# Mechanical influences to the resonance fluorescence of ions in the dressed standing waves

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***Abstract* — This report is devoted to the problem of the resonance fluorescence of an atomic (or ion) system in the resonance with driving the standing wave of the optical cavity. It is shown that in this case resonance fluorescence depends on the location of atoms (or ions) relative to the positions of the nodes and antinodes of standing waves. It is demonstrated that if the atoms perform mechanical oscillations relative to the equilibrium position like in Paul traps, the distance between the Mollow type resonance fluorescence triplets is changed as a function of the frequency of these oscillations. This effect is possible for two and three level system placed in two transversal standing waves of the resonator. The dependence of the photon statistics on the applied mechanical oscillation is studied.**

***Index Terms* — mechanical influence; resonance fluorescence;**

## I. INTRODUCTION

We consider the interaction of three level ion in interaction with two standing waves in the resonance with the transitions of this ion. As two level system, can be obtained from the three level system than one dipole active transition is considered equal to zero, we focus on the description of three levels of Lambda type system in resonance with the two transversal dressed standing waves of the resonator. Considering that the three level atoms interacts with two modes of the standing waves $E_\beta = E_{0\beta}\cos_\beta t\sin\left(\mathbf{k}_\beta,\ \mathbf{r}\right), \qquad \beta = 1,\ 2$ in resonance with the transitions $d_{3\beta}$, we can represent the dressed Hamiltonian of this system by the expression

$$H_0 = \hbar\sum_{\alpha=1.2}\sum_{j=1}^{N}(\omega_\alpha + \omega_{3\alpha})\hat{a}^+_{\alpha j}\hat{a}_{\alpha j} + \hbar\sum_{j=1}^{N}\sum_{\beta=1}^{2}\Omega_{j\beta}\{\hat{a}^+_{3j}\hat{a}_{\beta j}\exp[i(k_{0\beta},\ r_j)(1-\Lambda)] + H.c.\}, \quad (1)$$

where the Rabi frequencies depend on the position of " $j$ " atom in the transversal standing wave

$$\Omega_{j\beta} = \begin{cases} \Omega_{0\beta}\sin(\mathbf{k}_{0\beta},\ \mathbf{r}_j) & for\ \ \Lambda = 1 \\ \Omega_{0\beta} = (\mathbf{d}_{3\beta},\ \mathbf{E}_{0\beta})/\hbar & for\ \ \Lambda = 0 \end{cases} \quad (2)$$

$\hat{a}^+_{\alpha j}$ and $\hat{a}_{\alpha j}$ are the creation and annihilation atomic operators on the $\alpha$ state of three level system, $\hbar\omega_{3\alpha}$ represent the difference of energy between the level $3$ and $\alpha$, $\hbar\omega_\alpha$ is the energy of the electromagnetic field, $\Lambda$ takes $1$ and $0$ values which correspond to the the standing and travelling waves respectively. The cooperative spontaneous emission of the dressed Lambda type three level radiator in the single mod traveling wave was studied in the paper [Enaki & Svera, 1988] As was demonstrated recently the cooperative behavior of spontaneous emission in standing wave have new peculiarities which depend on the atomic positions in the standing wave [Enaki et all, 2011]. The resonance fluorescence of three level atom in the two standing transversal wave opens the attractive applications due to the fact that the stopped atom (or ion) can oscillates around the equilibrium position changing its interaction with the dressed field. Considering that the radiator suffers the mechanical oscillation around the equilibrium position in the antinode $r_j = r_{0j}\cos\omega_m t$ with frequency $\omega_m$ ($\omega_m << \Omega_{j\beta} << \omega_j$), we can obtain the dressed states in the Hamiltonian (1) using the Bogoliubov transformation

$$a_{1j}\exp[i(k_{01},\ r_j)(1-\Lambda)] = \frac{\Omega_{j2}}{\tilde{\Omega}_j}A_{2j} - \frac{1}{\sqrt{2}}\frac{\Omega_{j1}}{\sqrt{\tilde{\Omega}_j^2 + \frac{\Delta}{2}\tilde{\Omega}_j}}A_{1j} + \frac{1}{\sqrt{2}}\frac{\Omega_{j1}}{\sqrt{\tilde{\Omega}_j^2 - \frac{\Delta}{2}\tilde{\Omega}_j}}A_{3j},$$

$$a_{2j}\exp[i(k_{02},\ r_j)(1-\Lambda)] = -\frac{\Omega_{j1}}{\tilde{\Omega}_j}A_{2j} - \frac{1}{\sqrt{2}}\frac{\Omega_{j2}}{\sqrt{\tilde{\Omega}_j^2 + \frac{\Delta}{2}\tilde{\Omega}_j}}A_{1j} + \frac{1}{\sqrt{2}}\frac{\Omega_{j2}}{\sqrt{\tilde{\Omega}_j^2 - \frac{\Delta}{2}\tilde{\Omega}_j}}A_{3j},$$

$$a_{3j} = \frac{\sqrt{1+\Delta/\tilde{\Omega}_j}}{\sqrt{2}}(A_{1j} + A_{3j})$$

Here $\Delta = \Delta_\alpha = \omega_\alpha - |\,\omega_{3\alpha}\,|$, $\tilde{\Omega} = \sqrt{\Omega_1^2+\Omega_2^2}$ . After this transformation it is obtained three new dressed Hamiltonian with the quasi-levels "i" with energies $\hbar\lambda_{ji}$

$$H_0 = \hbar\sum_{j=1}^{N}\lambda_{j1}A_{j1}^{+}A_{j1} + \lambda_{j2}A_{j2}^{+}A_{j2} + \lambda_{j3}A_{j3}^{+}A_{j3}, \quad (3)$$

which depends on the vibration time $\lambda_{j1(3)} = \frac{\Delta}{2} \mp \sqrt{\frac{\Delta^2}{4} + \Omega_{j1}^2(t) + \Omega_{j2}^2(t)}, \ \lambda_{j2} = \Delta$ through mechanical oscillations of the nuclei in the standing wave $\Omega_{0\beta}\sin[(\mathbf{k}_{0\beta},\ \mathbf{r}_{j0})\cos\omega_m t]$. As follows from the expression (3) the quasi-levels energy depends on the position of the radiators relative the nodes and antinodes of the standing waves. So energy will depend on the frequency and amplitude of oscillations of atoms relative to the position of equilibrium $r_j = r_{0j}\cos\omega_m t$ and the position of this mechanical oscillator in the standing wave. Now we can introduce the interaction of these dressed atomic systems with vacuum of electromagnetic field (EMF)

$$H_I = \sum_{\beta=1}^{2}\sum_{k}\sum_{j=1}^{N}\frac{(\mathbf{g}_k,\ \mathbf{d}_{3\beta})}{2\tilde{\Omega}_j}$$
$$\times\{[\Omega_{j\beta}(U_{3j}^3 - U_{1j}^1 + U_{3j}^1 - U_{1j}^3)$$
$$+\sqrt{2}\Omega_{j(3-\beta)}(-1)^{\beta+1}(U_{2j}^1 + U_{2j}^3)]$$
$$\times b_{k\beta}\exp[i(\mathbf{k},\ \mathbf{r}_j)] + H.c.\}.$$

Here we have introduced the new operators of the transitions between the dressed states $U_{bj}^a = A_{ja}^{+}A_{jb}$. For simplicity, the interaction Hamiltonian is obtained for deviation of resonance $\Delta = 0$. Using the method of elimination of the Bose operators [1,2], in the Born-Marcoff limits it is obtained the following generalized equation for atomic subsystems

$$\frac{d\left\langle \hat{O}(t)\right\rangle}{dt} - \frac{i}{\hbar}\left\langle [\hat{H}_0, \hat{O}(t)]\right\rangle =$$
$$\frac{1}{\hbar^2}\sum_{\beta=1}^{2}\sum_{k}\sum_{j,j'=1}^{N}\{\frac{(\mathbf{g}_k,\mathbf{d}_{3\beta})^2}{4\tilde{\Omega}_j\tilde{\Omega}_{j'}}$$
$$\times[\Omega_{j\beta}\Omega_{j'\beta}\left\langle [(U_{3j}^3 - U_{1j}^1 + U_{3j}^1 - U_{1j}^3), \hat{O}(t)]\right.$$
$$\left.\times(\zeta_{11}^{\beta j'}U_{3j'}^3 - \zeta_{11}^{\beta j'}U_{1j'}^1 + \zeta_{31}^{\beta j'}U_{1j'}^3 - \zeta_{13}^{\beta j'}U_{3j'}^1)\right\rangle$$
$$+2\Omega_{j(3-\beta)}\Omega_{j'(3-\beta)}\left\langle [(U_{2j}^3 + U_{2j}^1), \hat{O}(t)]\right.$$
$$\left.\times(\zeta_{23}^{\beta j}U_{3j}^2 + \zeta_{21}^{\beta j}U_{1j}^2)\right\rangle + (-1)^{\beta+1}\sqrt{2}\Omega_{j\beta}\Omega_{j'(3-\beta)}$$
$$\times\left\langle [(U_{3j}^3 - U_{1j}^1 + U_{3j}^1 - U_{1j}^3), \hat{O}(t)]\right.$$
$$\left.\times(\zeta_{23}^{\beta j}U_{3j}^2 + \zeta_{21}^{\beta j}U_{1j}^2)\right\rangle + (-1)^{\beta+1}\sqrt{2}\Omega_{j(3-\beta)}\Omega_{j'\beta}$$
$$\times\left\langle [(U_{2j}^3 + U_{2j}^1), \hat{O}(t)](\zeta_{11}^{\beta j'}U_{3j'}^3\right.$$
$$\left.-\zeta_{11}^{\beta j'}U_{1j'}^1 + \zeta_{21}^{\beta j'}U_{1j'}^3 - \zeta_{12}^{\beta j'}U_{3j'}^1)\right]$$
$$\times\exp[i(\mathbf{k}, \mathbf{r}_j - \mathbf{r}_{j'})] + H.c.\}$$

From this equation it is easy to obtain the system of equation for the population operators for new quasi-levels of energy

$$\frac{d\left\langle U_{1j}^1\right\rangle}{dt} = -\gamma_1(t,r_j)\left\langle U_{1j}^1(t) - U_{3j}^3(t)\right\rangle$$
$$-\gamma_2(t,r_j)\left\langle 2U_{1j}^1(t) + U_{3j}^1(t) + U_{1j}^3(t)\right\rangle,$$
$$\frac{d\left\langle U_{3j}^3\right\rangle}{dt} = -\gamma_1(t,r_j)\left\langle U_{3j}^3(t) - U_{1j}^1(t)\right\rangle$$
$$-\gamma_2(t,r_j)\left\langle 2U_{3j}^3(t) + U_{3j}^1(t) + U_{1j}^3(t)\right\rangle,$$
$$\frac{d\left\langle U_{2j}^2\right\rangle}{dt} = \gamma_2(t,r_j)$$
$$\times\left\langle U_{3j}^3(t) + U_{1j}^1(t) + U_{1j}^3(t) + U_{3j}^1(t)\right\rangle.$$

Here $\gamma_{1(2)} = \gamma_{31}\frac{\Omega_{1(2)}^2}{2\tilde{\Omega}^2} + \gamma_{32}\frac{\Omega_{2(1)}^2}{2\tilde{\Omega}^2}$,

$$\gamma_3 = \frac{\sqrt{2}\Omega_1\Omega_2}{2\tilde{\Omega}^2}\left(\gamma_{31} - \gamma_{32}\right)$$

This system of equation is solved exactly. It describes the interference of two Mollow triplet in the resonance fluorescence at frequencies $\omega_{1(2)}$, $\omega_{1(2)} \pm \Omega_{j1(2)}$. This interference appears then the radiators are situated in the anti-nodes of transversal standing waves with frequencies $\omega_1$ and $\omega_2$. The problem appears when the individual atoms begin to mechanically oscillate around the equilibrium position

## CONCLUSION

We have obtained the Mollow triplets of the two dimensional lattice of trapped ion in the anti-nodes of this standing wave. In the process of mechanical oscillations the position of ions relative the center of anti-nodes are possible. In this case all information about this oscillation can be obtained measuring the distances between the Mollow triplet peaks of resonance fluorescence. The in-phase and anti-phase oscillations of this ion array is studied trough the interferences of resonance fluorescent light from this system of radiators.

## REFERENCES

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