# RING CAVITY EXCITABILITY AND COHERENCE RESONANCE. APPLICATIONS TO COMMUNICATION SYSTEMS

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Abstract. We discuss the phenomena of excitability and coherence resonance of a nonlinear ring cavity where excitons and biexcitons are created. A bifurcation analysis of the ring cavity dynamics demonstrates both the existence of self-pulsation and excitability. In addition, we show that an excitable exciton-biexciton system in a ring cavity can display coherence resonance. Finally, the optimum conditions for these phenomena are investigated and the possibilities for applications in communication systems are discussed.

Key words: excitability, coherence resonance, communications, excitons, biexcitons

### **1. INTRODUCTION**

The phenomena of excitability and coherence resonance are new and rapidly expanding topics in optics having initially receiving much attention in biology [1-3] and chemistry [4]. Excitability can be illustrated in the biological context by the all or nothing behavior of neurons; a sub-threshold stimulus implies only a local (i.e. non-propagated) response, while a stimulus above the threshold leads to a pulse propagating along the axon. However, excitability can also occur in optical systems and has been predicted to occur under certain conditions in a nonlinear ring cavity [5, 6], a laser with a saturable absorber [7], and a semiconductor laser subjected to delayed optical feedback [8]. This phenomenon has been studied both theoretically and experimentally for a laser with dispersive reflector [9,10]. Excitability in semiconductor-based optics is attracting substantial interest because it offers prospects for practical applications in optoelectronic devices.

The influence of noise on nonlinear systems is especially interesting. The noise can have quite different effects [11] when acting on oscillatory, excitable or bistable systems, but here we concentrate on the phenomenon of coherence resonance in the excitable ring cavity. Coherence resonance occurs when there is a near-periodic response to the effect of noise, and this can be associated with an almost periodic phase space trajectory for the perturbed system. The first theoretical evidence of coherence resonance in FitzHugh-Nagumo model was given in ref. [3]. The

phenomenon has also been studied [12] in the Lang-Kobayashi model of a semiconductor laser with optical feedback. Recently, we predicted that a system of excitons and biexcitons in a nonlinear ring cavity can be excitable [6]. This paper extends our work of [6] and shows the first theoretical evidence of coherence resonance in the same device. Finally we discuss the possibility of implementation of an excitable element as a source for communications systems.

#### 2. RING CAVITY DYNAMICS

We consider a system of excitons and biexcitons in a nonlinear ring cavity. The structure of the ring cavity studied is shown in Fig. 1. The optical nonlinearity is considered to be due to the

creation of biexcitons by the interaction of excitons and photons [13]. For simplicity we use the giant oscillator strength, three-level model to describe the microscopic dynamics, and study the response to the simultaneous action of two independent optical pulses. The photons of the first pulse  $Y_1$  are in resonance with a transition in the exciton spectral range while the photons of the second pulse  $Y_2$  exciton - biexciton conversion. The dynamics of system is described by the dimensionless equations [6]



Fig. 1. A schematic diagram of the ring cavity.  $Y_1$  and  $Y_2$  are the incident fields and  $X_1$  and  $X_2$  are the transmitted fields.

$$\frac{dX_{1}}{d\tau} = \sigma(-X_{1} + 2CU + Y_{1}), \quad \frac{dX_{2}}{d\tau} = \sigma(-X_{2} - 2CUV + Y_{2} + Z),$$

$$\frac{dU}{d\tau} = -\gamma U - \gamma(X_{1} + X_{2}V), \quad \frac{dV}{d\tau} = -V + X_{2}U, \quad \frac{dZ}{d\tau} = -\frac{1}{\tau_{c}}Z + \frac{(2D)^{1/2}}{\tau_{c}}\xi(\tau), \quad (1)$$

where  $X_j$  and  $Y_j$  are the normalized field amplitudes, as defined in Fig. 1, while U and V are the normalized exciton and biexciton amplitudes, respectively.  $\tau$  is the dimensionless time (for which unity corresponds to 1ps),  $\sigma$  describes the damping of the electric-field amplitude in the cavity,  $\gamma$  is the decay rate of an exciton relative to that of a biexciton and C is a constant. Z is the external colored noise with autocorrelation function  $\langle Z(\tau)Z(\tau')\rangle = (D/\tau_c) e^{-(\tau-\tau')/\tau_c}$  with correlation time  $\tau_c$ , and  $\xi(\tau)$  is Gaussian white noise with zero average and unit standard deviation [14].

First we have analyzed the dynamics of the system in the absence of noise by considering its bifurcations. Fig. 2 shows the different types of system behaviour found in the  $\sigma - Y_2$  plane for two values of *C*. When C = 10 (Fig. 2(a)) we observe only regions corresponding to the existence of a stable stationary state and to self-pulsation. Fig. 2(b), for C = 5, is more complicated. In addition to stationary and pulsation regions, there are also regions corresponding to bistability and excitability. For the excitability region, a linear stability analysis shows the existence of a saddle close to a



stable node in the  $X_1 - X_2$  phase space, which as pointed out earlier provides a mechanism for excitability.

## Fig 2 The different regimes of behaviour

# *Fig 3 Left: time evolution of the variable X*<sub>1</sub> *Right: the power spectrum*

We have also studied the influence of noise on the ring cavity dynamics. We integrated the equations (1) numerically for the case where the system parameters were  $Y_1 = 10$ ,  $\gamma = 0.1$ ,  $\sigma = 0.1$ and  $Y_2 = 9.0$  so that the operating point is situated in the excitable region of Fig. 2(b). In the absence of noise, the transmitted amplitudes are constant and after some relaxation oscillations the trajectories tend to the stable states. When noise is introduced with D = 0.7 and a correlation time of  $\tau_c = 0.1$ , a random sequence of output pulses similar to those in Fig. 3(a) is obtained. When D is increased, the pulses become more frequent and it is apparent that increasing the noise level affects not only the duration and separation of the pulses but also their amplitudes. In particular, for D =0.9 the response has the regularity shown in Fig. 3(b) that is characteristic of the phenomenon of coherence resonance as described in section 1. However, further increase of the noise level to D =2.0 leads to an irregular output signal of the type shown in Fig. 3(c). Thus we conclude that the response of the system is more regular around a critical level of noise similar to that relating to Fig.3(b), and hence an optimum external noise level exists to achieve coherence resonance. These qualitative observations are supported by the power spectra of  $X_1$ , shown on the right hand side of Fig. 3. Thus we predict that a system of excitons and biexcitons in a ring cavity can display coherence resonance. If such an effect could be produced experimentally it might find application in communications systems where a source of regular pulses is required.

# **3. CONCLUSIONS**

In conclusion we have shown that an excitable exciton-biexciton system in a ring cavity can display the phenomenon of coherence resonance. Bifurcation analysis has been used to identify a range of system parameters for which excitability occurs. It is found that the excitability is the result of a saddle point and a stable node in close proximity in the relevant phase space. We have demonstrated that a low level of noise applied to the excitable system results in a sequence of random output pulses. However, when the noise level is increased to an appropriate value, the response becomes almost periodic in a manner characteristic of coherence resonance and it could have possible applications as a functional optoelectronic element.

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