Nonlinear cooperative phenomena caused by Fröhlich phonons in biological objects

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***Abstract* — Nonlinear cooperative stationary phenomena are studied for the interaction of Bose-condensed phonons with millimeter electromagnetic radiation in the biological media. The real and imaginary parts of the dielectric susceptibility and permeability, refraction and reflection indexes determined by Bose-condensed phonons were obtained. Finally, the possibility of occurrence of the polariton effect was predicted in biological objects.**

***Index Terms* — millimeter waves, biological media, Bose-Einstein phonons, complex dielectric functions.**

I. INTRODUCTION

The investigation of the nonlinear cooperative phenomena in complex system is one of the most important and up-to-date problems of the modern science. During recent years the interest for the investigation of the interaction of the millimeter range coherent electromagnetic radiation with medical-biological objects has attracted much attention, due to many applications of the millimeter waves in the applied medicine, biotechnology and agriculture [1-8].

The concept of generation of phonons in alive media was proposed by Fröhlich [9]. He suggested theoretically that biological systems can generate collective vibrational modes (phonons) in the GHz frequency range and these phonons might play a basic role in the active biological systems. The idea is based on the assumption that if the generated energy is supplied to the phonon with a rate greater than a certain critical value, then the phenomenon of Bose condensation to the lowest excitation of a single mode can occur. Such kind of condensation that occurs at the lowest excitation mode can serve as method for energy storage as well as channel for specific bio-processes like cell division or macromolecule synthesis. This idea stimulated further investigations in the domain with an aim of studying and prediction of new biophysical phenomena and their effective use in biomedicine [10-15].

Even the number of experimental and theoretical papers in the field continuously grows up to present there is no a clear understanding of the mechanism of generation and interaction of the millimeter radiation with the biological media. Thus, a consistent theory of the low-intensity millimeter wave interaction with biological media was not elaborated, and a lot of cooperative nonlinear phenomena are not investigated yet. It is well known that due to the metabolism processes in the organisms, the excitation of the polarizing coherent waves is possible in the frequency range 10^{11}-10^{12} Hz. Such oscillations are supposed to cover parts of biological membranes and as was demonstrated, it can be equivalent to Bose-Einstein condensation of phonons in such a media. For example, Dovydov investigated the possibility of the collective state excitation in one-dimensional molecular structures and in 2-spiral protein molecules in the form of solitons [16, 17], further developed for dissipative structures [18]. Nonlinear, collective and stochastic phenomena of Bose-type quasiparticles in condensed media and in particular the nonlinear dynamics of the immune system interaction with the bilocal cancer tumor were developed in [19]. A new simple model of local interaction between two spaced cancer cells and cytotoxical T-lymphocytes is proposed.

This paper is devoted to the investigation of nonlinear cooperative phenomena at the interaction of Bose-condensed phonon with the millimeter electromagnetic radiation generated in a biological object. We start in Section II with a Hamiltonian of the interaction of millimeter electromagnetic radiation with coherent phonons in the square form for the low concentrations of Bose-condensed phonons. The collective nonlinear properties of the coherent phonons are determined, in particular, by the phonon-phonon interaction character, the energetic spectrum of the coherent phonons and their interaction with the non-condensed quasiparticles. Section III is devoted to the study of the real and imaginary parts of dielectric susceptibility and permeability, refraction and reflection indexes determined by Bose-condensed phonons. This study represents a major interest in modern biophysics. In Section IV the numerical results on calculations of system characteristics are presented. The possibility of polariton occurrence is also discussed. Conclusions are given in section V.

II. MODEL

In what follows based on the Fröhlich idea that coherent phonons excited in biological objects turn into coherent internal photons and form the self-correlated interval millimeter electromagnetic field we derive a set of equations that describe the evolution of such photons and phonons. As was demonstrated previously, the coherent state of the phonon in the biological objects can be similar to the coherent state of the exciton in the condensed media [20-23]. These coherent states of the elementary excitations can appear in different regimes; i.e. in regime of ultrashort pulses existing a time period shorter that the relaxation times, as well as in conditions of Bose-Einstein condensation during a time period longer that the relaxation times, but shorter that the quasiparticle life time. In both cases the corresponding elementary excitations are characterized by certain phase, wave vector and macroscopic value of the single-particle state, i.e. the number of excitations in condensate $N_V \sim V$, where V is the system volume.

The Hamiltonian of the system has the form

$$H=\sum_{\vec{k}}\hbar\Omega a_{\vec{k}}^{+}a_{\vec{k}}+\frac{1}{8\pi}\sum_{\vec{k}}\int\left(\varepsilon E_{\vec{k}}^{2}+\mu H_{\vec{k}}^{2}\right)dv_{\vec{k}}$$
$$+\frac{1}{2V}\sum_{\vec{k}_1,\vec{k}_2,\vec{k}_1',\vec{k}_2'}g\left(\vec{k}_1-\vec{k}_1'\right)\delta_{kp}\left(\vec{k}_1+\vec{k}_2,\vec{k}_1'+\vec{k}_2'\right)a_{\vec{k}_1}^{+}a_{\vec{k}_2}^{+}a_{\vec{k}_1'}a_{\vec{k}_2'} \quad (1)$$
$$-\sum_{\vec{k}}d_{\vec{k}}\left(E_{\vec{k}}^{+}a_{\vec{k}}^{+}+E_{\vec{k}}^{-}a_{\vec{k}}\right),$$

where $\hbar\Omega$ is the dipole-active phonon energy. $E_{\vec{k}}=E_{\vec{k}}^{+}+E_{\vec{k}}^{-}$, $H_{\vec{k}}=H_{\vec{k}}^{+}+H_{\vec{k}}^{-}$ are the intensities of the electrical and magnetic fields, respectively. $E_{\vec{k}}^{\pm}$ and $H_{\vec{k}}^{\pm}$ are the positive or negative frequency parts of the variable electromagnetic field. $a_{\vec{k}}^{+}(a_{\vec{k}})$ are the operators of creation (annihilation) of the dipole-active phonons that satisfy the commutation relation $[a_{\vec{k}},a_{\vec{k}'}^{+}]=\delta_{kk'}$, $[a_{\vec{k}},a_{\vec{k}}^{+}]=0$. ε and μ are the dielectric and magnetic permeability of the biological medium. $g\left(\vec{k}\right)$ is the phonon-phonon interaction constant. $d_{\vec{k}}$ is the dipole momentum of the transition into phonon state.

In the Hamiltonian (1), both the field and the Bose-condensed phonons are supposed to have the same wave vector $\vec{k}\neq 0$, polarization and phase. The amplitudes are considered to be macroscopically large, i.e. proportional to the system effective volume V. We mention that the effects of spatial dispersion were neglected being insignificant in the actual spectrum range. Thus, we consider the case of phonon mass $m\approx\infty$.

The Heisenberg motion equation for the operator $a_{\vec{k}}$ has the form

$$i\frac{da}{dt}=\Omega a+\frac{g}{\hbar}\frac{|a|^2}{V}a+\frac{d}{\hbar}E^{+}-i\gamma a \quad (2)$$

where the wave vector indexes are omitted. We mention that, we introduced phenomenologically in (2) the term $i\gamma a$, which takes into account the Bose-condensed phonons leaving from condensed state. On the other hand we mention that the attenuation term can be considered strictly within the framework of the quantum theory of attenuations and fluctuations using the Fokker-Plank master equation [24].

The polarization of the biological media is given by

$$P^{+}=-\frac{1}{V_O}\frac{\partial H}{\partial E^{-}},\; P^{-}=-\frac{1}{V_O}\frac{\partial H}{\partial E^{+}} \quad (3)$$

where $V_O=\frac{V}{N_d}$, and N_d is the atomic dipole number.

Taking account the Hamiltonian (1) and (3) we obtain for polarization the following form

$$P^{+}=\frac{da}{V_O},\; P^{-}=\frac{da^{+}}{V_O} \quad (4)$$

We assume that the Fröhlich electromagnetic millimeter field is spread along the axe *x*. Thus, the equation of the positive frequency component of the electromagnetic field is equivalent to

$$\frac{\partial^2E^{+}}{\partial x^2}-\frac{1}{c^2}\frac{\partial^2E^{+}}{\partial t^2}=\frac{4\pi}{c^2}\frac{\partial^2P^{+}}{\partial t^2} \quad (5)$$

where *c* is the light velocity in the biological media.

Equations (2) and (5) describe completely the spatial – temporal evolution of the Bose-condensed phonons in the biological media, taking into account self-interaction and the interaction with the Fröhlich field.

We write the macroscopic amplitudes of the phonons and the field in the form of modulated plane waves with the frequency ω and the wave vector k

$$a(x,t)=\sqrt{V}\tilde{a}(x,t)e^{-i\omega t+ikx},$$
$$E^{+}(x,t)=\sqrt{V}\tilde{E}(x,t)e^{-i\omega t+ikx} \quad (6)$$

where $\tilde{a}(x,t)$ and $\tilde{E}(x,t)$ are the slowly-varying amplitudes satisfying the following conditions

$$\left|\frac{\partial\tilde{a}}{\partial t}\right|<<\omega|\tilde{a}|,\quad \left|\frac{\partial\tilde{a}}{\partial x}\right|<<k|\tilde{a}|,$$
$$\left|\frac{\partial\tilde{E}}{\partial t}\right|<<\omega|\tilde{E}|,\quad \left|\frac{\partial\tilde{E}}{\partial x}\right|<<k|\tilde{E}|.$$

Taking into account the above assumption, the equation of motion (2) is transformed in

$$\frac{d\tilde{a}}{dt}=i\left[\Delta+i\gamma-\frac{g}{\hbar}n\right]\tilde{a}+i\frac{d}{\hbar}\tilde{E} \quad (7)$$

where $\Delta=\omega-\Omega$ is the resonance detuning between the Fröhlich field frequency and the phonon frequency. $n=|\tilde{a}|^2$ represents the concentration of the Bose-condensed phonons.

STATIONARY STATES

In this Section we consider the steady states ($d\tilde{a}/dt=0$) of the equation of motion (7). We obtain the dependence of amplitudes of phonons on photons

$$\tilde{a}=-\frac{d/\hbar}{\Delta-gn/\hbar+i\gamma}\tilde{E} \quad (8)$$

The dielectric susceptibility of the biological media is given by the relation $\chi=P^{+}/E^{+}$. Taking into account (4) and (8) for the real and imaginary parts of the dielectric susceptibility we obtain

$$\chi'=-\frac{d^2}{V_O\hbar}\frac{\Delta-gn/\hbar}{\left(\Delta-gn/\hbar\right)^2+\gamma^2}$$

$$\chi'' = \frac{d^2\gamma}{V_O\hbar}\frac{1}{(\Delta - gn/\hbar)^2 + \gamma^2}$$

(9)

Thus the dielectric permeability, that takes into account the phonon level contribution, is described by the expression

$$\varepsilon = \varepsilon_\infty + 4\pi\chi = \varepsilon' + i\varepsilon'',$$

(10)

where $\varepsilon', \varepsilon''$ are the real and imaginary parts of the dielectric permeability

$$\varepsilon' = \varepsilon_\infty - \frac{4\pi d^2}{V_O\hbar}\frac{\Delta - gn/\hbar}{(\Delta - gn/\hbar)^2 + \gamma^2}$$

$$\varepsilon'' = \frac{4\pi d^2\gamma}{V_O\hbar}\frac{1}{(\Delta - gn/\hbar)^2 + \gamma^2}.$$

Here, ε_∞ is the phonon dielectric permeability that takes into account the dielectric permeability of all excitations besides phonons of the biological media. The real and imaginary parts of the permeability determine the frequency dispersion. From (10) we conclude that the dielectric permeability of the biological media depends significantly on Bose-condensed phonon density n.

In what follows it is convenient to switch to the following dimensionless quantities

$$N = \frac{g}{\gamma\hbar}n,\ f = \frac{\tilde{E}}{E_s},\ \frac{1}{E_S^2} = \left(\frac{d}{\hbar}\right)^2\frac{g}{\hbar\gamma^3},$$

$$\Omega_0 = \frac{4\pi d^2}{\hbar V_0},\ \delta = \frac{\Delta}{\gamma}.$$

(11)

Taking account (11) from (10) we obtain

$$\varepsilon' = \varepsilon_\infty - \frac{(\Omega_O/\gamma)(\delta - N)}{[(\delta - N^2) + 1]},$$

$$\varepsilon'' = \frac{\Omega_O/\gamma}{[(\delta - N)^2 + 1]},$$

$$N[(\delta - N)^2 + 1] = f^2. \quad (12)$$

For a plane wave, where the surfaces of the field components are planes, the dielectric permeability, perpendicular to the spreading direction, is connected with the refraction index $\bar{n}$ and with the absorption index κ by the expression [25]

$$\varepsilon = \bar{n}^2 - \kappa^2 + 2i\bar{n}\kappa \quad (13)$$

Thus, we can write the refraction and absorption indexes

$$\bar{n} = \sqrt{\frac{\varepsilon' + \sqrt{\varepsilon'^2 + \varepsilon''^2}}{2}},$$

$$\kappa = \sqrt{\frac{-\varepsilon' + \sqrt{\varepsilon'^2 + \varepsilon''^2}}{2}}$$

(14)

It is well known that the reflection coefficient R is defined as the ratio of the time average energy flow reflected from surface to the incident one. For the case of perpendicular flow the reflection coefficient can be written [25, 26]

$$R = \frac{(\bar{n} - 1)^2 + \kappa^2}{(\bar{n} + 1)^2 + \kappa^2}$$

(15)

In what follows let consider that the electromagnetic field is uniformly distributed in the biological media $(\partial\tilde{E}/\partial x = 0)$. From (5) we obtain the equation for the field temporal evolution in the slowly-varying amplitude approximation

$$\frac{d\tilde{E}}{dt} = i\left[\frac{\omega^2 - c^2k^2}{2\omega} - \frac{4\pi d^2}{V_O\hbar}\right]\tilde{E} + \frac{2\pi d\omega i}{V_O}\left[1 - \frac{2}{\omega}\left[\omega - \Omega - \frac{g}{\hbar}n\right] - \frac{2i\gamma}{\omega}\right]\tilde{\text{э}}. \quad (16)$$

In the stationary case $\partial E/\partial t = 0$ and the dispersion limit, when the attenuation of the dipole-active Bose-condensed phonons can be neglected, from (16) we obtain

$$k^2 = \frac{\omega^2}{c^2}\left[1 + \frac{\Omega_O}{\Omega + \frac{g}{\hbar}n - \omega}\right]. \quad (17)$$

We mention that the equation (17) determines the dispersion law of the nonlinear polaritons, i.e. the energy values of the elementary excitations in biological medium at the Fröhlich radiation interaction with dipole-active Bose-condensed phonons can be realized at the same value of the wave vector. The position and the form of the polariton branches depend not only on the parameters of the phonons and of the electrical field, but also on the stationary concentration of the Bose-condensed phonons. The concentration depends on the Fröhlich field intensity according to the equation (12). Thus, the polariton appearance is caused by the intersection of the coherent photon dispersion curves with Bose-condensed phonon for small wave vectors. Close to the intersection point the energies and pulses of both excitations coincide and the bond between them becomes significant. As result neither photon, nor phonon could be considered as independent elementary excitations in biological objects but a new elementary excitation - polariton – appears in the biological objects.

III. NUMERICAL RESULTS

A conclusion The expression (17), as we mention above, coincides with that of dispersion law of the polariton in case of soliton appearance in the excitonic spectrum range [27]. Using the following dimensionless variables $\bar{k} = k/k_O$, $\bar{\omega} = \omega/(ck_O)$, $\bar{\omega}_O = \Omega_O/(ck_O)$, where $ck_O = \Omega + gn/\hbar$, from (17) we obtain the expression for dispersion law

$$\bar{k}^2 = \bar{\omega}^2 \left[1 + \frac{\bar{\omega}_0}{1 - \bar{\omega}} \right]. \qquad (18)$$

According to (12), for the small values of the field intensities, the dependence of the phonon concentration and the field is linear. While excitation increases, the phonon-phonon interaction processes play an important role. It is easy to observe that the relation between N and f is linear for $\delta < \sqrt{3}$ (see Fig.1).

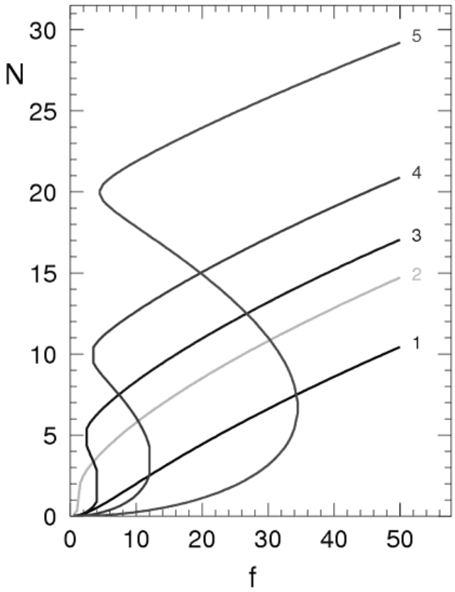

Fig.1 (color online) The dependence of the phonon concentration on the field intensity for different values of resonance detuning: (1) for δ =-5.0, (2) for δ =1.7, (3) for δ =5.0, (4) for δ =10.0 and (5) for δ =20.0.

For $\delta > \sqrt{3}$, the dependence $N(f)$ is not any more linear and is characterized by hysteresis region where three values of the dipole-active Bose-condensed phonon density correspond to one values of the Fröhlich field intensity. Figure 1 shows the dependence of phonon concentration on the field intensity for different values of resonance detuning δ. As it is shown in the figure, the growth of the field intensity leads to a jump increasing of the Bose-condensed phonon density at $\delta > \sqrt{3}$. While field decreases along the upper curve, a jump fall of phonon density occurs at $\delta > \sqrt{3}$. Thus, when the resonance detuning value is larger than the critical one $\delta_c = \sqrt{3}$, the forward and backward alteration of the Fröhlich field intensity leads to the jump changes of the phonon density and to the formation of the amplitude hysteresis loop in the $N(f)$ dependence. The frequency hysteresis can be shown to occur also when one resonance detuning value corresponds to three values of the phonon density, values that are larger than the critical field intensity value $f_c = (4/3)^{3/4}$, one of which is unstable. We mention that that the phenomena of nonlinear hysteresis present in the dependency of the Bose-condensed phonon density on Fröhlich field intensity influences all dielectric function: the real and imaginary parts of the dielectric susceptibility and permeability the refraction, absorption and reflection indexes. Figure 2 shows the dependences of the real (left) and imaginary (right) parts of the dielectric permeability of the biological object on field intensity, caused by Bose-condensed phonons, for $\varepsilon_\infty = 5.2, \bar{\omega} = 5.0$ and different values of the resonant detuning. When the resonance detuning values are smaller than the critical δ_c, the real and imaginary parts of the dielectric permeability are single-valued functions of the field intensity. With increasing of the resonance detuning, the curves $\varepsilon'(f)$, and $\varepsilon''(f)$ become deformed, and hysteresis dependences $\varepsilon'(f), \varepsilon''(f)$ appear at $\delta > \delta_C$.

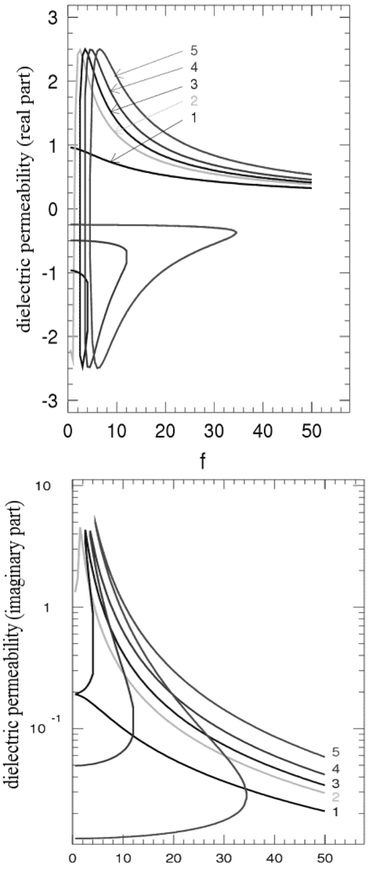

Fig.2 (color online)The dependences of the real and imaginary parts of the dielectric permeability on the amplitude of the field for ε_∞ =5.2, $\bar{\omega}$ =5.0 and the values of the resonant detuning similar to that of Fig. 1

Figure 3 shows the dependences of the refraction, absorption and reflection indexes on the field amplitude for different values of resonance detuning. From this figure, we conclude that for high values of the resonance detuning the dependencies $n(f), \kappa(f)$ and $R(f)$ become more complicated. Thus, these functions display hysteresis behavior caused by the nonlinear dependence of the Bose-condensed phonon density on the coherent millimeter electromagnet field amplitude, when the resonance detuning exceeds the critical value δ_C.

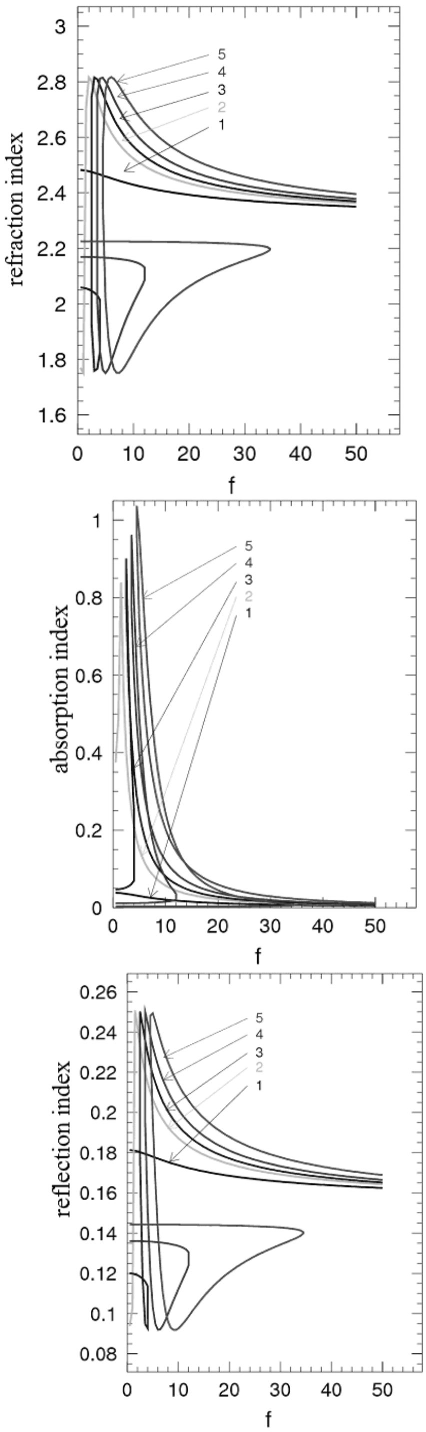

Fig.3 (color online) The dependences of the refraction, absorption and reflection indexes on the field amplitude for the same parameters as in Fig. 2

IV. CONCLUSION

In this paper we developed a model to describe the interaction of Bose-condensed phonons with millimeter electromagnetic radiation in the biological media. We obtain the real and imaginary parts of the dielectric susceptibility, as well as, the permeability, refraction and reflection indexes determined by Bose-condensed phonons. The dielectric functions as well as absorption and reflection coefficients strictly depend on the coherent millimeter electromagnet field amplitude and resonance detuning. For the values of the field intensity larger than the critical f_c, the hysteresis appears in the system. The dispersion law shows that at the interaction of Fröhlich radiation with dipole-active Bose-condensed phonons in biological media the polaritons can occur. We show that the position and the form of the polariton branches depend on the stationary concentration of the Bose-condensed phonons. We mention that for finite values of the phonon mass, the equations (2) and (5) describe the electromagnetic field propagation and the generation of the coherent photons in biological medium, and can be represented by a system equivalent to Ginzburg-Landau-Keldysh. Neglecting the interaction of the coherent phonons with the thermostat γ=0, the above equations have soliton solutions. We believe that our work provides a good basis for future study and, in particular, provides some pointers for more detailed investigations of dynamics of Bose-condensed phonons with millimeter electromagnetic radiation in the biological media.

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