# THE DETERMINATION OF THE LINK EXPRESSIONS BETWEEN SOME GEOMETRIC PARAMETERS OF THE ROMASCON MILLING CUTTERS PART II 

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$$
\begin{equation*}
\operatorname{cotg} \rho=\operatorname{cotg} \Phi \cdot \cos \theta_{V} \tag{15}
\end{equation*}
$$

In triangle $B V V^{\prime}$ (figures 6a and e):

$$
\begin{equation*}
\operatorname{tg} \theta_{V}^{\prime}=\frac{V V^{\prime}}{B V^{\prime}} \tag{16}
\end{equation*}
$$

where $\theta^{\prime} v$ is the angle which sees the point of tooth $V$, being considered from the milling cutter axis in section G-G (figures 5 and 6).

In triangle $A V V^{\prime}$ (figure 6a and c):

$$
\begin{equation*}
V V^{\prime}=A V^{\prime} \cdot \operatorname{tg} \theta_{V} \tag{17}
\end{equation*}
$$

In triangle $A B V^{\prime}$ (figure 6a and d):

$$
\begin{equation*}
B V^{\prime}=\frac{A V^{\prime}}{\sin \Phi} \tag{18}
\end{equation*}
$$

By replacing relations (17) and (18) into relation (16) it results that:

$$
\begin{equation*}
\operatorname{tg} \theta_{V}^{\prime}=\operatorname{tg} \theta_{V} \cdot \sin \Phi \tag{19}
\end{equation*}
$$

Taking into consideration that the expression (8) of the angle $\rho$ has been determined, (relation 14), the angles of the milling cutter are found through particularizing in relations (8) and (9) angles which are defined in transverse and longitudinal planes to tooth axis, as follows:

- for $\eta_{1}=90^{\circ}+\rho$ it is obtained $\gamma_{s}=\gamma_{x f}^{\prime}$ respectively $\alpha_{s}=\alpha_{x f}^{\prime}$ therefore

$$
\begin{align*}
& \operatorname{tg} \gamma_{x f}^{\prime}=\operatorname{tg} \gamma_{N f} \cdot \cos \left(\rho-K_{f}\right)- \\
& \operatorname{tg} \lambda_{f} \cdot \sin \left(\rho-K_{f}\right)  \tag{20}\\
& \operatorname{cotg} \alpha_{x f}^{\prime}=\operatorname{cotg} \alpha_{N f} \cdot \cos \left(\rho-K_{f}\right)- \\
& \operatorname{tg} \lambda_{f} \cdot \sin \left(\rho-K_{f}\right) \tag{21}
\end{align*}
$$

- for $\eta_{2}=\rho$ it is obtained $\gamma_{s}=\gamma_{y f}^{\prime}$ respectively $\alpha_{s}=\alpha_{y f}^{\prime}$, therefore

$$
\begin{align*}
& \operatorname{tg} \gamma^{\prime}{ }_{y f}=\operatorname{tg} \gamma_{N f} \cdot \sin \left(\rho-K_{f}\right)+ \\
& \operatorname{tg} \lambda_{f} \cdot \cos \left(\rho-K_{f}\right)  \tag{22}\\
& \operatorname{cotg} \alpha^{\prime}{ }_{y f}=\operatorname{cotg} \alpha_{N f} \cdot \sin \left(\rho-K_{f}\right)+  \tag{23}\\
& \operatorname{tg} \lambda_{f} \cdot \cos \left(\rho-K_{f}\right)
\end{align*}
$$

Relations (20), (21), (22), (23) serve to determine the geometric parameters of the milling cutter in transverse plane and longitudinal plane respectively to tooth axis.

The connection between angles $\alpha_{x f}^{\prime}, \gamma_{x f}^{\prime}, \alpha_{y f}^{\prime}, \gamma_{y f}^{\prime}$ and $\alpha^{\prime \prime}{ }_{x f}, \gamma^{\prime \prime}{ }_{x,}, \alpha^{\prime \prime}{ }_{y f} \gamma^{\prime \prime}{ }_{y f}$ is established by means of angle $\omega^{\prime \prime}$ which is measured in section D-D (figure 5), between the tooth axis projection on the axis plane and the projection of the normal on the perpendicular plane to the tooth axis which passes through the point of the tooth $V$, being actually the projection of the angle $\omega^{\prime}$ on section D-D.

In figure 7 a detailed view is presented which is taken from figure 5 in order to determine angle $\omega^{\prime \prime}$.


Figure 7. Determination of the angle $\omega^{\prime \prime}$

In figure 7 a the straight line CV ' represents the tooth axis in section G-G and the straight line $A B$ represents its projection in section D-D respectively (figure 5).

In triangle $V^{\prime} C V$ (right-angled in C - figure 7 a and b) we have relation:

$$
\begin{equation*}
C V^{\prime}=\sin \omega^{\prime} \cdot V V^{\prime} \tag{24}
\end{equation*}
$$

In triangle $V A V^{\prime}$ (right-angled in $A$ - figure 7a and c):

$$
\begin{equation*}
A V=V V^{\prime} \cdot \cos \theta_{V}^{\prime} \tag{25}
\end{equation*}
$$

In triangle $V A B$ (right-angled in $B$-figure 7a and d):

$$
\begin{equation*}
A B=A V \cdot \sin \omega^{\prime \prime} \tag{26}
\end{equation*}
$$

The segment $A B$ represents the orthogonal projection of the segment CV' (figure 5a), therefore:

$$
\begin{equation*}
A B=C V^{\prime} \tag{27}
\end{equation*}
$$

By replacing relations (24), (25), (26) in (27) the result is:

$$
\begin{equation*}
\sin \omega^{\prime \prime}=\frac{\sin \omega^{\prime}}{\cos \theta_{V}^{\prime}} \tag{28}
\end{equation*}
$$

From figure 5 section D-D, taking into account the sign agreement for $\omega^{\prime \prime}$, the expressions of the angles $\alpha^{\prime \prime}{ }_{y f}$ and $\gamma^{\prime \prime}{ }_{y f}$ can be determined directly:

$$
\begin{align*}
\alpha^{\prime \prime}{ }_{y f} & =\alpha^{\prime}{ }_{y f}+\omega^{\prime \prime}  \tag{29}\\
\gamma^{\prime \prime}{ }_{y f} & =\gamma^{\prime}{ }_{y f}-\omega^{\prime \prime} \tag{30}
\end{align*}
$$

Inasmuch as the section F-F (figure 5) in which the angles $\gamma^{\prime \prime}{ }_{x f}$ and $\alpha^{\prime \prime}{ }_{x f}$ are measured is normal to tooth axis, the angle $\theta^{\prime \prime}{ }_{V}$ has a different value from the angle $\theta_{V}^{\prime}$ measured in a normal plane to the


Figure 8. Relation between $\theta^{\prime \prime}{ }_{V}$ and $\theta_{V}^{\prime}$ projection of the tooth axis. The relation between the
two parameters is established on the basis of figure 8 which represents a detail from figure 5 pertaining to section F-F.

In triangle $L V^{\prime} V$ (right-angled in $V^{\prime}$ ), figure 8 :

$$
\begin{equation*}
L V=\frac{V V^{\prime}}{\sin \theta^{\prime}{ }_{V}} \tag{31}
\end{equation*}
$$

In triangle $L E V$ (right-angled in $E$ ), figure 8:

$$
\begin{equation*}
L V=\frac{V E}{\sin \theta^{\prime \prime}{ }_{V}}=\frac{V V^{\prime} \cdot \cos \omega^{\prime}}{\sin \theta^{\prime \prime}{ }_{V}} \tag{32}
\end{equation*}
$$

By making relations (31) and (32) equal it has been obtained:

$$
\begin{equation*}
\sin \theta_{V}^{\prime \prime}=\sin \theta_{V}^{\prime} \cdot \cos \omega^{\prime} \tag{33}
\end{equation*}
$$

From section F-F (figure 5) it can be directly concluded that:

$$
\begin{align*}
& \gamma_{x d}=\gamma_{x f}^{\prime \prime}+\theta_{V}^{\prime \prime}  \tag{34}\\
& \alpha_{x d}=\alpha_{x f}^{\prime \prime}-\theta_{V}^{\prime \prime} \tag{35}
\end{align*}
$$

In relations (34) and (35) the expressions of the angles $\gamma^{\prime \prime}{ }_{x f}$ and $\alpha^{\prime \prime}{ }_{x f}$ are unknown and they are to be determined in what follows.

In order to determine the relation between the geometric parameters of the milling cutter and the tooth in longitudinal plane and transverse plane respectively, it is written the equation of the tangential plane to the face through points $V, P$ (section D-D figure 5) and $Q$ (section E-E - figure 5), all these having in the trihedron $V x_{f}^{\prime} y^{\prime} z_{f}^{\prime}$ the following coordinates:

$$
\begin{aligned}
& \mathrm{V}(0 ; 0 ; 0) \\
& \mathrm{P}\left(0 ; 1 ;-\operatorname{tg} \gamma_{\mathrm{yf}}^{\prime}\right) \\
& \mathrm{Q}\left(-1 ; 0 ;-\operatorname{tg} \gamma_{\mathrm{xf}}\right)
\end{aligned}
$$

The determinant which gives the equation of the tangential plane to the face of the tooth is:

$$
\left|\begin{array}{cccc}
x_{f}^{\prime} & y^{\prime}{ }_{f} & z_{f}^{\prime} & 1  \tag{36}\\
0 & 0 & 0 & 1 \\
0 & 1 & -\operatorname{tg} \gamma^{\prime}{ }_{y f} & 1 \\
-1 & 0 & -\operatorname{tg} \gamma^{\prime}{ }_{x f} & 1
\end{array}\right|=0
$$

This determinant can also be written as:

$$
\left|\begin{array}{ccc}
x_{f}^{\prime} & y_{f}^{\prime} & z_{f}^{\prime}  \tag{37}\\
0 & 1 & -\operatorname{tg} \gamma^{\prime} \\
-1 & 0 & \operatorname{tg} \gamma^{\prime}{ }_{x f}
\end{array}\right|=0
$$

By expanding the determinant (37) it is obtained:

$$
\begin{equation*}
-x_{f} \cdot \operatorname{tg} \gamma_{x f}^{\prime}+y_{f}^{\prime} \cdot \operatorname{tg} \gamma_{y f}^{\prime}+z_{f}^{\prime}=0 \tag{38}
\end{equation*}
$$

The relation (38) represents the equation of the tangential plane to the face of the tooth in relation to the reference trihedron $V x^{\prime} f y^{\prime} z^{\prime}$, , attached to the milling cutter, with the axes so defined:

- the axis $V x_{f}^{\prime}$ in the forward motion of tool;
- the axis $V z_{f}^{\prime}$ tangential to the trajectory described by the nose $V$ of tool, normal to the principal plane of milling cutter;
- the axis $V y_{f}^{\prime}$ specially selected so that the trihedron $V x^{\prime} y^{\prime} y^{\prime} z z^{\prime}$, should be right-angled triorthogonal, parallel to the axis of spinning of milling cutter.

The trihedron $V x_{f}^{\prime} y y^{\prime} z^{\prime} f$, in which are defined the parameters of the longitudinal and transverse planes to milling cutter axis is rotated around the axis $V x_{f}^{\prime}$ with angle $\omega^{\prime \prime}$ (considered in positive sense of axis $V x_{f}^{\prime}$ in section D-D (figure 5), in relation to the trihedron $V x_{f y}^{\prime} y_{f}^{\prime} z^{\prime}$, and thus the matrix of transition between the two systems is:

$$
\left\|\begin{array}{ccc}
1 & 0 & 0  \tag{39}\\
0 & \cos \omega^{\prime \prime} & \sin \omega^{\prime \prime} \\
0 & -\sin \omega^{\prime \prime} & \cos \omega^{\prime \prime}
\end{array}\right\|
$$

from which it is obtained that:

$$
\left\{\begin{array}{l}
x_{f}^{\prime}=x^{\prime \prime}{ }_{f}  \tag{40}\\
y_{f}^{\prime}={y^{\prime \prime}}_{f} \cdot \cos \omega^{\prime \prime}-z^{\prime \prime} \cdot \sin \omega^{\prime \prime} \\
z_{f}^{\prime}=y^{\prime \prime}{ }_{f} \cdot \sin \omega^{\prime \prime}+z^{\prime \prime}{ }_{f} \cdot \cos \omega^{\prime \prime}
\end{array}\right.
$$

By replacing relation (40) into (38) it results that:

$$
\begin{align*}
& -x^{\prime \prime} \cdot \operatorname{tg}{\gamma^{\prime}}_{x f}+y_{f}^{\prime \prime}\left(\cos \omega^{\prime \prime} \cdot \operatorname{tg}{\gamma^{\prime}}_{y f}+\right. \\
& \left.\sin \omega^{\prime \prime}\right)+z^{\prime \prime}{ }_{f}\left(\cos \omega^{\prime \prime}-\sin \omega^{\prime \prime \cdot} \cdot \operatorname{tg} \gamma_{y f}^{\prime}\right)=0 \tag{41}
\end{align*}
$$

Relation (41) represents the equation of the tangential plane to the face of the tooth in the system $V x_{f}^{\prime} y^{\prime} z_{z}^{\prime} z_{z}^{\prime}$.

In order to determine the angle $y^{\prime \prime}{ }_{x x}$, the tangential plane to the face has been intersected with
the transverse plane, that is, in relation (41) $y^{\prime \prime}=0$, therefore:

$$
\left\{\begin{array}{l}
-x^{\prime \prime}{ }_{f} \cdot \operatorname{tg} \gamma_{x f}^{\prime}+y^{\prime}{ }_{f}\left(\cos \omega^{\prime \prime} \cdot \operatorname{tg} \gamma_{y f}^{\prime}{ }_{y}{ }^{+}\right. \\
\left.\sin \omega^{\prime \prime}\right)+z^{\prime \prime}{ }_{f}\left(\cos \omega^{\prime \prime}-\sin \omega^{\prime \prime} \cdot \operatorname{tg} \gamma^{\prime}{ }_{y f}=0\right. \\
y^{\prime \prime}{ }_{f}=0
\end{array}\right.
$$

having as a result:

$$
\begin{align*}
& -x_{f} \cdot \operatorname{tg} \gamma_{x f}^{\prime}+z_{f f}^{\prime \prime}\left(\cos \omega^{\prime \prime}-\right. \\
& \left.\sin \omega^{\prime \prime \prime} \cdot \operatorname{tg} \gamma_{y f}^{\prime}\right)=0 \tag{42}
\end{align*}
$$

Relation (42) represents the equation of the


Figure 9. Determination of $\operatorname{tg} \gamma{ }^{\prime \prime}{ }_{x f}$
tangent to the face in relation to which the rake angle $\gamma^{\prime \prime}{ }_{x f}$ has been measured (figure 9). The gradient for the straight line of equation (42) is given by the equation:

$$
\begin{equation*}
\operatorname{tg} \gamma_{x f}^{\prime \prime}=\frac{-d z_{f}^{\prime \prime}}{-d x_{f}^{\prime \prime}}=\left[z_{f\left(x_{f}^{\prime \prime}\right)}\right] \tag{43}
\end{equation*}
$$

By making the calculations in relation (43) it is found that:

$$
\begin{equation*}
z_{f}^{\prime \prime}=\frac{\operatorname{tg} \gamma_{x f}^{\prime}}{\cos \omega^{\prime \prime}-\sin \omega^{\prime \prime} \cdot \operatorname{tg} \gamma_{y f}^{\prime}} \cdot x^{\prime \prime}{ }_{f} \tag{44}
\end{equation*}
$$

By calculating the first derivative of function (44), according to relation (43) it results that:

$$
\begin{equation*}
\operatorname{tg} \gamma_{x f}^{\prime \prime}=\frac{\operatorname{tg} \gamma_{x f}^{\prime}}{\cos \omega^{\prime \prime}-\sin \omega^{\prime \prime} \cdot \operatorname{tg} \gamma_{y f}^{\prime}} \tag{45}
\end{equation*}
$$

In relation (45) by making the trigonometric calculation for tgy'yf and taking into account relation (30) it is obtained that:

$$
\begin{equation*}
\operatorname{tg} \gamma_{x f}^{\prime \prime}=\operatorname{tg} \gamma_{x f}^{\prime} \cdot \frac{\cos \gamma_{y f}^{\prime}}{\cos \gamma^{\prime \prime}} \tag{46}
\end{equation*}
$$

In order to determine the angle of clearance $\alpha^{\prime \prime}{ }_{x f}$ it is proceeded similarly as shown above, but to make calculations easier, the lip angle $\beta=0^{\circ}$, which implies:

$$
\begin{align*}
& \gamma^{\prime \prime}{ }_{x f}=90^{\circ}-\alpha^{\prime \prime}{ }_{x f} \\
& \gamma_{x f}^{\prime}=90^{\circ}-\alpha_{x f}^{\prime}  \tag{47}\\
& \gamma_{y f}^{\prime}=90^{\circ}-\alpha_{y f}^{\prime} \\
& \boldsymbol{\operatorname { t g }} \gamma^{\prime \prime}{ }_{x f}=\operatorname{cotg} \alpha^{\prime \prime}{ }_{x f} \\
& \boldsymbol{\operatorname { t g }} \boldsymbol{\gamma}_{x f}^{\prime}=\boldsymbol{\operatorname { c o t g }} \alpha_{x f}^{\prime}  \tag{48}\\
& \boldsymbol{\operatorname { t g }} \gamma_{y y}^{\prime}=\boldsymbol{\operatorname { c o t g }} \alpha_{y f}^{\prime}
\end{align*}
$$

By replacing relation (48) into (45), (46) and taking into consideration relation (29), it is obtained:

$$
\begin{equation*}
\operatorname{cotg} \alpha^{\prime \prime}{ }_{x f}=\frac{\operatorname{cotg} \alpha^{\prime}{ }_{x f}}{\cos \omega^{\prime \prime}-\sin \omega^{\prime \prime} \cdot \operatorname{cotg} \alpha^{\prime}{ }_{y f}} \tag{49}
\end{equation*}
$$

and (50) respectively:

$$
\begin{equation*}
\operatorname{cotg} \alpha^{\prime \prime}{ }_{x f}=\operatorname{cotg} \alpha_{x f}^{\prime} \cdot \frac{\sin \alpha^{\prime} y f}{\sin \alpha^{\prime \prime} y f} \tag{50}
\end{equation*}
$$

In order to determine the angle $\gamma_{y d}$ it is necessary to know the equation of the tangential plane to the face of the tooth in the system $V x_{f}^{\prime} f y^{\prime} z^{\prime} f$. The equation of this surface can be written in points $V, P$ and $Q$, but it is easier to take into account relations (30), (46) which will be replaced into relation (41), and after making the trigonometric calculations it is obtained that:

$$
\begin{equation*}
-x^{\prime \prime}{ }_{f} \cdot \operatorname{tg} \gamma_{x f}^{\prime \prime}{ }_{x f}+y_{f}^{\prime \prime} \cdot \operatorname{tg} \gamma_{y f}^{\prime \prime}+z^{\prime \prime}{ }_{f}=0 \tag{51}
\end{equation*}
$$

The geometric parameters of tooth are defined in system $V x_{d} y_{d} z_{d}$ (figure 5) whose axis $V y_{d}$ is parallel to tooth axis and axis $V z_{d}$ is normal to it, system which has been rotated in relation to the system $V x_{f}{ }_{f y}{ }^{\prime \prime} f_{f} z_{f}$, around the axis $V y^{\prime \prime}{ }_{f}$ with angle $\left(-\theta^{\prime \prime}{ }_{V}\right.$ - figure 5 , section F-F) and figure 10.

The matrix of transition from the system $V x^{\prime \prime}{ }_{f} y_{f}{ }_{f} z_{f}$ to the system $V x_{d} y_{d} z_{d}$ is:


Figure 10. The position of the $V x^{\prime \prime} f_{f}{ }^{\prime \prime} z^{\prime \prime}{ }_{f}$ system regarding the system $V x_{d} y_{d} z_{d}$

$$
\left\|\begin{array}{ccc}
\cos \theta^{\prime \prime} V & 0 & -\sin \theta^{\prime \prime} V  \tag{52}\\
0 & 1 & 0 \\
\sin \theta^{\prime \prime} V & 0 & \cos \theta^{\prime \prime} V
\end{array}\right\|
$$

and thus it is obtained:

$$
\left\{\begin{array}{l}
x_{f}^{\prime \prime}=x_{d} \cdot \cos \theta_{V}^{\prime \prime}+z_{d} \cdot \sin \theta_{V}^{\prime \prime}  \tag{53}\\
y_{f}^{\prime \prime}=y_{d} \\
z_{f}^{\prime \prime}=-x_{d} \cdot \sin \theta_{V}^{\prime \prime}+z_{d} \cdot \cos \theta_{V}^{\prime \prime}
\end{array}\right.
$$

By replacing relation (53) into relation (51) and by making the trigonometric calculations and taking into consideration relation (34), it results relation.

$$
\begin{align*}
& -x_{d} \cdot \frac{\sin \gamma_{x d}}{\cos \gamma^{\prime \prime}}+y_{d f} \cdot \operatorname{tg} \gamma_{y f}^{\prime \prime}+  \tag{54}\\
& z_{d} \cdot \frac{\cos \gamma_{x d}}{\cos \gamma^{\prime \prime}{ }_{x f}}=0
\end{align*}
$$

Relation (54) represents the equation of the tangential plane to the face of the tooth in system $V x_{d} y_{d z d}$. By intersecting this plane with the longitudinal plane $y_{d} V z_{d}$, namely in relation (54) $x_{d}=0$ and accordingly:

$$
\left\{\begin{array}{l}
-x_{d} \cdot \frac{\sin \gamma_{x d}}{\cos \gamma^{\prime \prime}}+y_{d f} \cdot \operatorname{tg} \gamma^{\prime \prime}{ }_{y f}+  \tag{55}\\
z_{d} \cdot \frac{\cos \gamma_{x d}}{\cos \gamma^{\prime \prime}{ }_{x f}}=0 \\
x_{d}=0
\end{array}\right.
$$

having as a result the straight line equation:

$$
\begin{equation*}
y_{d} \cdot \operatorname{tg} \gamma_{y f}^{\prime \prime}+z_{d} \cdot \frac{\cos \gamma_{x d}}{\cos \gamma_{x f}^{\prime \prime}}=0 \tag{56}
\end{equation*}
$$

whose gradient according to figure 11 is:

$$
\begin{equation*}
\operatorname{tg} \gamma_{x d}=\frac{-d z_{d}}{d y_{d}}=-\left[z_{d\left(y_{d}\right)}\right] \tag{57}
\end{equation*}
$$

By making the calculation in relation (56) it results that:

$$
\begin{equation*}
z_{d}=-\frac{\operatorname{tg} \gamma_{y f}^{\prime \prime}}{\cos \gamma_{x d}} \cdot \cos \gamma_{x f}^{\prime \prime} \cdot y_{d} \tag{58}
\end{equation*}
$$



Figure 11. Determination of $\operatorname{tg} \gamma_{y d}$
By calculating the first derivative of the function given by relation (58), according to relation (57), it results that:

$$
\begin{equation*}
\operatorname{tg} \gamma_{y d}=\operatorname{tg} \gamma^{\prime \prime}{ }_{y f} \cdot \frac{\cos \gamma_{x f}^{\prime \prime}}{\cos \gamma_{x d}} \tag{59}
\end{equation*}
$$

In order to determine angle $\alpha_{y d}$ it has been considered that $\beta=0^{\circ}$ which implies:

$$
\begin{gather*}
\gamma_{y d}=90^{\circ}-\alpha_{y d} \\
\gamma^{\prime \prime}, y=90^{\circ}-\alpha^{\prime \prime}{ }_{y f}  \tag{60}\\
\gamma^{\prime \prime} x=90^{\circ}-\alpha^{\prime \prime}{ }_{x f} \\
\gamma_{x d}=90^{\circ}-\alpha_{x d}
\end{gather*}
$$

respectively relation:

$$
\begin{gather*}
\operatorname{tg} \gamma_{y d}=\operatorname{cotg} \alpha_{y d} \\
\operatorname{tg} \gamma^{\prime \prime}{ }_{y f}=\operatorname{cotg} \alpha^{\prime}{ }_{y f}  \tag{61}\\
\cos \gamma^{\prime}{ }_{x f}=\sin \alpha^{\prime \prime}{ }_{x f} \\
\cos \gamma_{x d}=\sin \alpha_{x d}
\end{gather*}
$$

By replacing relation (61) into relation (59) it results relation

$$
\begin{equation*}
\operatorname{tg} \alpha_{y d}=\operatorname{tg} \alpha^{\prime \prime}{ }_{y f} \cdot \frac{\sin \alpha_{x d}}{\sin \alpha^{\prime \prime}{ }_{x f}} \tag{62}
\end{equation*}
$$

Relations (46), (34), (50), (35), (29), (59), (30), (62) enable to determine agles $\gamma_{x d}, \alpha_{x d}, \gamma_{y d}, \gamma_{y d}$ of the transverse plane and the longitudinal one of the tooth, but it is also necessary to determine the inclination angle of the main cutting edge $\lambda_{d}$ and of the main angle of approach $K_{d}$.

In the literature [2] are demonstrated the expressions of the angles of the longitudinal and the transverse planes, which are applicable to the present paper, both for geometric parameters of the milling cutter and for the geometric parameters of tooth,
relations which lead to the expressions of the inclination angle of cutting edge:

$$
\begin{gather*}
\operatorname{tg} \lambda=\operatorname{tg} \gamma_{y} \cdot \sin K-\operatorname{tg} \gamma_{x} \cdot \cos K  \tag{63}\\
\operatorname{tg} \lambda=\operatorname{cotg} \alpha_{y} \cdot \sin K-\operatorname{cotg} \alpha_{x} \cdot \cos K \tag{64}
\end{gather*}
$$

Inasmuch as relation (63) and relation (64) refer to the inclination angle of the main cutting edge $\lambda$, they can be equated after making the calculation and therefore it results equation:

$$
\begin{equation*}
\operatorname{tg} K_{d}=\frac{\operatorname{tg} \gamma_{x d}-\operatorname{cotg} \alpha_{x d}}{\operatorname{tg} \gamma_{y d}-\operatorname{cotg} \alpha_{y d}} \tag{65}
\end{equation*}
$$

Relations (63), (64) and (65) are general and can be applied to edges of any cutting edge of any tool, respectively they can be particularized to determine the geometric parameters of the milling cutter and of its inserts.

If the angles of the milling cutter are known of the longitudinal and transverse planes to the projections of the tooth axis, then the expression of the main angle of approach of milling cutter can be deduced under the expression:

$$
\begin{equation*}
\operatorname{tg}\left(\rho-K_{f}\right)=\frac{\operatorname{tg} \gamma_{y f}^{\prime}-\operatorname{cotg} \alpha^{\prime}{ }_{y f}}{\operatorname{tg} \gamma_{x f}^{\prime}-\operatorname{cotg} \alpha_{x f}^{\prime}} \tag{66}
\end{equation*}
$$

The relation above will be demonstrated in the same way as relation (55), but relations (20), (21), (22) and (23) have been taken into consideration as well.

In papers [3] and [9] it is defined the position of the point of the tooth $V$ through distances $a_{V}, h_{V}, r_{V}$, $l_{C}$ (figure 2) and angle $\varphi_{v}$ [3], [9], between these elements the following relations ensue:

$$
\begin{gather*}
r_{V}=\left(a_{V}^{2}+h_{V}^{2}\right)^{\frac{1}{2}}  \tag{67}\\
\operatorname{tg} \varphi_{V}=\frac{h_{V}}{a_{V}} \tag{68}
\end{gather*}
$$

In the sketch of milling cutter body, the position of tooth axis is defined through angles $\Phi$ and $\omega$, diameters $D_{l}$ and $D_{2}$ respectively (figure 1 ), the connection between them being determined by relations:

$$
\begin{align*}
& \frac{D_{1}}{2}=\frac{D_{f}}{2} \cdot \cos \theta_{V}-a_{V} \cdot \sin \Phi  \tag{69}\\
& D_{2}=D_{2}+2 l_{C} \cdot \cos \Phi \cdot \cos \omega^{\prime} \tag{70}
\end{align*}
$$

In Table 1 the demonstrated relations are shawn syntetically, which enable to make the necessary calculations, by means of the method of succesive equations, with the view of constructing a milling cutter with inserts and the tooth axis space positioned in relation to milling cutter body, starting from the geometric parameters of the milling cutter and those of the teeth.

Table 1.

|  | Considered known: $D_{f} ; K_{f} ; \alpha_{N f} ; \gamma_{N f} ; \lambda_{f} ;$ $\Phi ; \omega ; a_{V} ; h_{V} ; l_{C} ; d_{;} a$ |  |
| :---: | :---: | :---: |
| Crt. <br> nr. | Relation | Rel. nr. |
| 1. | $\operatorname{tg} \omega^{\prime}=\operatorname{tg} \omega \cdot \sin \Phi$ | (5) |
| 2. | $\begin{aligned} & \sin \theta_{V}=2\left(l_{C} \cdot \sin \omega^{\prime} \pm\right. \\ & \left.h_{V} \cdot \cos \omega^{\prime}\right) / D_{f} \end{aligned}$ | (7) |
| 3. | $\operatorname{tg} \varphi_{V}=\frac{h_{V}}{a_{V}}$ | (68) |
| 4. | $r_{V}=\left(a_{V}^{2}+h_{V}^{2}\right)^{\frac{1}{2}}$ | (67) |
| 5. | $\frac{D_{1}}{2}=\frac{D_{f}}{2} \cdot \cos \theta_{V}-a_{V} \cdot \sin \Phi$ | (69) |
| 6. | $\mathrm{D}_{2}=\mathrm{D}_{2}+2 l_{C} \cdot \cos \Phi \cdot \cos \omega^{\prime}$ | (70) |
| 7. | $\operatorname{cotg} \rho=\operatorname{cotg} \Phi \cdot \cos \theta_{V}$ | (15) |
| 8. | $\boldsymbol{t g} \theta^{\prime}{ }_{V}=\operatorname{tg} \theta_{V} \cdot \sin \Phi$ | (19) |
| 9. | $\sin \omega^{\prime \prime}=\frac{\sin \omega^{\prime}}{\cos \theta_{V}^{\prime}}$ | (28) |
| 10. | $\sin \theta^{\prime \prime}{ }_{V}=\sin \theta^{\prime}{ }_{V} \cdot \cos \omega^{\prime}$ | (33) |
| 11. | $A=\rho-K_{f}$ (notation) |  |
| 12. | $\alpha^{\prime \prime}{ }_{y f}=\alpha^{\prime}{ }_{y f}+\omega^{\prime \prime}$ | (29) |
| 13. | $\gamma^{\prime \prime}{ }_{y f}=\gamma^{\prime}{ }_{y f}-\omega^{\prime \prime}$ | (30) |
| 14. | $\operatorname{tg} \gamma_{x f}^{\prime \prime}=\operatorname{tg} \gamma_{x f}^{\prime} \cdot \frac{\cos \gamma^{\prime}{ }_{y f}}{\cos \gamma^{\prime \prime}{ }_{y f}}$ | (46) |
| 15. | $\operatorname{cotg} \alpha^{\prime \prime}{ }_{x f}=\operatorname{cotg}{\alpha^{\prime}}_{x f} \cdot \frac{\sin \alpha^{\prime}{ }_{y f}}{\sin \alpha^{\prime \prime}{ }_{y f}}$ | (50) |
| 16. | $\gamma_{x d}=\gamma^{\prime \prime}{ }_{x f}+\theta^{\prime \prime}{ }_{V}$ | (34) |
| 17. | $\alpha_{x d}=\alpha^{\prime \prime}{ }_{x f}-\theta^{\prime \prime}{ }_{V}$ | (35) |
| 18. | $\operatorname{tg} \gamma_{y d}=\operatorname{tg} \gamma^{\prime \prime}{ }_{y f} \cdot \frac{\cos \gamma^{\prime \prime}{ }_{x f}}{\cos \gamma_{x d}}$ | (59) |


| 19. | $\operatorname{tg} \alpha_{y d}=\operatorname{tg}{\alpha^{\prime \prime} y f} \cdot \frac{\sin \alpha_{x d}}{\sin \alpha^{\prime \prime} x f}$ | (62) |
| :--- | :--- | :--- |
| 20. | $\operatorname{tg} K=\frac{\operatorname{tg} \gamma_{x}-\operatorname{cotg} \alpha_{x}}{\operatorname{tg} \gamma_{y}-\operatorname{cotg} \alpha_{y}}$ | (65) |
| 21. | $z \leq \frac{\pi \cdot D_{f}}{d+a}$ |  |

Observation: $a$ represents the minimal tooth distance
In the case when the geometric parameters of teeth are known ( $K_{d}, \alpha_{N d}, \gamma_{N d}, \lambda_{d}$ ) being given $a_{V}, H_{V}$, $l_{C}, \Phi, \omega, D_{f}$, the previous calculations are made with relations (1)...(10) from table 1, after that the geometric parameters of milling cutter will be made with relations given in Table 2.

The relations of Table 2 are valid for any value of the geometric parameters of teeth, including when the main cutting edge is parallel to the tooth axis, namely $K_{d}=90^{\circ}$ and $K_{d}=0^{\circ}$.

Table 2.

|  | Considered known: $K_{d} ; \alpha_{N d} ; \gamma_{N d} ; \lambda_{d} ; \Phi ; \omega ; a_{V}$ $h_{V} ; l_{C} ; d_{i} D_{f}$ |
| :---: | :---: |
| Crt. <br> nr. | Relation |
| 1. | $\operatorname{cotg} \alpha_{x d}=\operatorname{cotg} \alpha_{N d} \cdot \sin K_{d}-\operatorname{tg} \lambda_{d} \cdot \cos K_{d}$ |
| 2. | $\operatorname{tg} \gamma_{x d}=\operatorname{tg} \gamma_{N d} \cdot \sin K_{d}-\operatorname{tg} \lambda_{d} \cdot \cos K_{d}$ |
| 3. | $\operatorname{cotg} \alpha_{y d}=\operatorname{cotg} \alpha_{N d} \cdot \cos K_{d}+\operatorname{tg} \lambda_{d} \cdot \sin K_{d}$ |
| 4. | $\operatorname{tg} \gamma_{y d}=\operatorname{tg} \gamma_{N d} \cdot \cos K_{d}+\operatorname{tg} \lambda_{d} \cdot \sin K_{d}$ |
| 5. | $\gamma^{\prime \prime}{ }_{x f}=\gamma_{x d}-\theta^{\prime \prime}{ }_{V}$ |
| 6. | $\alpha^{\prime \prime}{ }_{x f}=\alpha_{x d}+\theta^{\prime \prime} V_{V}$ |
| 7. | $\operatorname{tg} \gamma_{y f}^{\prime \prime}=\frac{\operatorname{tg} \gamma_{y d} \cdot \cos \gamma_{x d}}{\cos \gamma^{\prime \prime}}$ |
| 8. | $\operatorname{tg} \alpha^{\prime \prime}{ }_{y f}=\frac{\operatorname{tg} \alpha_{y d} \cdot \sin \alpha^{\prime \prime}{ }_{x f}}{\sin \alpha_{x d}}$ |
| 9. | $\alpha^{\prime}{ }_{y f}=\alpha^{\prime \prime}{ }_{y f}-\omega^{\prime \prime}$ |
| 10. | $\gamma^{\prime}{ }_{y f}=\gamma^{\prime \prime}{ }_{y f}+\omega^{\prime \prime}$ |
| 11. | $\operatorname{tg} \gamma^{\prime}{ }_{x f}=\frac{\operatorname{tg} \gamma^{\prime \prime}{ }_{x f} \cdot \cos \gamma^{\prime \prime}{ }_{y f}}{\cos \gamma^{\prime}{ }_{y f}}$ |
| 12. | $\operatorname{cotg} \alpha_{x f}^{\prime}=\frac{\operatorname{cotg} \alpha^{\prime \prime}{ }_{x f} \cdot \sin \alpha^{\prime \prime}{ }_{y f}}{\sin \alpha^{\prime}{ }_{y f}}$ |
| 13. | $K_{f}=\rho-\operatorname{arctg} \frac{\left(\operatorname{tg} \gamma^{\prime}{ }_{y f}-\operatorname{cotg} \alpha^{\prime}{ }_{y f}\right)}{\left(\operatorname{tg} \gamma^{\prime}{ }_{x f}-\operatorname{cotg} \alpha^{\prime}{ }_{x f}\right)}$ |

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