# The Interconnection of Polarization Singular Structure and Mueller-matrix Images of Biological Tissues in the Tasks of Cancer Changes Diagnostics

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Abstract – The paper deals with investigating the processes of laser radiation transformation by biological crystal networks using the singular optics techniques. The obtained results showed a distinct correlation between the points of "characteristic" values of coordinate distributions of Mueller matrix ( $M_{ik} = 0, \pm 1$ ) elements and polarization singularities (*L*- and *C*-points) in laser images of biological crystal networks with

elements and polarization singularities (*L*- and *C*-points) in laser images of biological crystal networks with the following possibility of Mueller-matrix selection of polarization singularity. It has been proposed the technique of Mueller-matrix singular diagnostics of pathological changes in woman reproductive sphere tissue (myometrium).

Index Terms – Mueller matrix, singularity, polarization, birefringence, biological tissue, statistics.

### I. INTRODUCTION

Laser polarimetry (polarization sensitive optical coherence tomography, Mueller matrix decomposition formalism, coherency matrix analysis, etc.) [1-29] is able to obtain information about optical anisotropy (linear retardance, optical activity, dichroism, etc.) of biological tissues (BT). It has the potential to be an important technology for non-invasive diagnostics of organic phase-inhomogeneous layers. One model approach underlies these researches, which generalizes the optical properties of BT [5, 6, 14, 21, 30]:

- 1. all the variety of human BT can be represented by four main types connective, muscular, epithelial, and neural tissues;
- 2. morphological structure of any BT type is regarded as a two-component amorphous-crystalline structure (optical isotropic and optical anisotropic);
- 3. the crystalline component is characterized by Mueller matrix  $\{M\}$  operators of an optical anisotropy

$$\{M\} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & M_{22} & M_{23} & M_{24} \\ 0 & M_{32} & M_{33} & M_{34} \\ 0 & M_{42} & M_{43} & M_{44} \end{vmatrix}$$
(1)

Specifically, the above mentioned model was used for finding and substantiating the interrelations between the ensemble of statistic moments of the  $1^{st}$  to  $4^{th}$  orders that characterize the orientation-phase structure (distribution of optical axes and phase shifts for directions of protein fibril networks) of birefringent BT architectonics and of 2D distributions for azimuths and ellipticities in their laser images [5]. It was determined [6, 7, 14, 15, 20, 23, 27, 28] that the  $3^{rd}$  and  $4^{th}$  statistic moments for coordinate distributions of ellipticities are the most sensitive to the change (caused by dystrophic and oncological processes) of

optical anisotropy inherent to protein crystals. On this basis, the criteria for early diagnostics of muscle dystrophy, precancer states of connective tissue, collagenoses etc. were determined.

In parallel with traditional statistical investigations, formed in recent 10 to 15 years is the new optical approach to describe a structure of polarization inhomogeneous fields in the case of scattered coherent radiation. The main feature of this approach is the analysis of definite (in contrast to continuous 2D distributions) polarization states to determine the whole structure of coordinate distributions for azimuths and ellipticities of polarization. The so-called polarization singularities are common used as these states [31-37]:

- states with linear polarization, when the direction of rotation for the electric field vector is indefinite, the socalled *L*-points;
- circularly polarized states, when the azimuth of polarization for the electric field vector is indefinite, the so-called *C*-points.

Investigations of polarization inhomogeneous object fields for BT with different morphology allowed ascertaining that they possess a developed network of *L*- and *C*-points [36]. For example in [37], the authors found interrelations between conditions providing formation of polarization singular points and particularity of the orientation-phase structure of biological crystals present in territorial matrix of human tissue architectonic network. These interrelations served as a base to make statistical and fractal analyses of distribution densities for the number of singular points in BT images. As a result, the authors confirmed the efficiency of this method for investigation of object fields to differentiate optical properties of BT with a different morphological structure and physiological state.

It is worth to note that the singular approach is mainly used out of the analysis of the mechanisms providing formation of polarization inhomogeneous laser images of BT by an extracellular matrix. Thus, development of laserpolarimetric techniques based on determination of singular interrelations "object - field" in order to find new methods of diagnostics of transformation of the BT extracellular matrix orientation-phase structure related with pre-cancer changes in their physiological state is very important.

To solve this problem, we should revert to the analysis of optical properties of biological crystal networks as these properties are comprehensively described by the Mueller matrix within the framework of the singular approach.

### II. BRIEF THEORY OF THE SINGULAR APPROACH IN THE ANALYSIS OF BIOLOGICAL TISSUE BIREFRINGENT NETWORKS

To analytically describe the L- and C-states of polarization, the most suitable is to use the extreme values of the fourth Stokes vector parameter. It is a widely applied mean and appears to be reasonable to represent these singularities as follows:

$$\begin{cases} V_4 = \sin 2\beta; \\ V_4 = 0 \leftrightarrow L(\beta = 0); \\ V_4 = \pm 1 \leftrightarrow \pm C(\beta = \pm \frac{\pi}{4}). \end{cases}$$
(2)

Here,  $\beta$  is the value of polarization ellipticity; the + C point is assumed to designate a right-circulated polarization state, which can be characterized by the phase shift  $\delta = \frac{\pi}{2}$  between the orthogonal components of laser beam amplitude; the -C point is assumed to designate a left-circular polarization state ( $\delta = -\frac{\pi}{2}$ ). Thus,  $\pm C$  points are the orthogonal states of circularly polarized wave. For the L points, the phase shift reaches  $\delta = 0; \pi$ .

Using the relations (1) and (2), it is possible to determine the interrelations between the characteristic values  $M_{ik}^*$  of Mueller matrix elements, which correspond to the "extreme" values of BT optical anisotropy, and to the extreme values of  $V_4$ . Thus, in order to characterize the BT structure we obtain the coordinate network generated by a finite number of characteristic values of Mueller matrix elements (1) (see Table 1).

Thus, measuring the coordinate distributions of the characteristic values  $(M_{ik}^* = 0, \pm 1)$  of the BT Mueller matrix elements enables not only to foresee the scenario  $(M_{ik}^* \rightarrow V_4^*)$  of forming the ensemble of polarization singularities  $(V_4 = 0, \pm 1)$  of its image, but also to additionally realize their differentiation, conditioned by the specificity of orientation structure of biological crystals.

Here, +L point corresponds to a linear polarization state with  $\delta = 0$ ; -L point corresponds to a linear polarization state with  $\delta = \pi$ . In this case, as for  $\pm C$ points, we will identify  $\pm L$  points as orthogonal.

# TABLE I. INTERRELATION BETWEEN THE CHARACTERISTIC VALUES OF MUELLER MATRIX $\boldsymbol{M}_{ik}$ elements of

BIOLOGICAL TISSUES AND POLARIZATION SINGULARITIES DESCRIBED BY THE FOURTH PARAMETER OF THE STOKES  $_{\rm VECTOP}V$ 

VLETOK V <sub>4</sub>			
M <sub>ik</sub>		$V_4$	Polarization state
M <sub>22;33;44</sub>	0	±1	$\pm C$
	1	0	+L
	-1	0	-L
$M_{{}_{24;42;34;43;23;32}}$	0	0	$\pm L$
	1	+1	+ <i>C</i>
	-1	-1	-C

## III. SCHEME AND METHODS OF EXPERIMENTAL INVESTIGATIONS

Fig. 1 shows the traditional optical scheme of polarimeter for measuring the elements of Mueller matrix of the BT histological sections [5, 20].



Fig. 1. Optical scheme of the polarimeter. 1 – He-Ne laser; 2 – collimator; 3 – stationary quarter-wave plates; 5, 8 – mechanically movable quarter-wave plates; 4, 9 – polarizer and analyzer, correspondingly; 6 – object of investigation; 7 – micro-objective; 10 – CCD camera; 11 – personal computer.

The parallel ( $\emptyset = 10^4 \ \mu m$ ) beam of He-Ne laser ( $\lambda = 0.6328 \ \mu m$ , W = 5.0  $\mu$ W) was used as an illuminator. Polarization illuminator consists of the quarter-wave plates 3, 5 and polarizer 4, and it sequentially forms a series of linearly polarized ( $I_0$ ,  $I_{45}$ ,  $I_{90}$ ,  $I_{135}$ ) with azimuths 0°, 90°, 45°, 135°, and right-hand ( $I_{\otimes}$ ) and left-hand ( $I_{\oplus}$ ) circularly polarized probing BT laser beams. The BT images made by the micro-objective 7 (4×) were projected into the plane of a light-sensitive area ( $800 \times 600$  pixels) of the CCD-camera 10.

Polarization images of BT were projected by means of the micro-objective 7 (focal distance - 1.5 cm , aperture - 0.2, magnification -4x) into the plane of light-sensitive area of the CCD camera (overall amount of pixels - 800x600, light sensitive area size - 4000x3000 µm, deviation of photosensitive characteristics from the linear one was no more than 15%), which provided the range of measuring the structural elements of BT with the resolution  $2-2000 \mu m$ . Maximal resolution verification (2  $\mu m$ ) were performed using the stage micrometer (linear scale), which image was projected into the light sensitive area of CCD camera using the micro-objective 7. Minimal resolution ( $2000 \mu m$ ) corresponds to the situation when the light sensitive area of CCD camera is entirely filled by two equal sized structural elements (light and dark) of stage micrometer. The experimental conditions were chosen in such a way that it enabled to reduce the space-angular aperture filtering when forming the BT images. It was ensured by conformance of angular characteristics of the

indicatrix of light scattering by the BT samples ( $\Omega \approx 16^{\circ}$ ) and the angular aperture of micro-objective ( $\Delta \omega = 20^{\circ}$ ). Here,  $\Omega$  is the solid angle within which 98% of all the energy of light-scattered radiation is concentrated.

Polarization analysis of the BT images was performed using the polarizer 9 and quarter-wave plate 8 according to the following technique:

• within the section of illuminating laser beam, the array ( $m \times n = 800 \times 600$ ) of values for the Stokes vector  $V_{j=1,2,3,4}$  parameters and elements of Mueller matrix  $M_{ik}(m \times n)$  were determined in accord with the following algorithms

$$\begin{cases} V_1 = I_0 + I_{90}, & V_2 = I_0 - I_{90}, & V_3 = I_{45} - I_{135}, & V_4 = I_{\odot} - I_{\odot}, \\ M_{i1} = 0.5 \begin{bmatrix} V_i^{(1)} + V_i^{(2)} \end{bmatrix}, & M_{i2} = 0.5 \begin{bmatrix} V_i^{(1)} - V_i^{(2)} \end{bmatrix}, & M_{i3} = V_i^{(3)} - M_{i1}, & M_{i3} = V_i^{(4)} - M_{i1}, i = 1, 2, 3, 4. \end{cases}$$
(3)

• in each array  $M_{ik}(m \times n)$  and  $V_4(m \times n)$ , coordinate distributions of characteristic (singular) values 0,  $\pm 1$  were determined.

At the first stage, the interrelations  $(M_{ik}^* \rightarrow V_4^*)$  of matrix and polarization singularities were investigated on the sample of a histological section prepared from healthy skin derma.

Fig. 2 represents coordinate distributions of matrix elements  $M_{44,24,34}(m \times n)$  corresponding to the skin derma histological section and the fourth Stokes vector parameter  $V_4(m \times n)$  of its image with the characteristic values  $(0, \pm 1)$  plotted on them (within the marked 100 pix × 100 pix sampling plot).

It can be seen from the data obtained that there is direct correlation between the coordinate  $(k, g \ 1 \le k \le m, 1 \le g \le n)$  positions of characteristic values of the matrix element  $M_{44}^*$  for skin derma and the network of L and C points in its laser image  $\left\{M_{44}^*(k,g) = \begin{cases} 0 \\ \pm 1 \end{cases} \Leftrightarrow V_4^*(k,g) = \begin{cases} \pm 1 & -\pm C \\ 0 & -L \end{cases}\right\}$  (Fig. 2a, d).

Coordinate distributions of characteristic values of matrix elements  $M_{24,42}^*(m,n)$ ,  $M_{34,43}^*(m,n)$  and corresponding networks of orthogonal  $\pm L$  and  $\pm C$  points (Table 1) possess the individual structure (Fig. 2b, c).

Analytically substantiated and experimentally proven interrelations between the matrix and polarization singularities were used as the basis for Mueller-matrix singular diagnostics of oncological changes in the tissues of woman reproductive sphere.

of matrix elements  $M_{44,24,34}$  (a, b, c) and singularities of polarization image inherent to the skin derma layer histological section  $V_4$  ("d"):

$$\begin{array}{rcl} - & ``\pm C"\text{-points} (\Box) (M_{44} & - & ``+L"\text{-points} (\Delta) (M_{44} \\ & = 0); & & = +1); \\ - & ``+C"\text{-points} (\Box) & & - & ``-L"\text{-points} (\nabla) (M_{44} \\ & & (M_{24,34} = +1, V_4 = +1); & & = -1); \\ & & & - & ``\pm L"\text{-points} (\diamond) \end{array}$$

- "-C"- points (
$$\square$$
)  
( $M_{24,34}=0, V_4=0$ ).  
( $M_{24,34}=0, V_4=0$ ).



Fig.2. Networks of characteristic values  $M^*_{44,24,34}(m imes n)$ 

### IV. MUELLER-MATRIX SINGULAR DIAGNOSTICS AND DIFFERENTIATION OF PATHOLOGICAL CHANGES IN THE TISSUES OF WOMAN REPRODUCTIVE SPHERE

Three groups of histological sections of the main tissue of woman reproductive sphere – myometrium – were used as the objects of investigation:

• biopsy of the healthy tissue of woman reproductive sphere (type "A" – Fig. 3a);

biopsy of the inflamed tissue (ectonia) (type "B" – Fig. 3b);

• biopsy of the tissue in the state of dysplasia (precancer state) (type "C" – Fig. 3c).



Fig. 3. Polarization images of woman reproductive sphere tissue – myometrium "A" (a), "B" (b) and "C" (c) types in the scheme of coaxial polarizer and analyzer.

To determine the criteria of Mueller-matrix singular diagnostics of myometrium oncological state and differentiate its severity degree, the following technique was used:

• coordinate networks of characteristic values for matrix elements  $M^*_{44,24,34}(m \times n) = 0$ ,  $\pm 1$  were scanned in the direction  $x \equiv 1$ , ..., m with the step  $\Delta x = 1$ pixel;

• within the obtained sampling  $(1_{pix} \times n_{pix})^{(k=1, 2, ..., m)}$  for coordinate distribution of the element  $M_{44}(m \times n)$ , the total amount  $(N^{(k)})$  of characteristic points  $(0, \pm 1)$  that set the complete ensemble of singular points was calculated, and the

dependences  $N(x) \equiv (N^{(1)}, N^{(2)}, ..., N^{(m)})$  were determined;

• distributions of the number of singular  $\pm L$  and  $\pm C$ -points were determined using the following expressions

$$\begin{cases} N^{(+)}(x) = N_C (M_{34,43} = +1) + N_L (M_{24,42} = 0), \\ N^{(-)}(x) = N_L (M_{34,43} = 0) + N_C (M_{24,42} = -1). \end{cases}$$
(4)

• statistical moments of the 1st to 4th orders for the N(x)

obtained distributions of N(x) amount of singularities were calculated according to the algorithms

$$Z_{1} = \frac{1}{m \times n} \sum_{i=1}^{m \times n} |N(x)|,$$

$$Z_{2} = \sqrt{\frac{1}{m \times n}} \sum_{i=1}^{m \times n} [N(x)]^{2},$$

$$Z_{3} = \frac{1}{Z_{2}^{3}} \frac{1}{m \times n} \sum_{i=1}^{m \times n} [N(x)]^{3},$$

$$Z_{4} = \frac{1}{Z_{2}^{4}} \frac{1}{m \times n} \sum_{i=1}^{m \times n} [N(x)]^{4}.$$
(5)

It follows from the data obtained that:

• efficiency of laser polarimetry for diagnostics and differentiation of early oncological changes of myometrium tissue is insufficient – the difference between the values of statistical moments  $(Z_{1,2,3,4}(M_{44,34,24}(m \times n))))$  of samples "A", "B"

and "C" - types is insignificant and does not exceed 20% - 45%;

 technique of polarization-correlation mapping is efficient for differentiation of optical properties of healthy and oncological changed myometrium tissue

- the skewness  $(Z_3)$  and the kurtosis  $(Z_4)$  of distribution of the number of singular points of "A"and "B"-types of laser images differ by 1.53 and 2.15 times;

- technique of Mueller-matrix singular diagnostics is efficient for differentiation of optical properties of all types of samples statistical moments of the 3rd and 4th orders of distributions N(x) for the samples "A", "B" and "C" types differ by 1.7 and 2.5 times, respectively.
- for distributions  $N^{-}(x)$ ) of myometrium tissue of "A" and "B" types, the maximal difference (from 2.2 to 4.1 times) is observed between all the statistical  $Z_{j=1,2,3,4}$  moments.

#### V. CONCLUSION

It has been ascertained correlation between coordinate locations of characteristic points for 2D elements of the Mueller matrix corresponding to an optically thin layer of biological tissue and the network of L and C points in its laser image. Shown is the potentiality of Mueller-matrix sampling for polarization singularities formed by biological crystals. It has been demonstrated the efficiency of Muellermatrix singular diagnostics not only for oncological changes of myometrium tissue but also for differentiating their severity degree.

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