frac{1 + ap}{1 + b_1 p + b_2 p^2} \quad (2)$$

where:

$a = (1+n)\alpha\tau,$

$b_1 = [0,333 + \alpha(1+n) - \gamma\beta]\tau,$

$b_2 = [0,333(1+n) + \beta n]\alpha\tau^2$ and other notations are the same as in the expression (1).

To pass the canonical form of transfer function (2) is necessary to make the changes of *p* variable in *s* plan . Then transfer function (2) may be transcribed in the following form:

$$T_2(s) = \frac{1 + As}{1 + Bs + s^2} \quad (3)$$

where:

s=j$\lambda (0,333\,\alpha(1+n) + \alpha\beta n)^{1/2}$,

A=α(1+n)[0,333*α(1+n)+$\alpha\beta$n]$^{-1/2}$,

$$B = \frac{0,333 + \alpha(1+n) - \gamma\beta}{\sqrt{0,333\,\alpha(1+n) + \alpha\beta n}}.$$

III. LPF SYNTHESIS BASED ON $\overline{RC}$-ELEMENT. CALCULATIONS AND EXPERIMENTAL RESULTS

We describe the method of synthesis of second degree active LPF based on an example.

Example. It is necessary to perform the synthesis of LPF with maximally flat amplitude-frequency characteristic (AFC) with cutting frequency f_t = 290 Hz and guaranteed attenuation at frequencies above that $2 f_t$ no less than 12 dB.

For second degree LPF, based on $\overline{RC}$ - elements the normalized cutoff frequency λ_t may vary within from 3,3 up to 7,2 [4]. If we choose $\lambda_t = 4$, the time constant τ of the $\overline{RC}$ - elements can be found from the expression

$$\tau = \lambda_t/2\pi f_t = 4/2\pi f_t = 2,1926*10^{-3}s \quad (4)$$

Made from coaxial microwire with linear resistance on $60 \div 90$ kOm/m and linear capacity of $300 \div 350$ pF/m, the $\overline{RC}$ - element having the following parameters $\overline{R}$ =765,572 kOm and $\overline{C}$ =2,864*10^{-9} F.

Because in the *s* plane would realized maximally flat second degree AFC it is required, that in equation (3) to satisfy the conditions A = 0 and B = 1,4142. But the condition A = 0 corresponds to the condition α (1 + n) = 0, what is not feasible for this scheme. Therefore to decrease the action of the coefficient A on the behavior of AFC in the passband and stopband will use a value of A in ten times lower than the counter value of the constant term expression (3), ie we use a A = 0,1. Besides this, taking into consideration the fact, that the normalized cutoff frequency (in this case $\lambda_t = 4$) term in the square a normal characteristic equation must be equal to 1, compose the following system of equations

$$\begin{cases} (\alpha K)^{1/2} \lambda_t = 1 \\ \alpha(1+n)\{\alpha[0,333(1+n) + \beta n] \}^{-1/2} = 0,1 \\ [0,333 + \alpha(1+n) - \gamma\beta](\alpha K)^{-1/2} = 1,4142 \end{cases} \quad (5)$$

where

K= $0,333\alpha(1+n) + \beta n$.

The system of equations (5) consists of three equations with four unknown. Therefore in such form system (5) has no solution. If the reasons unification scheme component values in fig. 3, we impose the condition C1=C3 and then get n=1, the system of equations (5) may be transcribed as follows:

$$\begin{cases} [\alpha(0,666 + \beta)]^{1/2} = 0,25 \\ 2\alpha[\alpha(0,666 + \beta)]^{-1/2} = 0,1 \\ (0,333 + 2\alpha - \gamma\beta)[(0,666 + \beta)]^{-1/2} = 1,4142 \end{cases} \quad (6)$$

In the result of solving system (6) is obtain $\alpha = 0,0125$, $\beta = 4,333$, $\gamma = 0,001$. When elements of the scheme in fig. 3 should have the following values: $C_1 = C_3 = \beta \overline{C} = 12,41*10^{-9}$ F; $R_2 = \alpha\tau/C_1 = 2,208$ kOm and fixing the value of $R_5 = 100$ kOm obtained as $R_4 = 110$ Om. To calculate the AFC of the synthesed filter it is necessary to calculate the coefficients A and B of equation (3) in *s* and *p* plans that are related to the parameters of the scheme elements indicated in fig. 3 with relations indicated in the notation of the equations (1) and (3). For the parameter values indicated above scheme components and calculated as a result of solving system equations (6) the coefficients A and B have the values A = 0,1 B = 1,4142.

The module of the transfer function (or AFC) to LPF synthesis is calculated by equation

$$|T_2(\Omega)| = \sqrt{\frac{1 + (A\Omega)^2}{1 + (B^2 - 2)\Omega^2 + \Omega^4}} \qquad (7)$$

Results of calculations of AFC of the synthesed LPF are shown in Table 1. In this table are given to AFC Butterworth LPF grade 2 and AFC experimental high on experimental models of filter synthesis.

TABLE 1. RESULTS OF CALCULATIONS AND MEASUREMENTS AFC OF A SYNTHEZED LPF

F, Hz	29	58	87	116	145	174	203	232	261	290	580
Ω	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0	2,0
$T(\Omega)_B$	0,9999	0,9992	0,9959	0,9874	0,9701	0,9409	0,898	0,8423	0,7770	0,7071	0,2425
$T(\Omega)calc$	0,9999	0,9994	0,9964	0,9881	0,9713	0,9425	0,9000	0,8448	0,7800	0,7100	0,2470
$T(\Omega)_{exp}$	1,000	0,999	0,998	0,996	0,980	0,951	0,905	0,847	0,780	0,707	0,242
δ_{calc}, %	0	0,02	0,05	0,07	0,12	0,17	0,22	0,29	0,38	0,41	1,84
δ_{exp}, %	0	-0,02	0,21	0,87	1,02	1,07	0,78	0,56	0,38	0,014	-0,20

It must specify that, to obtain the necessary values of capacitors C_1 and C_3 of synthesis filter in assembly model used two capacitors each type K73-11, joined in parallel with nominals $10*10^{-9}$ F and $2,4*10^{-9}$ F with error $\pm$ 10%. Experimental model AFC filter was measured only after increasing by 0,8% the value of R2 and the value of R4 with 0,35% to obtain the cutting frequency of the experimental filter on amplitude with value equal to 0,7071 (-3 dB) to the value AFC at $\Omega = 0$, where the value of AFC = 1. In the last two lines of the table are calculated deviations δ_c AFC LPF synthesis filter of the formula (7) to the AFC classic Butterworth LPF second degree (penultimate line) and the corresponding deviation δ_e AFC experimental model of LPF to AFC Butterworth classic (last line). The data table shows that with growing frequency it is increasing the value of δ_c and at the bandwidth limit reaches 0,41%, and at the beginning of the stopband the value of $\delta_c = 1,84\%$, ie attenuation of the AFC for synthesis LPF at the beginning of the stopband is with less with 1,84% than the Butterworth filter attenuation. Because the AFC experimental models built by the filter was partially given by the method described above, the data table is noted that at the cutting frequency and at the starting frequency of the stopband the AFC of the experimental model of synthesized filter completely coincides with the AFC of the Butterworth filter. In the passband the value δ_e varies from -0,02 % to 1,07% and in the stopband has value not exceeding -0,02%.

IV. CONCLUSIONS

In conclusion we mention that the results obtained in this paper confirm the possibility to using the described method for the synthesis of second degree LPF based $\overline{RC}$ - elements with the error not greater than $1\div2$ % on the classical Butterworth filters.

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