Superconducting state in the twofold degenerate Anderson impurity model

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***Abstract* — The twofold degenerated Anderson impurity model in superconducting state is investigated and the strong electronic correlations of impurity ion are taken into account by elaborating suitable diagram technique. We obtain the Dyson type equations between impurity electron propagators and normal and anomalous correlation functions and by summing infinite series of ladder diagrams have established the approximation for correlation functions. Criterion for appearance of superconducting state of the model is discussed.**

***Index terms* — Correlation function, electron propagator, Dyson type equation, twofold degenerated Anderson impurity model, strong electronic correlations**

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INTRODUCTION

The technological progress in nanoscale electronic circuits, such as quantum dots, stimulates the studies of quantum-impurity models. One of them is the twofold degenerate Anderson impurity model.

In previous our paper [1], quoted below as I, we have investigated the properties of the twofold degenerated Anderson impurity model in normal state.

Now we shall discuss the properties of this model in superconducting state.

Our investigation is based on the diagrammatic theory elaborated for strongly correlated electron systems both in non-degenerated [2-6,8-11] and in twofold degenerated systems [1,7].

In Section 2 the main equations are formulated and necessary approximations elaborated. In the next Section 3 the correlation function $\bar{Y}$ is determined in both cases when triplet or singlet superconductivity is realized. Section 4 is devoted to the analysis of the conditions which determine the critical temperature. Last Section 5 contains the conclusions.

We use the method of Matsubara Green's functions defined by equations (21) and (22) of paper I with Dyson-type equations formulated in (31) paper I. These last equations are:

$$
\begin{aligned}
&g_{\sigma}^{ll'}(i\omega_n) = \Lambda_{\sigma}^{ll'}(i\omega_n) + \\
&+\Lambda_{\sigma}^{ll_1}(i\omega_n)\mathcal{G}_{\sigma}^{l_1(0)}(i\omega_n)g_{\sigma}^{l_1l'}(i\omega_n) - \\
&-Y_{\sigma\bar{\sigma}}^{ll_1}(i\omega_n)\mathcal{G}_{\bar{\sigma}}^{l_1(0)}(-i\omega_n)\bar{f}_{\bar{\sigma}\sigma}^{l_1l'}(i\omega_n), \\
&\bar{f}_{\bar{\sigma}\sigma}^{ll'}(i\omega_n) = \bar{Y}_{\bar{\sigma}\sigma}^{ll'}(i\omega_n) + \\
&+\Lambda_{\bar{\sigma}}^{l_1l}(-i\omega_n)\mathcal{G}_{\bar{\sigma}}^{l_1(0)}(-i\omega_n)\bar{f}_{\bar{\sigma}\sigma}^{l_1l'}(i\omega_n) + \\
&+\bar{Y}_{\bar{\sigma}\sigma}^{ll_1}(i\omega_n)\mathcal{G}_{\sigma}^{l_1(0)}(i\omega_n)g_{\sigma}^{l_1l'}(i\omega_n).
\end{aligned}
\tag{1}
$$

Here $\Lambda_{\sigma}^{ll'}(i\omega_n)$, $Y_{\sigma\bar{\sigma}}^{ll'}(i\omega_n)$ and $\bar{Y}_{\bar{\sigma}\sigma}^{ll'}(i\omega_n)$ are correlation functions of superconducting state and $g_{\sigma}^{ll'}(i\omega_n)$ and $\bar{f}_{\bar{\sigma}\sigma}^{ll'}(i\omega_n)$ full normal and anomalous one-particle Green's functions.

II. THE MAIN EQUATIONS

Because the orbital quantum number l takes, in our model, two values $l = 1,2$ we can rewrite the equation (1) in the form:

$$
\begin{aligned}
&g_{\sigma}^{11}(i\omega_n) = \Lambda_{\sigma}^{11}(i\omega_n) + \Lambda_{\sigma}^{11}(i\omega_n)\mathcal{G}_{1}^{(0)}(i\omega_n)g_{\sigma}^{11}(i\omega_n) \\
&+\Lambda_{\sigma}^{12}(i\omega)\mathcal{G}_{2}^{(0)}(i\omega_n)g_{\sigma}^{21}(i\omega_n) - \\
&-Y_{\sigma\bar{\sigma}}^{11}(i\omega_n)\mathcal{G}_{1}^{(0)}(-i\omega_n)\bar{f}_{\bar{\sigma}\sigma}^{11}(i\omega_n) - \\
&-Y_{\sigma\bar{\sigma}}^{12}(i\omega_n)\mathcal{G}_{2}^{(0)}(-i\omega_n)\bar{f}_{\bar{\sigma}\sigma}^{21}(i\omega_n), \\
&g_{\sigma}^{21}(i\omega_n) = \Lambda_{\sigma}^{21}(i\omega_n) + \\
&+\Lambda_{\sigma}^{21}(i\omega_n)\mathcal{G}_{1}^{(0)}(i\omega_n)g_{\sigma}^{11}(i\omega_n) + \\
&+\Lambda_{\sigma}^{22}(i\omega)\mathcal{G}_{2}^{(0)}(i\omega_n)g_{\sigma}^{21}(i\omega_n) \\
&-Y_{\sigma\bar{\sigma}}^{21}(i\omega_n)\mathcal{G}_{1}^{(0)}(-i\omega_n)\bar{f}_{\bar{\sigma}\sigma}^{11}(i\omega_n) - \\
&-Y_{\sigma\bar{\sigma}}^{22}(i\omega_n)\mathcal{G}_{2}^{(0)}(-i\omega_n)\bar{f}_{\bar{\sigma}\sigma}^{21}(i\omega_n), \\
&\bar{f}_{\bar{\sigma}\sigma}^{11}(i\omega_n) = \bar{Y}_{\bar{\sigma}\sigma}^{11}(i\omega_n) + \bar{Y}_{\bar{\sigma}\sigma}^{11}\mathcal{G}_{1}^{(0)}(i\omega_n)g_{\sigma}^{11}(i\omega_n) + \\
&+\Lambda_{\bar{\sigma}}^{11}(-i\omega_n)\mathcal{G}_{1}^{(0)}(-i\omega_n)\bar{f}_{\bar{\sigma}\sigma}^{11}(i\omega_n) + \\
&+\bar{Y}_{\bar{\sigma}\sigma'}^{12}(i\omega_n)\mathcal{G}_{2}^{(0)}(i\omega_n)g_{\sigma}^{21}(i\omega_n) + \\
&+\Lambda_{\bar{\sigma}}^{21}(-i\omega_n)\mathcal{G}_{2}^{(0)}(-i\omega_n)\bar{f}_{\bar{\sigma}\sigma}^{21}(i\omega_n), \\
&\bar{f}_{\bar{\sigma}\sigma}^{21}(i\omega_n) = \bar{Y}_{\bar{\sigma}\sigma}^{21}(i\omega_n) + \bar{Y}_{\bar{\sigma}\sigma}^{21}\mathcal{G}_{1}^{(0)}(i\omega_n)g_{\sigma}^{11}(i\omega_n) + \\
&+\Lambda_{\bar{\sigma}}^{12}(-i\omega_n)\mathcal{G}_{1}^{(0)}(-i\omega_n)\bar{f}_{\bar{\sigma}\sigma}^{11}(i\omega_n) + \\
&\bar{Y}_{\bar{\sigma}\sigma}^{22}(i\omega_n)\mathcal{G}_{2}^{(0)}(i\omega_n)g_{\sigma}^{21}(i\omega_n) + \\
&+\Lambda_{\bar{\sigma}}^{22}(-i\omega_n)\mathcal{G}_{2}^{(0)}(-i\omega_n)\bar{f}_{\bar{\sigma}\sigma}^{21}(i\omega_n).
\end{aligned}
\tag{2}
$$

The other system of four equations for quantities g_{σ}^{12}, g_{σ}^{22}, $\bar{f}_{\bar{\sigma}\sigma}^{12}$, and $\bar{f}_{\bar{\sigma}\sigma}^{22}$ can be formulated.

We introduce the definition

$$Q_{\sigma}^{l}(i\omega_n) = 1 - \Lambda_{\sigma}^{ll}(i\omega_n)\mathcal{G}_{\sigma}^{l(0)}(i\omega_n), \tag{3}$$

and determine the determinant of fourth order $D_4(i\omega_n)$:

$$D_4(i\omega_n)=\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix}, \qquad (4)$$

where

$$\begin{aligned}
&a_{11}=Q^1_\sigma(i\omega_n),\ a_{12}=-\Lambda^{12}_\sigma(i\omega_n)\mathcal{G}^{(0)}_2(i\omega_n),\\
&a_{13}=Y^{11}_{\sigma\bar\sigma}(i\omega_n)\mathcal{G}^{(0)}_1(-i\omega_n),\ a_{14}=Y^{12}_{\sigma\bar\sigma}(i\omega_n)\mathcal{G}^{(0)}_2(-i\omega_n),\\
&a_{21}=-\Lambda^{21}_\sigma(i\omega_n)\mathcal{G}^{(0)}_1(i\omega_n),\ a_{22}=Q^2_\sigma(i\omega_n),\\
&a_{23}=Y^{21}_{\sigma\bar\sigma}(i\omega_n)\mathcal{G}^{(0)}_1(-i\omega_n),\ \ a_{24}=Y^{22}_{\sigma\bar\sigma}(i\omega_n)\mathcal{G}^{(0)}_2(-i\omega_n),\\
&a_{31}=-\bar Y^{11}_{\bar\sigma\sigma}(i\omega_n)\mathcal{G}^{(0)}_1(i\omega_n),\ a_{32}=-\bar Y^{12}_{\bar\sigma\sigma}(i\omega_n)\mathcal{G}^{(0)}_2(i\omega_n),\\
&a_{33}=Q^1_{\bar\sigma}(-i\omega_n),\ \ a_{34}=-\Lambda^{21}_{\bar\sigma}(-i\omega_n)\mathcal{G}^{(0)}_2(-i\omega_n),\\
&a_{41}=-\bar Y^{21}_{\bar\sigma\sigma}(i\omega_n)\mathcal{G}^{(0)}_1(i\omega_n),\ a_{42}=-\bar Y^{22}_{\bar\sigma\sigma}(i\omega_n)\mathcal{G}^{(0)}_2(i\omega_n),\\
&a_{43}=-\Lambda^{12}_{\bar\sigma}(-i\omega_n)\mathcal{G}^{(0)}_1(-i\omega_n),\ \ a_{44}=Q^2_{\bar\sigma}(-i\omega_n).
\end{aligned}$$

These equations are the Dyson-type equations and they establish the relations between propagators $\mathcal{g}$, f and $\bar f$ and correlation functions Λ, Y and $\bar Y$. Anomalous correlation functions have the properties of the order parameters Y and $\bar Y$ of the superconducing state.

The system of equation (2) permits us to obtain for $T=T_c$ such linear dependencies:

$$\begin{aligned}
&\Delta_4(i\omega_n)\,\bar f^{11}_{\bar\sigma\sigma}=\bar Y^{11}_{\bar\sigma\sigma}Q^2_\sigma(i\omega_n)Q^2_{\bar\sigma}(-i\omega_n)+\\
&+\bar Y^{12}_{\bar\sigma\sigma}(i\omega_n)Q^2_{\bar\sigma}(-i\omega_n)\Lambda^{21}_\sigma(i\omega_n)\mathcal{G}^{(0)}_2(i\omega_n)+\\
&+\bar Y^{21}_{\bar\sigma\sigma}(i\omega_n)\Lambda^{21}_{\bar\sigma}(-i\omega_n)\mathcal{G}^{(0)}_2(-i\omega_n)Q^2_\sigma(i\omega_n)+\\
&+\bar Y^{22}_{\bar\sigma\sigma}(i\omega_n)\Lambda^{21}_{\bar\sigma}(-i\omega_n)\Lambda^{21}_\sigma(i\omega_n)\mathcal{G}^{(0)}_2(i\omega_n)\mathcal{G}^{(0)}_2(-i\omega_n),\\
&\Delta_4(i\omega_n)\,\bar f^{21}_{\bar\sigma\sigma}(i\omega_n)=Q^2_\sigma(i\omega_n)\bar Y^{11}_{\bar\sigma\sigma}(i\omega_n)\Lambda^{12}_{\bar\sigma}(-i\omega_n)\mathcal{G}^{(0)}_1(-i\omega_n)\\
&+Q^2_\sigma(i\omega_n)\bar Y^{21}_{\bar\sigma\sigma}(i\omega_n)Q^1_{\bar\sigma}(-i\omega_n)+\\
&+\bar Y^{12}_{\bar\sigma\sigma}(i\omega_n)\Lambda^{12}_{\bar\sigma}(-i\omega_n)\Lambda^{21}_\sigma(i\omega_n)\mathcal{G}^{(0)}_2(i\omega_n)\mathcal{G}^{(0)}_1(-i\omega_n)+\\
&+\bar Y^{22}_{\bar\sigma\sigma}(i\omega_n)Q^1_{\bar\sigma}(-i\omega_n)\Lambda^{21}_\sigma(i\omega_n)\mathcal{G}^{(0)}_2(i\omega_n),\\
&\Delta_4(i\omega_n)\,\bar f^{22}_{\bar\sigma\sigma}(i\omega_n)=\bar Y^{22}_{\bar\sigma\sigma}Q^1_\sigma(i\omega_n)Q^1_{\bar\sigma}(-i\omega_n)+\\
&+\bar Y^{21}_{\bar\sigma\sigma}(i\omega_n)Q^1_{\bar\sigma}(-i\omega_n)\Lambda^{12}_\sigma(i\omega_n)\mathcal{G}^{(0)}_1(i\omega_n)+\\
&+\bar Y^{12}_{\bar\sigma\sigma}(i\omega_n)\Lambda^{12}_{\bar\sigma}(-i\omega_n)\mathcal{G}^{(0)}_1(-i\omega_n)Q^1_\sigma(i\omega_n)+\\
&+\bar Y^{22}_{\bar\sigma\sigma}(i\omega_n)\Lambda^{12}_{\bar\sigma}(-i\omega_n)\Lambda^{12}_\sigma(i\omega_n)\mathcal{G}^{(0)}_1(i\omega_n)\mathcal{G}^{(0)}_1(-i\omega_n),\\
&\Delta_4(i\omega_n)\,\bar f^{12}_{\bar\sigma\sigma}=Q^1_\sigma(i\omega_n)\bar Y^{22}_{\bar\sigma\sigma}(i\omega_n)\Lambda^{21}_{\bar\sigma}(-i\omega_n)\mathcal{G}^{(0)}_2(-i\omega_n)+\\
&+Q^1_\sigma(i\omega_n\bar Y^{12}_{\bar\sigma\sigma}(i\omega_n)Q^2_{\bar\sigma}(-i\omega_n)+\\
&+\bar Y^{21}_{\bar\sigma\sigma}(i\omega_n)\Lambda^{21}_{\bar\sigma}(-i\omega_n)\Lambda^{12}_\sigma(i\omega_n)\mathcal{G}^{(0)}_1(i\omega_n)\mathcal{G}^{(0)}_2(-i\omega_n)+\\
&+\bar Y^{11}_{\bar\sigma\sigma}(i\omega_n)Q^2_{\bar\sigma}(-i\omega_n)\Lambda^{12}_\sigma(i\omega_n)\mathcal{G}^{(0)}_1(i\omega_n),
\end{aligned}\qquad (5)$$

where $\Delta_4(i\omega_n)$ is equal to $D_4(i\omega_n)$ with equated to zero the order parameters Y and $\bar Y$:

$$\begin{aligned}
&\Delta_4(i\omega_n)=(Q^1_\sigma(i\omega_n)Q^2_\sigma(i\omega_n)-\\
&-\Lambda^{12}_\sigma(i\omega_n)\Lambda^{21}_\sigma(i\omega_n)\mathcal{G}^{(0)}_1(i\omega_n)\mathcal{G}^{(0)}_2(i\omega_n))\times\\
&\times(Q^1_{\bar\sigma}(-i\omega_n)Q^2_{\bar\sigma}(-i\omega_n)-\\
&-\Lambda^{12}_{\bar\sigma}(-i\omega_n)\Lambda^{21}_{\bar\sigma}(-i\omega_n)\mathcal{G}^{(0)}_1(-i\omega_n)\mathcal{G}^{(0)}_2(-i\omega_n)).
\end{aligned}\qquad (6)$$

The system of equations (5) is not closed because up till now we have not the dependence of the correlation functions Λ, Y and $\bar Y$ on the electron propagators.

Such dependence can be the result of infinite summation of the diagrams and is, of course, consequence of some approximations. Our main approximations are depicted on the Fig. 3 paper I.

III. SELF-CONSISTENCY CONDITION

Now we shall make more precise our approximation (see Fig. 1) which determine the correlation function $\bar Y$ as a result of summing class of ladder diagrams

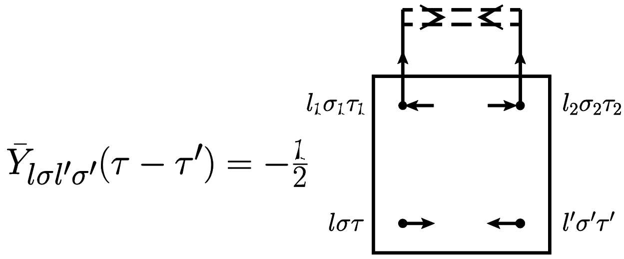

Figure 1: The ladder approximation for $\bar Y$ correlation function. Double dashed line is full anomalous Green's function of impurity electrons. The solid thin lines are conduction electron Green's functions. The rectangle depicts the simplest irreducible Green's function.

The analytical form of the result of this approximation is

$$\begin{aligned}
&\bar Y^{ll'}_{\sigma\sigma'}(i\omega)\approx-\frac{1}{2\beta}\sum_{\omega_1}\sum_{l_1\sigma_1l_2\sigma_2}\\
&(\tilde G^{(0)irr}_2[l_1,\sigma_1,-i\omega_1;l_2,\sigma_2,i\omega_1\,|\,l,\sigma,-i\omega;l',\sigma',i\omega]\times\\
&\times\mathcal{G}^{(0)}_{l_1\sigma_1}(-i\omega_1)\,\bar f^{\,l_1l_2}_{\sigma_1\sigma_2}(i\omega_1)\mathcal{G}^{(0)}_{l_2\sigma_2}(i\omega_1)),
\end{aligned}\qquad (7)$$

where

$$\begin{aligned}
&G^{irr}_2[l_1,\sigma_1,i\omega_1;l_2,\sigma_2,i\omega_2\,|\,l_3,\sigma_3,i\omega_3;l_4,\sigma_4,i\omega_4]=\\
&\beta\delta(\omega_1+\omega_2-\omega_3-\omega_4)\times\\
&\times\tilde G^{irr}_2[l_1,\sigma_1,i\omega_1;l_2,\sigma_2,i\omega_2\,|\,l_3,\sigma_3,i\omega_3;l_4,\sigma_4,i\omega_4],
\end{aligned}\qquad (8)$$

$$\begin{aligned}
&\tilde G^{(0)irr}_2[l_1,\sigma_1,i\omega_1;l_2,\sigma_2,i\omega_2\,|\,l_3,\sigma_3,i\omega_3;l_4,\sigma_4,i\omega_4]=\\
&\frac{1}{6}p(i\omega_1)p(i\omega_2)(\beta\delta(\omega_1-\omega_4)\times\\
&\times\delta_{l_1l_4}\delta_{l_2l_3}[2\delta_{\sigma_1,-\sigma_4}\delta_{\sigma_2,-\sigma_3}\delta_{\sigma_2\sigma_4}+\\
&+\delta_{\sigma_1\sigma_3}\delta_{\sigma_1\sigma_4}\delta_{\sigma_2\sigma_3}-\delta_{\sigma_1\sigma_4}\delta_{\sigma_2\sigma_3}\delta_{\sigma_3,-\sigma_1}]-\\
&-\beta\delta(\omega_1-\omega_3)\delta_{l_1l_3}\delta_{l_2l_4}[2\delta_{\sigma_1,-\sigma_3}\delta_{\sigma_2,-\sigma_4}\delta_{\sigma_2\sigma_3}+\\
&+\delta_{\sigma_1\sigma_3}\delta_{\sigma_2\sigma_4}\delta_{\sigma_1\sigma_4}-\delta_{\sigma_1\sigma_3}\delta_{\sigma_2\sigma_4}\delta_{\sigma_4,-\sigma_1}]).
\end{aligned}\qquad (9)$$

By using the antisymmetry property which is the consequence of the Pauli principle

$$\bar{f}^{ll'}_{\sigma\sigma'}(i\omega_n) = -\bar{f}^{l'l}_{\sigma'\sigma}(-i\omega_n), \tag{10}$$

we can transform the above equation and to obtain more simple one:

$$\begin{aligned} \bar{Y}^{ll'}_{\sigma\sigma'}(i\omega_n) = \frac{1}{6}[&\delta_{\sigma\sigma'} q_{l'\sigma}(i\omega_n) q_{l\sigma}(-i\omega_n)\,\bar{f}^{ll'}_{\sigma\sigma}(i\omega_n) - \\ &-\delta_{\sigma'\bar{\sigma}} q_{l\sigma}(-i\omega_n) q_{l'\bar{\sigma}}(i\omega_n)\,\bar{f}^{ll'}_{\sigma\bar{\sigma}}(i\omega_n) + \\ &+2\delta_{\sigma'\bar{\sigma}} q_{l'\sigma}(i\omega_n) q_{l\bar{\sigma}}(-i\omega_n)\,\bar{f}^{ll'}_{\bar{\sigma}\sigma}(i\omega_n)], \end{aligned} \tag{11}$$

where

$$q_{l\sigma} = p(i\omega_n)\mathcal{G}^{(0)}_{l\sigma}(i\omega_n). \tag{12}$$

As we can see there are two different possibilities for correlation function $\bar{Y}$:
one diagonal by spin indices

$$\bar{Y}^{ll'}_{\sigma\sigma}(i\omega_n) = \frac{1}{6} q_{l\sigma}(-i\omega_n) q_{l'\sigma}(i\omega_n)\,\bar{f}^{ll'}_{\sigma\sigma}(i\omega_n), \tag{13}$$

and second non diagonal by spin indices:

$$\begin{aligned} \bar{Y}^{ll'}_{\sigma\bar{\sigma}}(i\omega_n) &= \frac{1}{6} q_{l\sigma}(-i\omega_n) q_{l'\bar{\sigma}}(i\omega_n) \times \\ &\times (2\,\bar{f}^{ll'}_{\bar{\sigma}\sigma}(i\omega_n) - \bar{f}^{ll'}_{\sigma\bar{\sigma}}(i\omega_n)). \end{aligned} \tag{14}$$

The diagonal solution belongs to the triplet superconductivity and non diagonal to the singlet case.

We suppose that in the last case the changing of the order of the spin indices is accompanied by changing of the sigh of the function.

In such a way we obtain

$$\bar{Y}^{ll'}_{\bar{\sigma}\sigma}(i\omega_n) = \frac{1}{2} q_l(-i\omega_n) q_{l'}(i\omega_n)\,\bar{f}^{ll'}_{\bar{\sigma}\sigma}(i\omega_n). \tag{15}$$

The both possibilities can be jointed in the form

$$\bar{Y}^{ll'}(i\omega_n) = \lambda q_l(-i\omega_n) q_{l'}(i\omega_n)\,\bar{f}^{ll'}(i\omega_n), \tag{16}$$

where $\lambda = -\frac{1}{2}$ for singlet and $\lambda = \frac{1}{6}$ for triplet superconductivity.

IV. CRITICAL TEMPERATURE

Now we come back to the system of linearized equations (5) and substitute the propagators $\bar{f}^{ll'}_{\bar{\sigma}\sigma}$ by their values obtained from equation (16). The result of such substitution is the following system of linear equations for the components of order parameter $\bar{Y}^{ll'}_{\bar{\sigma}\sigma}$:

$$\begin{aligned}
&\left[Q^2_\sigma(i\omega_n)Q^2_{\bar{\sigma}}(-i\omega_n) - \frac{\Delta_4}{\lambda q_1(-i\omega_n)q_1(i\omega_n)}\right]\bar{Y}^{11}_{\bar{\sigma}\sigma}(i\omega_n) + \\
&+\mathcal{G}^{(0)}_2(i\omega_n)Q^2_{\bar{\sigma}}(-i\omega_n)\Lambda^{21}_\sigma(i\omega_n)\bar{Y}^{12}_{\bar{\sigma}\sigma}(i\omega_n) + \\
&+\mathcal{G}^{(0)}_2(-i\omega_n)Q^2_\sigma(i\omega_n)\Lambda^{21}_{\bar{\sigma}}(-i\omega_n)\bar{Y}^{21}_{\bar{\sigma}\sigma}(i\omega_n) + \\
&+\mathcal{G}^{(0)}_2(-i\omega_n)\mathcal{G}^{(0)}_2(i\omega_n)\Lambda^{21}_\sigma(i\omega_n)\Lambda^{21}_{\bar{\sigma}}(-i\omega_n)\bar{Y}^{22}_{\bar{\sigma}\sigma}(i\omega_n) = 0, \\
&\mathcal{G}^{(0)}_1(i\omega_n)(Q^2_{\bar{\sigma}}(-i\omega_n)\Lambda^{12}_\sigma(i\omega_n)\bar{Y}^{11}_{\bar{\sigma}\sigma}(i\omega_n) + \\
&\left[Q^1_\sigma(i\omega_n)Q^2_{\bar{\sigma}}(-i\omega_n) - \frac{\Delta_4}{\lambda q_1(-i\omega_n)q_2(i\omega_n)}\right]\bar{Y}^{12}_{\bar{\sigma}\sigma}(i\omega_n)) + \\
&+\mathcal{G}^{(0)}_2(-i\omega_n)\mathcal{G}^{(0)}_1(i\omega_n)\Lambda^{12}_\sigma(i\omega_n)\bar{Y}^{21}_{\bar{\sigma}\sigma}(i\omega_n) + \\
&+\mathcal{G}^{(0)}_2(-i\omega_n)Q^1_\sigma(i\omega_n)\Lambda^{21}_{\bar{\sigma}}(-i\omega_n)\bar{Y}^{22}_{\bar{\sigma}\sigma}(i\omega_n) = 0, \\
&\mathcal{G}^{(0)}_1(-i\omega_n)Q^2_\sigma(i\omega_n)\Lambda^{12}_{\bar{\sigma}}(-i\omega_n)\bar{Y}^{11}_{\bar{\sigma}\sigma}(i\omega_n) + \\
&+\mathcal{G}^{(0)}_1(-i\omega_n)\mathcal{G}^{(0)}_2(i\omega_n)\Lambda^{21}_\sigma(i\omega_n)\Lambda^{12}_{\bar{\sigma}}(-i\omega_n)\bar{Y}^{12}_{\bar{\sigma}\sigma}(i\omega_n) + \\
&+\left[Q^2_\sigma(i\omega_n)Q^1_{\bar{\sigma}}(-i\omega_n) - \frac{\Delta_4}{\lambda q_2(-i\omega_n)q_1(i\omega_n)}\right]\bar{Y}^{21}_{\bar{\sigma}\sigma}(i\omega_n) + \\
&+\mathcal{G}^{(0)}_2(i\omega_n)Q^1_{\bar{\sigma}}(-i\omega_n)\Lambda^{21}_\sigma(i\omega_n)\bar{Y}^{22}_{\bar{\sigma}\sigma}(i\omega_n) = 0, \\
&\mathcal{G}^{(0)}_1(-i\omega_n)\mathcal{G}^{(0)}_1(i\omega_n)\Lambda^{12}_\sigma(i\omega_n)\Lambda^{12}_{\bar{\sigma}}(-i\omega_n)\bar{Y}^{11}_{\bar{\sigma}\sigma}(i\omega_n) + \\
&+\mathcal{G}^{(0)}_1(-i\omega_n)Q^1_\sigma(i\omega_n)\Lambda^{12}_{\bar{\sigma}}(-i\omega_n)\bar{Y}^{12}_{\bar{\sigma}\sigma}(i\omega_n) + \\
&+\mathcal{G}^{(0)}_1(i\omega_n)Q^1_{\bar{\sigma}}(-i\omega_n)\Lambda^{12}_\sigma(i\omega_n)\bar{Y}^{21}_{\bar{\sigma}\sigma}(i\omega_n) + \\
&+\left[Q^1_\sigma(i\omega_n)Q^1_{\bar{\sigma}}(-i\omega_n) - \frac{\Delta_4}{\lambda q_2(-i\omega_n)q_2(i\omega_n)}\right]\bar{Y}^{22}_{\bar{\sigma}\sigma}(i\omega_n) = 0,
\end{aligned} \tag{17}$$

where Δ_4 is equal to (6).

Determinant D_s of this linear system of equations must be equal to zero:

$$\begin{vmatrix} d_{11} & d_{12} & d_{13} & d_{14} \\ d_{21} & d_{22} & d_{23} & d_{24} \\ d_{31} & d_{32} & d_{33} & d_{34} \\ d_{41} & d_{42} & d_{43} & d_{44} \end{vmatrix} = 0, \tag{18}$$

where

$$\begin{aligned}
d_{11} &= Q^2_\sigma(k)Q^2_{\bar{\sigma}}(-k) - \frac{\Delta_4}{\lambda q_1(-k)q_1(k)}, \\
d_{12} &= \mathcal{G}^{(0)}_2(k)Q^2_{\bar{\sigma}}(-k)\Lambda^{21}_\sigma(k), \\
d_{13} &= \mathcal{G}^{(0)}_2(-k)Q^2_\sigma(k)\Lambda^{21}_{\bar{\sigma}}(-k), \\
d_{14} &= \mathcal{G}^{(0)}_2(-k)\mathcal{G}^{(0)}_2(k)\Lambda^{21}_\sigma(k)\Lambda^{21}_{\bar{\sigma}}(-k), \\
d_{21} &= \mathcal{G}^{(0)}_1(k)Q^2_{\bar{\sigma}}(-k)\Lambda^{12}_\sigma(k), \\
d_{22} &= Q^1_\sigma(k)Q^2_{\bar{\sigma}}(-k) - \frac{\Delta_4}{\lambda q_1(-k)q_2(k)}, \\
d_{23} &= \mathcal{G}^{(0)}_2(-k)\mathcal{G}^{(0)}_1(k)\Lambda^{12}_\sigma(k)\Lambda^{21}_{\bar{\sigma}}(-k), \\
d_{24} &= \mathcal{G}^{(0)}_2(-k)Q^1_\sigma(k)\Lambda^{21}_{\bar{\sigma}}(-k), \\
d_{31} &= \mathcal{G}^{(0)}_1(-k)Q^2_\sigma(k)\Lambda^{12}_{\bar{\sigma}}(-k), \\
d_{32} &= \mathcal{G}^{(0)}_1(-k)\mathcal{G}^{(0)}_2(k)\Lambda^{21}_\sigma(k)\Lambda^{12}_{\bar{\sigma}}(-k), \\
d_{33} &= Q^2_\sigma(k)Q^1_{\bar{\sigma}}(-k) - \frac{\Delta_4}{\lambda q_2(-k)q_1(k)}, \\
d_{34} &= \mathcal{G}^{(0)}_2(k)Q^1_{\bar{\sigma}}(-k)\Lambda^{21}_\sigma(k), \\
d_{41} &= \mathcal{G}^{(0)}_1(-k)\mathcal{G}^{(0)}_1(k)\Lambda^{12}_\sigma(k)\Lambda^{12}_{\bar{\sigma}}(-k), \\
d_{42} &= \mathcal{G}^{(0)}_1(-k)Q^1_\sigma(k)\Lambda^{12}_{\bar{\sigma}}(-k), \\
d_{43} &= \mathcal{G}^{(0)}_1(k)Q^1_{\bar{\sigma}}(-k)\Lambda^{12}_\sigma(k), \\
d_{44} &= Q^1_\sigma(k)Q^1_{\bar{\sigma}}(-k) - \frac{\Delta_4}{\lambda q_2(-k)q_2(k)}, \\
k &= i\omega_n.
\end{aligned}$$

This condition determines the free parameter of the theory and, as usual, defines the critical temperature T_c . In our case the critical temperature is present in dependence from T_c of the Matsubara frequencies ($\omega_n = (2n+1)\pi k_B T_c$).

We put λ equal to value $-\frac{1}{2}$ which correspond to the singlet state and preserve equation (18) for determination the value of T_c .

The other argument in favor of the choose $\lambda = -\frac{1}{2}$ is the approximation based on the equality to zero of the functions $Q_\sigma^l(i\omega_n) = 0$. In this special case equation (18) is reduced to the simple form:

$$D_s = (\Delta_4^{(0)})^2 [\frac{\Delta_4^{(0)}}{\lambda^2 q_1(-i\omega_n) q_1(i\omega_n) q_2(-i\omega_n) q_2(i\omega_n)} - 1]^2 \quad (19)$$

,

where $\Delta_4^{(0)}$ is Δ_4 with the condition: $Q_\sigma^l(i\omega_n) = 0$:

$$\Delta_4^{(0)} = \mathcal{G}_1^{(0)}(i\omega_n)\mathcal{G}_1^{(0)}(-i\omega_n)\mathcal{G}_2^{(0)}(i\omega_n)\mathcal{G}_2^{(0)}(-i\omega_n) \times$$
$$\times \Lambda_\sigma^{12}(i\omega_n)\Lambda_{\bar{\sigma}}^{12}(-i\omega_n)\Lambda_\sigma^{21}(i\omega_n)\Lambda_{\bar{\sigma}}^{21}(-i\omega_n).$$

By taking into account the value (56) paper I

$$\Lambda_\sigma^{12}(i\omega_n)\Lambda_\sigma^{21}(i\omega_n) = -\frac{p^2(i\omega_n)}{2}$$

and the definition of q_σ^l , we obtain:

$$D_s = (\Delta_4^{(0)})^2 \left(\frac{1}{(2\lambda)^2} - 1 \right)^2 = 0 \quad (20)$$

That is the condition $\lambda = -\frac{1}{2}$.

V. CONCLUSIONS

We have formulated the Dyson-type equations for full Green's functions of Anderson impurity model in superconducting state and established the relations between renormalized propagators and correlation functions.

We have summed infinite class of diagram and obtained the approximate expression for correlation functions and, in special, for superconductors order parameters.

We have investigated the linearized equations for order parameters and formulated the condition for realization of singlet superconductivity and determination of the critical temperature.

ACKNOWLEDGMENTS

Two of authors (V.M. and L.D.) would like to thank Dr. S. Cojocaru for a very helpful discussion.

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