THE MATHEMATICAL MODEL OF THE ELECTROCHEMICAL GRINDING PROCESS

C. Ungureanu
Technical University "Gh. Asachi" Iasi

INTRODUCTION

The technological procedure of electrochemical grinding belongs to unconventional technologies category. This method is used for machining hard and extremely hard materials. Electrochemical erosion machining is based on the phenomenon of anodic dissolving, which is accompanied by chemical reactions of electrolysis between the work piece and abrasive wheel. The machining process by electrochemical grinding is characterized by a very large number of working parameters, and this paper approaches some correlations between input and output parameters.

1.THE ELECTROCHEMICAL GRINDING PARAMETERS

As compared to conventional grinding process, working parameters are represented by: electric conditions (voltage, intensity, current density), mechanical conditions (abrasive wheel speed, longitudinal feed rate, abrasive wheel speed, longitudinal feed rate, length of oscillation stroke, cross feed, pressing force, dimensions of the contact surface), abrasive wheel electrode (nature of abrasive material, granulation, concentration, nature of binder), electrolyte solution (chemical composition, concentration, specific weight, specific heat, electric conductivity, temperature, flow rate), work piece material (composition, structure, physical and mechanical properties), machine tool, fixing devices.

The output parameters (evaluation criteria for electrochemical grinding performances) can be considered: total amount of removed material, total work time, volume of worn out abrasive wheel driving, consumed power by electrochemical process, roughness of machined surfaces, maximum form deviation, edge roughness.

Accounted to the dates from literature, the author has choose for experimental researches as working parameters to be optimized, some fundamental ones, whose variations have a strong influence on the evaluation criteria for electrochemical grinding performances. These input parameters are:

- C.c. source voltage, U [V];
- Specific contact pressure between abrasive wheel and work piece, p [daN/cm^2];
- Abrasive wheel peripheral speed, v [m/s];
- Longitudinal feed rate, S [cd/min];
- Height of contact surface between abrasive wheel and work piece, h [mm].

These parameters have the advantage that can be easy modelled applying the factorial programmatic experiment.

For the evaluation criteria of process performances, the following output parameters have been adopted

- Current intensity absorbed by the electrolytic cell, I [A];
- Power absorbed by the main motion engine, P [W];
- Machining productivity, Q [cm^3 / min];
- Abrasive wheel relative wear, Ur [cm3 / cm^3];
- Roughness of machined surface, Ra [μm];
- Edge roughness, ρ [μm].

2.THE MATHEMATIC MODEL PROPOSED

For the dependence of the output parameters versus vs. working conditions, based on literature, the equations (1) are proposed.

$$
\begin{aligned}
I &= C_I \cdot e^{a_1 U} \cdot e^{a_2 p} \cdot e^{a_3 v} \cdot e^{a_4 S} \cdot e^{a_5 h} \\
P &= C_P \cdot e^{b_1 U} \cdot e^{b_2 p} \cdot e^{b_3 v} \cdot e^{b_4 S} \cdot e^{b_5 h} \\
Q &= C_Q \cdot e^{c_1 U} \cdot e^{c_2 p} \cdot e^{c_3 v} \cdot e^{c_4 S} \cdot e^{c_5 h} \\
Ur &= C_{Ur} \cdot e^{d_1 U} \cdot e^{d_2 p} \cdot e^{d_3 v} \cdot e^{d_4 S} \cdot e^{d_4 h} \\
Ra &= C_{Ra} \cdot e^{e_1 U} \cdot e^{e_2 p} \cdot e^{e_3 v} \cdot e^{e_4 S} \cdot e^{e_5 h} \\
\rho &= C_{\rho} \cdot e^{f_1 U} \cdot e^{f_2 p} \cdot e^{f_3 v} \cdot e^{f_4 S} \cdot e^{f_5 h}
\end{aligned}
\tag{1}
$$

These equations are adequate for metal cutting phenomena model. In these proposed equations the constants will be determined through a precise method, with a work volume and research materials as small as possible.

3.EXPERIMENTAL DESIGN

The multifactor active experiment is widely used for determining the proposed equations coefficients, based on entry values modifications from a programming plan. There are some different experimental planning systems, each of them with advantages and disadvantages. For present research a compositive multifactor experiment, centred and routable, at five levels has been adopted. The total number of experiments N is presented by relation (2), in which r represents the number of entry parameters (r = 5) and n_0 – central experiments number (n_0 = 6).

$$N = 2^{r-1} + 2r + n_0 \qquad (2)$$

So, only 32 experiments are needed. By translation of experimental region origin in its centre, rate setting is making, and the calculus is simplified. In those conditions, independent centred and rate-setting variables takes -2, -1, 0, 1, 2, values. For studied process, the variables values, normally and rate setting centred, are shown in Table 1.

Table 1. Variables values

Independent Variables					Variables rate setting centred
U [V]	P [daN / cm^2]	v [m/s]	S [cd/min]	h [mm]	
2	2	10	4	2	-2
5	4	18	8	4	-1
8	6	26	12	6	0
11	8	34	16	8	1
14	10	42	20	10	2
3	2	8	4	2	**Variation unit**

Natural to codified values passing was made with relation (3), in which x_c represent codified variable, x_{nm} – natural medium value of independent variable, Δp – independent variable variation unit.

$$x_c = \frac{x - x_{nm}}{\Delta p} \qquad (3)$$

Applying logarithms to relations (1) and adopting notations (4), the functions (5) are obtained.

$$\begin{aligned} &Y_1 = \ln I \,;\; Y_2 = \ln P \,;\; Y_3 = \ln Q \,; \\ &Y_4 = \ln Ur \,;\; Y_5 = \ln Ra \,;\; Y_6 = \ln \rho ; \\ &x_1 = U \,;\; x_2 = p \,;\; x_3 = v \,;\; x_4 = S \,; \\ &x_5 = h \,;\; a_0 = \ln C_I \,;\; b_0 = \ln C_P \,; \\ &c_0 = \ln C_Q \,;\; d_0 = \ln C_{Ur} \,; \\ &e_0 = \ln C_{Ra} \,;\; f_0 = \ln C_\rho \,; \end{aligned} \qquad (4)$$

$$\begin{aligned} Y_1 &= a_0 + \sum_{i=1}^{5} a_i x_i \\ Y_2 &= b_0 + \sum_{i=1}^{5} b_i x_i \\ Y_3 &= c_0 + \sum_{i=1}^{5} c_i x_i \\ Y_4 &= d_0 + \sum_{i=1}^{5} d_i x_i \\ Y_5 &= e_0 + \sum_{i=1}^{5} e_i x_i \\ Y_6 &= f_0 + \sum_{i=1}^{5} f_i x_i \end{aligned} \qquad (5)$$

4. EXPERIMENTAL RESULTS PROCESSING

For experimental results determination, the next steps were followed:

- Coefficients of equations (5) were determined with relations (6), in which Y_n represents function values and x_i, x_j – independent variables codified values;

$$a_0 = 0{,}1590909 \cdot \sum_{n=1}^{32} Y_n - 0{,}0340909 \cdot \sum_{n=1}^{32} x_{ic}^2 Y_n ; i = \overline{1{,}5};$$

$$a_i = 0{,}0416666 \cdot \sum_{n=1}^{32} x_{ic} Y_n ; i = \overline{1{,}5};$$

$$a_{ij} = 0{,}0625 \cdot \sum_{n=1}^{32} x_{ic} x_{jc} Y_n ; i = \overline{1{,}5}; j = \overline{1{,}5}; i \prec j; \tag{6}$$

$$a_{ii} = 0{,}03125 \sum_{n=1}^{32} x_{ic}^2 Y_n + 0{,}002841 \sum_{\substack{n=1\\i=1}}^{\substack{n=32\\i=5}} x_{ic}^2 Y_n - 0{,}0340909 \sum_{n=1}^{32} Y_n$$

- Coefficients significations were tested with Student criterion and those that satisfied condition (7) were eliminated (t_{crt} = 2,015 for 5 liberty degrees, 32 experiments and 0,05 confidence level);

$$t \prec t_{crt} \tag{7}$$

- Equations coefficients were determined in natural coordinates.

So, the objective functions equations for Rp3 material are presented by relations (8) and for 205Cr115 by relations (9).

$$\begin{aligned}
&\ln I = 0{,}924426 + 0{,}252222 \cdot U + 0{,}143267 \cdot p - 0{,}109158 \cdot v - 0{,}012192 \cdot S - 0{,}009770 \cdot h - \\
&- 0{,}002979 \cdot U \cdot p - 0{,}01118 \cdot p \cdot v - 0{,}000426 \cdot p \cdot S - 0{,}008926 \cdot U^2 + 0{,}001270 \cdot v^2 + 0{,}007344 \cdot h^2 \\
&\ln P = 4{,}253021 + 0{,}007100 \cdot U + 0{,}098771 \cdot p + 0{,}056414 \cdot v + 0{,}025237 \cdot S + 0{,}061934 \cdot h - \\
&- 0{,}000606 \cdot p \cdot v - 0{,}000204 \cdot v \cdot S - 0{,}000871 \cdot U^2 - 0{,}004808 \cdot p^2 - 0{,}000454 \cdot v^2 - 0{,}000544 \cdot S^2 - \\
&- 0{,}000354 \cdot h^2 \\
&\ln Q = -6{,}061301 + 0{,}138979 \cdot U + 0{,}076532 \cdot p - 0{,}008292 \cdot v - 0{,}036301 \cdot S - 0{,}083026 \cdot h - \\
&- 0{,}013325 \cdot U \cdot p + 0{,}001466 \cdot U \cdot v - 0{,}009793 \cdot U \cdot h - 0{,}000718 \cdot p \cdot v + 0{,}016056 \cdot U^2 + \\
&+ 0{,}014945 \cdot p^2 + 0{,}000256 \cdot v^2 + 0{,}000932 \cdot S^2 + 0{,}008365 \cdot h^2 \\
&\ln Ur = -5{,}360768 - 0{,}229401 \cdot U + 0{,}130545 \cdot p - 0{,}063784 \cdot v + 0{,}024486 \cdot S + 0{,}057981 \cdot h + \\
&+ 0{,}010891 \cdot U^2 + 0{,}000924 \cdot v^2
\end{aligned} \tag{8}$$

$$\begin{aligned}
&\ln Ra = -1{,}896647 + 0{,}089858 \cdot U - 0{,}005574 \cdot p + 0{,}003655 \cdot v + 0{,}015405 \cdot S + 0{,}157257 \cdot h + \\
&+ 0{,}007249 \cdot U \cdot p + 0{,}002479 \cdot U \cdot v - 0{,}002878 \cdot p \cdot v + 0{,}006875 \cdot p \cdot S + 0{,}009244 \cdot p \cdot h - \\
&- 0{,}004690 \cdot S \cdot h - 0{,}006611 \cdot U^2 - 0{,}016330 \cdot p^2 - 0{,}000491 \cdot v^2 - 0{,}007924 \cdot h^2 \\
&\ln \rho = 4{,}113347 + 0{,}108715 \cdot U - 0{,}230374 \cdot p - 0{,}004290 \cdot v + 0{,}017920 \cdot S - 0{,}026339 \cdot h + \\
&+ 0{,}016535 \cdot U \cdot p - 0{,}006748 \cdot U \cdot S - 0{,}002149 \cdot p \cdot v + 0{,}004774 \cdot p \cdot S + 0{,}001579 \cdot S^2
\end{aligned}$$

$$\begin{aligned}
\ln I &= 0{,}582756 + 0{,}291447\cdot U + 0{,}165095\cdot p - 0{,}105818\cdot v - 0{,}001022\cdot S - 0{,}014388\cdot h - \\
&- 0{,}004387\cdot U\cdot p - 0{,}002033\cdot p\cdot v - 0{,}002708\cdot p\cdot S - 0{,}010514\cdot U^2 + 0{,}001034\cdot v^2 + 0{,}007328\cdot h^2 \\
\ln P &= 3{,}695826 - 0{,}000694\cdot U + 0{,}121619\cdot p + 0{,}065434\cdot v + 0{,}044525\cdot S + 0{,}084555\cdot h - \\
&- 0{,}000478\cdot p\cdot v - 0{,}000406\cdot v\cdot S - 0{,}000461\cdot U^2 - 0{,}000619\cdot p^2 - 0{,}000488\cdot v^2 - 0{,}000935\cdot S^2 - \\
&- 0{,}004304\cdot h^2 \\
\ln Q &= -6{,}680099 + 0{,}141560\cdot U + 0{,}200154\cdot p + 0{,}007148\cdot v - 0{,}021314\cdot S - 0{,}042259\cdot h - \\
&- 0{,}012669\cdot U\cdot p + 0{,}000784\cdot U\cdot v - 0{,}002279\cdot U\cdot h - 0{,}001891\cdot p\cdot v + 0{,}013720\cdot U^2 + \\
&+ 0{,}008302\cdot p^2 + 0{,}000146\cdot v^2 + 0{,}000586\cdot S^2 + 0{,}003098\cdot h^2 \\
\ln Ur &= -5{,}464419 - 0{,}233025\cdot U + 0{,}118586\cdot p - 0{,}064410\cdot v + 0{,}028436\cdot S + 0{,}040704\cdot h + \\
&+ 0{,}010596\cdot U^2 + 0{,}000969\cdot v^2 \\
\ln Ra &= -1{,}670263 + 0{,}119263\cdot U - 0{,}052709\cdot p - 0{,}004558\cdot v + 0{,}024075\cdot S + 0{,}141924\cdot h + \\
&+ 0{,}003587\cdot U\cdot p + 0{,}002262\cdot U\cdot v - 0{,}001074\cdot p\cdot v + 0{,}007601\cdot p\cdot S + 0{,}009504\cdot p\cdot h - \\
&- 0{,}006126\cdot S\cdot h - 0{,}007189\cdot U^2 - 0{,}014159\cdot p^2 - 0{,}000536\cdot v^2 - 0{,}006175\cdot h^2 \\
\ln \rho &= 4{,}688960 + 0{,}086827\cdot U - 0{,}214311\cdot p - 0{,}010145\cdot v - 0{,}055412\cdot S + 0{,}020943\cdot h + \\
&+ 0{,}008756\cdot U\cdot p - 0{,}001457\cdot U\cdot S - 0{,}000902\cdot p\cdot v + 0{,}006552\cdot p\cdot S + 0{,}002515\cdot S^2
\end{aligned} \qquad (9)$$

Finally, equations adequate were verified with Fisher test (relation 10), in which S_{rem} represents reminisced dispersion and S – objective function standard deviation.

$$F = \frac{S_{rem}^2}{S^2} \qquad (10)$$

Fisher criteria values are shown in Table 2.

Table 2. Fisher criteria values

Objective function	Fisher criteria	
	Rp3	**205Cr115**
I	2,857	3,014
P	4,486	2,420
Q	3,593	3,170
Ur	2,378	2,471
Ra	2,391	2,192
ρ	2,480	2,220

For each function condition (11) is materialised, where F_c = 4,95, for 95% confidence level.

$$F \prec F_C \qquad (11)$$

That means equations (8) may be considered adequate.

References

1. **Cochran W.G., Cox G.M**. *Experimental Design, New York, London, Sydney, 1968.*
2. **Constantinescu I., Golumbovici D., Militaru C.** *Prelucrarea datelor experimentale cu calculatoare numerice, Ed. Tehnică, Buc.,1980.*
3. **Raul M.** *Introducere în strategia experimentării cu aplicaţii în tehnologia chimică, Ed. Şt. Şi Enciclop., Buc., 1976.*

Recomandat spre publicare: 20.04.04