Statistic and Fractal Processing of Human Biological Fluids Phase-Inhomogeneous Images

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Abstract – Performed in this work are complex statistic and fractal analyses of phase properties inherent to birefringence networks of optically thin layers prepared from human bile. Within the framework of a statistic approach, the authors have investigated values and ranges for changes of statistic moments of the 1-st to the 4-th orders that characterize coordinate distributions for phase shifts between orthogonal components of amplitudes inherent to laser radiation transformed by human bile with various pathologies. The correlation criteria for differentiation of phase maps describing pathologically changed liquid-crystal networks are determined. In the framework of the fractal approach, determined are dimensions of self-similar coordinate phase distributions as well as features of transformation of logarithmic dependences for power spectra of these distributions for various types of human pathologies.

Index Terms – polarization, phase, fractal, biological fluid, statistic moments, birefringence.

I. INTRODUCTION

Among the methods for optical diagnostics of human biological tissues (BT), the methods of laser polarimetric diagnostics aimed at their optically-anisotropic structure are widely spread [1 - 31]. The main "information product" of these methods is availability of coordinate distributions for azimuths $\alpha(x, y)$ and ellipticity $\beta(x, y)$ of polarization (polarization maps) with the following types of analyses: statistical (statistical moments of the 1-st to 4-th orders [5, 6, 10, 14, 19, 25, 26, 30]), correlation (auto- and joint correlation function [12, 17, 18, 21, 26]), fractal (fractal dimensionalities [5, 6, 25]), singular (distributions of amounts of linearly and circularly polarized states [22, 28]). As a result, interrelations between the set of these parameters and distributions of optical axis directions as well as values of the birefringence characterizing the network of optically uniaxial protein (myosin, collagen, elastin, etc.) fibrils in optically anisotropic components of BT layers can be determined. Using this base, developed is a set of methods for early recognition and differentiation of pathological changes in BT structures related with their degenerativedystrophic and oncological changes [4-6, 12, 19, 20-22, 27, 29, 31].

It is noteworthy that there exists a widespread group of optically anisotropic biological objects for which the methods of laser polarimetric diagnostics are not so efficient. Optically-thin (the attenuation coefficient $\tau \le 0.1$) layers of various biological fluids (bile, urine, liquor, synovial fluid, blood plasma, etc.) can be related to these objects. All these layers possess considerably less optical anisotropy of the biological component matter as compared with birefringent BT structures [4]. As a consequence, these objects weakly

modulate polarization of laser radiation $\begin{pmatrix} \alpha(x, y) \approx const; \\ \beta(x, y) \rightarrow 0. \end{pmatrix}$.

On the other hand, the biological fluids are more available for a direct laboratory analysis as compared to traumatic methods of BT biopsy. From the above reasoning, it seems topical to search new, additional parameters for laser diagnostics of optically anisotropic structures in biological fluids.

This work is aimed at searching the possibilities to

perform diagnostics of structures inherent to liquid-crystal networks of human bile with various pathologies by using the method to determine the coordinate distributions of phase shifts (phase maps) between orthogonal components of laser radiation amplitudes with the following statistical, correlation and fractal analyses of these distributions.

II. THE OPTICAL MODEL OF HUMAN BILE

As a base for modeling the optical properties of human bile we use the conception of anisotropy observed in BT protein networks developed in [1-4, 7, 9, 14, 16, 23-27, 30]:

• human bile can be considered as a two-component amorphous-crystalline structure;

• optically isotropic - optically homogeneous micellar solution;

• optically anisotropic - liquid-crystalline phase, consisting of three types of liquid crystals: needle crystals of fatty acids, cholesterol monohydrate crystals, bilirubinate crystals of calcium.

The optical properties of amorphous $\{A\}$ and crystalline $\{C\}$ components of biological fluids can be exhaustively described using the following Jones operators [26]

$$\{A\} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = \begin{vmatrix} \exp(-\pi) & 0 \\ 0 & \exp(-\pi) \end{vmatrix}; \quad (1)$$

$$\{C\} = \begin{vmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{vmatrix} =$$

$$= \begin{vmatrix} \cos^{2} \rho + \sin^{2} \rho \exp(-i\delta) & \cos \rho \sin \rho [1 - \exp(-i\delta)] \\ \cos \rho \sin \rho [1 - \exp(-i\delta)] & \sin^{2} \rho + \cos^{2} \rho \exp(-i\delta) \end{vmatrix}.$$

Here, τ is the absorption coefficient for laser radiation in the biological fluid layer with the geometric thickness l; ρ - direction of the optical axis; $\delta = \frac{2\pi}{\lambda} \Delta nd$ - phase shift between the orthogonal components E_x and E_y of the amplitude of illuminating laser light with the wavelength λ ; Δn - index of birefringence.

The Jones matrix of the biological fluid layer, where isotropic and anisotropic creations lie in one plane, can be

(4)

expressed as a sum of operators $\{A\}$ and $\{C\}$

$$\{M\} = \{A\} + \{C\} = \begin{vmatrix} a_{11} + c_{11}; & a_{12} + c_{12}; \\ a_{21} + c_{21}; & a_{22} + c_{22}, \end{vmatrix},$$
(3)

Let us consider the process of transformation of the complex amplitude $(E \rightarrow U)$ of a laser wave that passed through the biological fluid layer $(\{M\})$ located between two crossed phase filters – quarter-wave plates $(\{\Phi_1\})$ and $\{\Phi_2\}$) and polarizers $(\{P_1\})$ and $\{P_2\}$, planes of transmission for which make +45⁰ and -45⁰ angles with axes of the highest velocity. The amplitude U of the transformed laser beam in this experimental setup can be determined from the following matrix equation

Here,

$$E = \begin{pmatrix} E_{x} \\ E_{y} \exp(-i\delta_{0}) \end{pmatrix}, \quad U = \begin{pmatrix} U_{x} \\ U_{y} \exp(-i\delta) \end{pmatrix}, \\ \{P_{1}\} = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}, \quad \{P_{2}\} = \begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix}, \\ \{\Phi_{1}\} = \begin{vmatrix} 1 & 0 \\ 0 & i \end{vmatrix}, \quad \{\Phi_{2}\} = \begin{vmatrix} i & 0 \\ 0 & 1 \end{vmatrix}.$$
(5)

 $U = 0.25 \{P_2\} \{\Phi_2\} \{M\} \{\Phi_1\} \{P_1\} E$.

In the special case of a plane-polarized wave $E(E_x = E_y; \delta_0 = 0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, Eq. (4) acquires the form $U = 0.25 \begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix} \begin{vmatrix} i & 0 \\ 0 & 1 \end{vmatrix} \times$

$$\times \begin{vmatrix} \cos^{2} \rho + \sin^{2} \rho \exp[-i\delta] & \cos \rho \sin \rho \{1 - \exp[-i\delta]\} \\ \cos \rho \sin \rho \{1 - \exp[-i\delta]\} & \sin^{2} \rho + \cos^{2} \rho \exp[-i\delta] \end{vmatrix} \times (6)$$
$$\times \begin{vmatrix} 1 & 0 \\ 0 & i \end{vmatrix} \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} \begin{pmatrix} 1 \\ 1 \end{vmatrix}.$$

The solution of the matrix equation (6) is the value of complex amplitude $U(\delta)$ that is determined exclusively by the phase shift δ and does not depend on orientation of the optical axis ρ for a laser image of biological fluid. Being based on it, one can write

$$I_{\delta}(r) = UU^* = I_0 \sin^2 \left[\frac{\delta}{2} \right]. \tag{7}$$

Here, I_0 is the intensity of a probing laser beam, $I_{\delta}(r)$ intensity of the laser image for the biological fluid layer in the point (r).

Interrelations (4) to (7) define the algorithm for direct experimental measuring the coordinate distribution of phase shifts $\delta(r)$ between orthogonal components of the amplitudes U_x, U_y in the laser image of an optically anisotropic biological fluid layer.

III. TABLES, FIGURES, EQUATIONS

As objects for experimental studying, we chose opticallythin layers of bile taken from a healthy patient (figure 1a) and patient suffering from insulin-independent diabetes (figure 1b).



Fig. 1. Polycrystalline networks of human bile taken from healthy patient (a) and patient suffering from insulinindependent diabetes (b).

The images of layers prepared from human bile (figure 1) are indicative of availability of two fractions – optically isotropic and liquid-crystal network (anisotropic one). As seen, geometric structure and sizes of separate elements in the polycrystalline network of the samples prepared from biological fluids are individual for physiological state.



Fig. 2. Coordinate (a, b) and quantitative (c, d) distributions δ of laser images for the samples of bile taken from healthy patient's (a, c, e) and with insulin-independent diabetes (b, d, f).

Shown in figure 2 are the phase maps (fragments (a), (b), (c), (d)) and histograms (fragments (e), (f)) for distributions of random values inherent to the phase shifts δ between orthogonal components of the laser radiation amplitude transformed inside layers of bile taken from healthy patient's (left column) and from patient suffering from insulin-independent diabetes (right column).

The obtained data show that the value of phase shifts δ for laser radiation transformed inside layers of human bile lies within the short range of changes $0 \le \delta \le \pi$. The weak

phase modulation is related with two factors. First, it is low geometric thickness ($d = 10...15 \ \mu m$) of the samples. Second, it is weak birefringence ($\Delta n \sim 10^{-4}...10^{-2}$) of liquid-crystal structures in human bile.

Our comparative analysis of histograms for distributions of random values inherent to phase shifts δ in laser images of both types human bile revealed availability of two dominant extreme ranges: $0 \le \delta \le 0.15$ and $0.85 \le \delta \le 1$. In our opinion, these features of probabilistic phase distributions are related with the influence of optically isotropic ($\delta \rightarrow 0$) and liquid-crystal ($\delta \rightarrow \pi$) components in the composition of biological fluid.

The results show that the differentiation phase maps of different groups is impossible - change of size and range of statistic moments of 1 - 4-th order almost coincide.

Being aimed at more specific investigation of phase features for both fractions, we used the following method to select information. From the available coordinate set of values $\delta(m \times n) = \begin{pmatrix} \delta_{11}, \dots, \delta_{1n} \\ \delta_{n1}, \dots, \delta_{nn} \end{pmatrix}$ in phase maps (figures 2(a)

and 2(b)), we found samples of extreme values $\delta(m \times n) = 0$ and $\delta(m \times n) = \pi$.

In what follows, by scanning along the direction $x=1 \div n$ we carried out calculation of the amount of extreme values for phase shifts within the column $m=n \times 1 pix$. Within the limits of each local sample $(1_{pix} \times n_{pix})^{(k=1,2,...,m)}$, we computed the amount (N) of extreme values $\delta(k)=0$ $(N_{\min}^{(k)})$ and $\delta(k)=\pi$ $(N_{\max}^{(k)})$. Thus, we found the dependences $N_{msn}(x) \equiv (N_{msn}^{(1)}, N_{\min}^{(2)}, ..., N_{\min}^{(m)})$ and $N_{\max}(x) \equiv (N_{\max}^{(1)}, N_{\max}^{(2)}, ..., N_{\max}^{(m)})$ for the amount of extreme values of phase shifts within the limits of laser image for bile.

Figures 3 and 4 show a set of coordinate distributions $\delta(m \times n) = 0, \pi$ (fragments (a, e)) for the dependences of the amount of extreme values $N_{\min;\max}(x)$ (fragments (b, f)), autocorrelation functions $K_{\min;\max}(\Delta x)$ (fragments (c, g)) and logarithmic dependences $\log J(N_{\min;\max}) - \log d^{-1}$ for power spectra of distributions $N_{\min;\max}(x)$ (fragments (d, h)) that characterize phase maps for the samples of bile belonging to a healthy patient (left column) and a patient suffering from insulin-independent diabetes (right column).

The comparative analysis of the obtained set of experimental data about statistic, correlation and fractal structures in dependences for the amount of extreme values $N_{\min,\max}(x)$ inherent to phase maps describing layers of bile of healthy patient and that sick with insulin-independent diabetes enabled to found:

• tendency to a decreasing (increasing) total amount of extreme values $\delta_{\min} \rightarrow 0$ ($\delta_{\max} \rightarrow \pi$) of the phase shifts in laser images of layers prepared from bile of a patient with insulin-independent diabetes (figures 3 and 4, fragments (b, f));

• fact that autocorrelation functions $K_{\min}(\Delta x)$ (figure 3, fragments (c, g)) monotonically drop with increasing the step of scanning Δx in dependences $N_{\min}(x)$;

• correlation structure of the distribution for the extreme sample $\delta(m \times n) = \pi$ in the phase map describing the polycrystalline component in bile of a sick patient changes: at the background of monotonic drop there arise oscillations of values in the dependence $K_{\max}(\Delta x)$ (see figure 4, fragment (g));

• logarithmic dependences for the power spectra of distributions $N_{\min}(x)$ for the optically isotropic component in bile of both types possess a stable slope angle (figure 3, fragments (d, f)) within the whole range of geometric sizes inherent to the laser image registered by the CCD camera (figure 1);

• fractal distributions $N_{\text{max}}(x)$ for phase maps of laser images describing the optically anisotropic fraction of bile a healthy man (figure 4, fragment (d)) are transformed into the statistic ones in the case of insulin-independent diabetes: approximating curve in the dependence $\log J(N_{\text{max}}) - \log d^{-1}$ has no stable slope (Fig. 4, fragment (h)).



Fig. 3. Coordinate $(m \times n)$ (a, e)), quantitative $N_{\min}(x)$ (b, f)), correlation $K_{\min}(\Delta x)$ (c, g)) and fractal $\log J(N_{\min}) - \log d^{-1}$ (d, h) parameters of the extreme sample $\delta(m \times n) = 0$ for phase maps of the samples of bile belonging to a healthy patient (a, b, c, d) and a patient with insulin-independent diabetes (e, f, g, h).

From the quantitative viewpoint, the dependences $N_{\min;\max}(x)$ illustrate statistic $M_{i=1-4}^{\delta}$, correlation S^{δ}, Q^{δ} and fractal F^{δ}, D^{δ} parameters determined within the limits of two patient groups, and they are summarized in Tables 2.

Our analysis of the parameters determined experimentally has shown that the following parameters are

diagnostically sensitive in observation of pathologic processes



Fig. 4. Coordinate $(m \times n)$ (a, e)), quantitative $N_{\max}(x)$ (b, f)), correlation $K_{\max}(\Delta x)$ (c, g)) and fractal $\log J(N_{\max}) - \log d^{-1}$ (d, h) parameters of the extreme sample $\delta(m \times n) = 1$ for phase maps of the samples of bile belonging to a healthy patient (a, b, c, d) and a patient with insulin-independent diabetes (e, f, g, h).

• statistic moments of the third (M_3^{δ}) and fourth (M_4^{δ}) orders in distributions for the amount of extreme values $N_{\max}(x)$ of phase shifts $\delta(m \times n) = 1$ in laser images for bile of both types – differences between them reach 2.4 and 4.1 times;

• the kurtosis (Q_4^{δ}) of autocorrelation functions $K_{\max}(\Delta x)$ related to distributions $N_{\max}(x)$ differ by 1.8 and 2.7 times;

• correlation area S° for the autocorrelation dependence $K_{\max}(\Delta x)$ of the distribution for the amount of extreme phase shifts in a laser image inherent to joint bile of a patient with osteoarthritis is 2.55 times less than that parameter determined for a healthy patient;

• distributions $N_{\text{max}}(x)$ for the phase maps describing bile for healthy and sick patients are, respectively, fractal and statistic;

• dispersion D^{δ} of the dependences $\log J(N_{\max}) - \log d^{-1}$ in the case of pathological changes in the polycrystalline structure of bile is 1.75 times decreased.

IV. CONCLUSION

Thus, one can draw the following conclusions:

➢ Human bile, independently of their physiological state, contains phase-modulating optically anisotropic network of biological crystals. TABLE 2. STATISTIC MOMENTS $M_{i=1-4}^{\delta}$, Correlation S^{δ}, Q^{δ}

AND FRACTAL F^{δ} , D^{δ} PARAMETERS THAT CHARACTERIZE THE DISTRIBUTIONS FOR AMOUNTS OF EXTREME VALUES IN COORDINATE DISTRIBUTIONS $\delta(m \times n)$ OF LASER IMAGES FOR HUMAN BILE

$\delta(m \times n)$	$\delta(m \times n) = 0$		$\delta(m \times n) = \pi$	
$M_{i=1-4}^{\delta}$	Healthy (21 patients)	Diabetes (19 patients)	Healthy (21 patients)	Diabetes (19 patients)
M_1^{δ}	0.51±0.063	0.54 ± 0.067	0.22±0.025	0.35±0.042
M_2^{δ}	0.13±0.018	0.08±0.011	0.25±0.031	0.14±0.017
M_3^{δ}	0.26±0.033	0.19±0.022	0.79±0.086	2.18±0.25
M_4^{δ}	0.48±0.054	0.55±0.068	0.83±0.098	3.11±0.42
Q_4^δ	0.14±0.016	0.12±0.015	0.56±0.069	2.21±0.31
S^{δ}	0.24±0.015	0.21±0.013	0.17 ± 0.021	0.08±0.012
\overline{F}^{δ}	2.42±0.12	2.49±0.11	2.58±0.15	Statistic
D^{δ}	0.21±0.028	0.24±0.027	0.34±0.042	0.18±0.023

> Ascertained and grounded is a set of criteria for phase diagnostics of inflammatory processes (diabetes, cholecystitis) as being based on statistic (statistic moments of the first to fourth orders), correlation (normalized fourth statistic moment of autocorrelation function, correlation area) and fractal (fractal dimension and dispersion for the distribution of extrema in log – log dependences of power spectra) analyses of phase distributions in laser images of human bile.

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