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PERFORMANCE REQUIRED FOR COMMON-USE COMPONENTS OF COMPUTER NETWORKS

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Abstract. Sometimes, in practice, simple solutions of quick preliminary estimation of basic characteristics of a computer network are needed. In this aim, the backbone subnet and server set of wide area computer networks are examined. Based on Jackson's partitioning theorem and considering the linear dependence of the costs of channels, routers, and servers on their performance, a simplified analytical model for these components of the network is defined. Using this model, two optimization problems are formulated: minimizing the average response time to user requests of data processing and minimizing the summary cost of servers, channels and routers of the computer network. For both problems, analytical solutions regarding the necessary performances of channels, routers and servers are obtained. As expected, in the obtained analytical solutions, the equations for the optimization criteria of the two problems coincide, only their form being different. Calculations of performances according to these solutions are simple and can be done, for example, in MS Excel. Because the obtained in this way performances are positive real numbers, and the allowed performances of concerned entities are discrete ones, further adjustment of the solution in question, depending of the case, may be necessary. For such an adjustment, two algorithms are proposed. One of them solves the problem by reducing it to that of backpack. Another solves the problem based on the use of resource concentration rule.

Keywords: *algorithm, channel, cost, optimization, response time, router, server.*

Rezumat. Uneori, în practică, sunt necesare soluții simple de estimare rapidă preliminară a caracteristicilor de bază ale unei rețele de calculatoare. În acest scop, se examinează subrețeaua și setul de servere ale rețelelor de calculatoare de arie largă. Pe baza teoremei de partiționare a lui Jackson și luând în considerare dependența liniară a costurilor canalelor, ruterelor și serverelor de performanța acestora, este definit un model analitic simplificat al acestor componente ale rețelei. Folosind acest model, se formulează două probleme de optimizare: minimizarea duratei medii de răspuns la solicitările utilizatorilor de prelucrare a datelor și minimizarea costului sumar al serverelor, canalelor și ruterelor rețelei de calculatoare. Pentru ambele probleme sunt obținute soluții analitice privind performantele necesare ale canalelor, ruterelor si serverelor. Cum era de așteptat, în soluțiile analitice obținute, ecuațiile pentru criteriile de optimizare ale celor două probleme coincid, fiind diferită doar forma lor. Calculele performanțelor conform acestor soluții sunt simple și se pot face, de exemplu, în MS Excel. Deoarece performanțele astfel obținute sunt numere reale pozitive, iar performanțele admise ale entităților rețelei sunt discrete, poate fi necesară, în funcție de caz, o ajustare ulterioară a soluției în cauză. Pentru o astfel de ajustare, sunt propuși doi algoritmi. Unul dintre aceștia rezolvă problema reducând-o la cea a rucsacului. Celălalt rezolvă problema pe baza aplicării regulii de concentrare a resurselor.

Cuvinte cheie: *algoritm, canal, cost, durată de răspuns, optimizare, ruter, server*.

1. Introduction

The creation and development of computer networks involves considerable expenses. Minimizing these expenses is important. For this purpose many researches are carried out.

In a general form, the problem of computer networks synthesis consists in determining of their topology, of the number, deployment, productivity, operating regimes and functions of servers and communication nodes, the capacity of data transfer channels, the distribution of user requests between servers and the ways of data transfer and routing in the network, which would ensure the extreme of the accepted optimization criterion for the required quality of serving requests.

For large networks, the mathematical formalization and overall solution of the problem is very difficult. That is why various simplifications are usually resorted to, including the disaggregation of the general problem into several sub-problems.

In the field, models and algorithms are developed, several analytical solutions are obtained. It is especially the case to mention the Jackson partitioning theorem [1], the conservation law and the square root law of L. Kleinrock [2], the Gordon-Newell theorem [3], the algorithm of J. Buzen [4], the BCMP networks [5], the MVA method proposed by H. Reiser and S. Lavenberg and later developed by J. McKenna [6], the MVAC algorithm proposed by A. Conway, De Souza E Silva and S.S. Lavenberg [7], and the DAC algorithm developed by De Souza E Silva and S.S. Lavenberg [8]. Cloud processing has involved separate research with data centers [9, 10]. For example, the reliability of data centers is discussed in [11], and the placement of virtual machines for cloud data centers is explored in [12]. Also, researches in soft defined networks (SDN) is gaining momentum [13], including the choosing of the SDN operating system for cloud data centers [14].

However, the proposed methods and algorithms are, as a rule, relatively laborious, and sometimes in practice simple solutions are useful for quick preliminary estimation of basic characteristics of a network. The purpose of this paper is to propose simple ways of preliminary determination of approximate necessary performances of the common-use components of computer networks.

2. A Simplified Model of Computer Networks

From the multitude of computer network synthesis aspects, the model includes those related to estimating the necessary capacity of their servers, data transfer channels (hereafter channels) and routers. The model is an extension of the one proposed by L. Kleinrock for channels [2].

The process of computer network operation has a stochastic character. That is why their research uses, as a rule, the stochastic approach with the application of methods and results of the theory of queues. For this purpose, computer networks are examined as networks of queues. Notes:

- rate β of the summary flow of requests to servers;
- rate γ of the summary flow of packets generated by user requests and the responses to them;
- the mean number g of packets per request, $g = \gamma/\beta$;
- the number of channels - k_c, routers - k_r and servers - k_s;
- packet rate through channel i - λ_{ci} and router j - λ_{rj};
- rate λ_{sl} of requests to server l;
- average size V_i of a packet transmitted through channel i;
- the average retention of a packet at channel i - T_{ci} and at router j - T_{rj};
- the average retention of a request at the server l - T_{sl} and in the network - T;
- summary cost C of channels, routers and servers.

Also, linear dependences are considered:

- of the channel i cost C_{ci} by its data transfer speed d_i [2]

$$C_{ci} = c_{ci}d_i,\ i = \overline{1, k_c}, \quad (1)$$

- of the ruter j cost C_{rj} by its packet processing rate μ_{rj}

$$C_{rj} = c_{rj}\mu_{rj},\ j = \overline{1, k_r}, \quad (2)$$

- of the server l cost C_{sl} by its request processing rate μ_{sl}

$$C_{sl} = c_{sl}\mu_{sl},\ l = \overline{1, k_s}, \quad (3)$$

where c_{ci}, c_{rj} and c_{sl} are coefficients of proportionality.

Moreover, it is considered that flows λ_{ci} ($i = \overline{1, k_c}$), λ_{rj} ($j = \overline{1, k_r}$) and λ_{sl} ($l = \overline{1, k_s}$) are elementary, and the times of packet transmission by channels, of packet processing by routers, and of request processing by servers are exponentially distributed.

It will be examined the steady-state operation of the network, which exists if the relationships occur:

$$\mu_{ci} > \lambda_{ci},\ i = \overline{1, k_c};\ \mu_{rj} > \lambda_{rj},\ j = \overline{1, k_r};\ \mu_{sl} > \lambda_{sl},\ l = \overline{1, k_s}, \quad (4)$$

where $\mu_{ci} = d_i/V_i$ is the packet transmission rate by channel i, $i = \overline{1, k_c}$.

Under the conditions and assumptions defined above, a computer network can be examined as an open exponential network of queues with a homogeneous flow of requests operating in steady state.

According to the Jackson partition theorem [1], any open exponential network of queues can be investigated as a set of isolated exponential queuing systems. Then the average time T_i of a request in the queuing system i of the network is determined [15] according to the relation $T_i = 1/(\mu_i - \lambda_i)$, where λ_i is the request flow rate, and μ_i – the request serving rate by station i. So, the relations take place:

$$T_{ci} = \frac{V_i}{d_i - \lambda_{ci}V_i}, i = \overline{1, k_c}; \quad (5)$$

$$T_{ri} = \frac{1}{\mu_{ri} - \lambda_{ri}}, i = \overline{1, k_r}; \quad (6)$$

$$T_{si} = \frac{1}{\mu_{si} - \lambda_{si}}, i = \overline{1, k_s}. \quad (7)$$

Taking into account Eq. (1)-(3) and Eq. (5)-(7), is obtained:

$$T = \frac{1}{\gamma}\left(\sum_{i=1}^{k_c}\lambda_{ci}T_{ci} + \sum_{i=1}^{k_r}\lambda_{ri}T_{ri}\right) + \frac{1}{\beta}\sum_{i=1}^{k_s}\lambda_{si}T_{si} = \frac{1}{\gamma}\left(\sum_{i=1}^{k_c}\frac{\lambda_{ci}V_i}{d_i - \lambda_{ci}V_i} + \sum_{i=1}^{k_r}\frac{\lambda_{ri}}{\mu_{ri} - \lambda_{ri}} + g\sum_{i=1}^{k_s}\frac{\lambda_{si}}{\mu_{si} - \lambda_{si}}\right); \quad (8)$$

$$C = C_c + C_r + C_s = \sum_{i=1}^{k_c} c_{ci}d_i + \sum_{i=1}^{k_r} c_{ri}\mu_{ri} + \sum_{i=1}^{k_s} c_{si}\mu_{si}. \quad (9)$$

The queuing network model is determined by Eq. (8) and Eq. (9) and conditions of InEq. (4). With this model, two problems of determining the performance required for common-use components of computer networks are usually of interest:

- minimizing the average time T in the network of the requests of data processing;
- minimizing the summary cost C of servers, channels and routers of the computer network.

These are explored in Sections 3 and 4, respectively.

3. Minimizing the Average Response Time to User Requests

Problem 1. Let's, for the model {Eq. (8), Eq. (9)} and conditions of InEq. (4), the parameters are known: C^*; β; γ; k_c; k_r; k_s; λ_{ci}, V_i, c_{ci}, $i = \overline{1, k_c}$; λ_{ri}, c_{ri}, $i = \overline{1, k_r}$; λ_{si}, c_{si}, $i = \overline{1, k_s}$, where C^* is the maximum allowed total cost of servers, channels and routers of the network. It is required to determine the necessary performances d_i, $i = \overline{1, k_c}$ of channels, those μ_{si}, $i = \overline{1, k_s}$ of routers and those μ_{si}, $i = \overline{1, k_s}$ of network servers that would minimize the average response time T to data processing requests at the total cost C of servers, channels and routers which would not exceed the given value C^*, i.e.:

$$T = \frac{1}{\gamma}\left(\sum_{i=1}^{k_c}\frac{\lambda_{ci}V_i}{d_i - \lambda_{ci}V_i} + \sum_{i=1}^{k_r}\frac{\lambda_{ri}}{\mu_{ri} - \lambda_{ri}} + g\sum_{i=1}^{k_s}\frac{\lambda_{si}}{\mu_{si} - \lambda_{si}}\right) \to \min \quad (10)$$

upon compliance with the restriction

$$C = \sum_{i=1}^{k_c} c_{ci}d_{ci} + \sum_{i=1}^{k_r} c_{ri}\mu_{ri} + \sum_{i=1}^{k_s} c_{si}\mu_{si} \le C^* \quad (11)$$

in conditions of InEq. (4).

Solving. From the essence of the problem, it is obvious that the solution of the problem corresponds to the limit value in Eq. (11), that is, $C = C^*$. Then the problem {Eq. (10), Eq. (11)} can be solved using the method of Lagrange multipliers. The respective Lagrangian L is

$$L = \frac{1}{\gamma}\left(\sum_{i=1}^{k_c}\frac{\lambda_{ci}V_i}{d_i - \lambda_{ci}V_i} + \sum_{i=1}^{k_r}\frac{\lambda_{ri}}{\mu_{ri} - \lambda_{ri}} + g\sum_{i=1}^{k_s}\frac{\lambda_{si}}{\mu_{si} - \lambda_{si}}\right) + \chi\left(\sum_{i=1}^{k_c} c_{ci}d_i + \sum_{i=1}^{k_r} c_{ri}\mu_{ri} + \sum_{i=1}^{k_s} c_{si}\mu_{si} - C^*\right), \quad (12)$$

the conditioned optimization problem {Eq. (10), Eq. (11)} reducing to an unconditioned optimization problem – minimizing L. Here χ is the Lagrange multiplier.

The Lagrangian L contains $k_c + k_r + k_s + 1$ unknowns: χ; d_i, $i = \overline{1, k_c}$, μ_{ri}, $i = \overline{1, k_r}$ and μ_{si}, $i = \overline{1, k_s}$. To minimize L, the partial derivatives of L with respect to the unknowns are determined; they are equalized to 0, resulting a system of $k_c + k_r + k_s + 1$ equations. By solving this system, the analytical expressions for the performances d_i, $i=\overline{1,k_c}$, μ_{ri}, $i=\overline{1,k_r}$ and μ_{si}, $i=\overline{1,k_s}$ can be obtained. One has:

$$\begin{cases} \dfrac{\partial L}{\partial d_{ci}} = \dfrac{1}{\gamma}\cdot\dfrac{-\lambda_{ci}V_i}{\left(d_i-\lambda_{ci}V_i\right)^2} + \chi c_{ci} = 0,\ i=\overline{1,k_c} \\ \dfrac{\partial L}{\partial \mu_{ri}} = \dfrac{1}{\gamma}\cdot\dfrac{-\lambda_{ri}}{\left(\mu_{ri}-\lambda_{ri}\right)^2} + \chi c_{ri} = 0,\ i=\overline{1,k_r} \\ \dfrac{\partial L}{\partial \mu_{si}} = \dfrac{g}{\gamma}\cdot\dfrac{-\lambda_{si}}{\left(\mu_{si}-\lambda_{si}\right)^2} + \chi c_{ri} = 0,\ i=\overline{1,k_r} \\ \dfrac{\partial L}{\partial \chi} = \sum_{i=1}^{k_c} c_{ci}d_i + \sum_{i=1}^{k_r} c_{ri}\mu_{ri} + \sum_{i=1}^{k_s} c_{si}\mu_{si} - C^* = 0. \end{cases} \tag{13}$$

From the equations of the first line of Eq. (13), are obtained $\left(d_i-\lambda_{ci}V_i\right)^2 = \dfrac{\lambda_{ci}V_i}{\gamma\chi c_{ci}},\ i=\overline{1,k_c},$ from where

$$d_i = \lambda_{ci}V_i \pm \sqrt{\frac{\lambda_{ci}V_i}{\gamma\chi c_{ci}}},\ i=\overline{1,k_c}. \tag{14}$$

For steady-state operation (see InEq. (4)), only the "+" sign should be considered in Eq. (14). Thus,

$$d_i = \lambda_{ci}V_i + \sqrt{\frac{\lambda_{ci}V_i}{\gamma\chi c_{ci}}},\ i=\overline{1,k_c}. \tag{15}$$

Similarly, from the equations of the second line of Eq. (13), taking into account InEq. (4), it is obtained

$$\mu_{ri} = \lambda_{ri} + \sqrt{\frac{\lambda_{ri}}{\gamma\chi c_{ri}}},\ i=\overline{1,k_r}. \tag{16}$$

Respectively, from the equations of the third line of Eq. (13), taking into account the conditions of InEq. (4), one has

$$\mu_{si} = \lambda_{si} + \sqrt{\frac{\lambda_{si}}{\gamma\chi c_{si}}},\ i=\overline{1,k_s}. \tag{17}$$

Substituting d_i ($i=\overline{1,k_c}$), μ_{ri} ($i=\overline{1,k_r}$) and μ_{si} ($i=\overline{1,k_s}$) from the last line of Eq. (13) with expressions of Eq. (15), Eq. (16) and Eq. (17), respectively, it is obtained

$$\frac{1}{\sqrt{\gamma\chi}} = \frac{C^* - \sum_{i=1}^{k_c} c_{ci}\lambda_{ci}V_i - \sum_{i=1}^{k_r} c_{ri}\lambda_{ri} - \sum_{i=1}^{k_s} c_{si}\lambda_{si}}{\sum_{i=1}^{k_c}\sqrt{c_{ci}\lambda_{ci}V_i} + \sum_{i=1}^{k_r}\sqrt{c_{ri}\lambda_{ri}} + \sum_{i=1}^{k_s}\sqrt{gc_{si}\lambda_{si}}}. \tag{18}$$

Substituting in Eq. (14)-(16) $1/\sqrt{\chi}$ by the expression of Eq. (16), are obtained:

$$d_{ci} = \lambda_{ci}V_i + H\sqrt{\lambda_{ci}V_i / c_{ci}}, \quad i = \overline{1, k_c}, \tag{19}$$

$$\mu_{ri} = \lambda_{ri} + H\sqrt{\lambda_{ri} / c_{ri}}, \; i = \overline{1, k_r}, \tag{20}$$

$$\mu_{si} = \lambda_{si} + H\sqrt{g\lambda_{si} / c_{si}}, \; i = \overline{1, k_s}, \tag{21}$$

where

$$H = \frac{C^* - \sum_{i=1}^{k_c} c_{ci}\lambda_{ci}V_i - \sum_{i=1}^{k_r} c_{ri}\lambda_{ri} - \sum_{i=1}^{k_s} c_{si}\lambda_{si}}{\sum_{i=1}^{k_c}\sqrt{c_{ci}\lambda_{ci}V_i} + \sum_{i=1}^{k_r}\sqrt{c_{ri}\lambda_{ri}} + \sum_{i=1}^{k_s}\sqrt{gc_{si}\lambda_{si}}}.$$

Substituting expressions of Eq. (19), Eq. (20) and Eq. (21) for d_{ci} ($i = \overline{1, k_c}$), μ_{ri} ($i = \overline{1, k_r}$) and μ_{si} ($i = \overline{1, k_s}$) in Eq. (10), the expression for the optimal value T^* of T is obtained

$$T^* = \frac{\left(\sum_{i=1}^{k_c}\sqrt{c_{ci}\lambda_{ci}V_i} + \sum_{i=1}^{k_r}\sqrt{c_{ri}\lambda_{ri}} + \sum_{i=1}^{k_s}\sqrt{gc_{si}\lambda_{si}}\right)^2}{\gamma\left(C^* - \sum_{i=1}^{k_c} c_{ci}\lambda_{ci}V_i - \sum_{i=1}^{k_r} c_{ri}\lambda_{ri} - \sum_{i=1}^{k_s} c_{si}\lambda_{si}\right)}. \tag{22}$$

The analytical solution of the problem is constituted by Eq. (19)-(22).■

It should be noted that the value of the time T can also be determined according to Eq. (10), that is, the relations {Eq. (10), Eq. (19)-(21)} can also be used as a solution. The expected values of quantities T^*, d_i ($i = \overline{1, k_c}$), μ_{ri} ($i = \overline{1, k_r}$) and μ_{si} ($i = \overline{1, k_s}$) can be calculated relatively easily, including in MS Excel.

Remark 1. Of course, the values of quantities d_i ($i = \overline{1, k_c}$), μ_{ri} ($i = \overline{1, k_r}$) and μ_{si} ($i = \overline{1, k_s}$), obtained according to Eq. (19)-(21), are positive real numbers, and the allowed performances of channels, routers and servers that could be used in the network are discrete ones, belonging to a finite set. Therefore, further adjustment of the solution in question, depending of the case, may be necessary. ■

It is useful to present the Eq. (22) in the form

$$T^* = \frac{A}{\gamma\left(C^* - B\right)}, \tag{23}$$

where

$$A = \left(\sum_{i=1}^{k_c}\sqrt{c_{ci}\lambda_{ci}V_i} + \sum_{i=1}^{k_r}\sqrt{c_{ri}\lambda_{ri}} + \sum_{i=1}^{k_s}\sqrt{gc_{si}\lambda_{si}}\right)^2 \tag{24}$$

and

$$B = \sum_{i=1}^{k_c} c_{ci}\lambda_{ci}V_i + \sum_{i=1}^{k_r} c_{ri}\lambda_{ri} + \sum_{i=1}^{k_s} c_{si}\lambda_{si}. \tag{25}$$

From Eq. (24) and Eq. (25), it can be seen that if conditions of InEq. (4) are not taken into account, then the expressions for determining the values of the quantities *A* and *B* do not depend on the capacities of channels, routers and servers. Also, the value of the average response time T^* to data processing requests is inversely proportional to the maximum allowed summary cost C^* of channels, routers and servers. So, the smaller the value of the difference $\Delta C = C^* - C^{**}$ will be, the smaller the value of the difference $\Delta T = T^{**} - T^*$ will be, where C^{**} and T^{**} are the values of quantities *C* and *T* at allowed values of performances of channels, routers and servers of the network. From here, the essence of the problem of the necessary adjustment of the solution of Problem 1 mentioned in Remark 1 it follows.

Problem 2 – adjusting the solution of Problem 1. Let the initial data of Problem 1 and also the increasing strings of performances of common-use components that can be used in the network are known: D_{ci}, $i = \overline{1, n_c}$ of the given data transfer speeds of channels; D_{ri}, $i = \overline{1, n_r}$ of packet processing rates by routers and D_{si}, $i = \overline{1, n_s}$ of request processing rates by servers. It is required to adjust the solution of problem {Eq. (10), Eq. (11)} obtained according to Eq. (19)-(21) so that, at allowed values of performances of common-use entities (channels, routers and servers) of the network and respecting the conditions of InEq. (4), to minimize the value $\Delta C = C^* - C^{**}$.

Solving the problem by reducing it to that of backpack. Obviously, the adjusted solution (performances allowed for common-use components) should be as close as possible to the original one. Therefore, it is unlikely to be appropriate to increase the performance of a component by more than the next value in the range of allowed performances.

This aspect is the basis of **Algorithm 1** for the heuristic solution of Problem 2:

1. Initial data: C^*, T^*; D_{ci}, $i = \overline{1, n_c}$; D_{ri}, $i = \overline{1, n_r}$; D_{si}, $i = \overline{1, n_s}$; c_{ci}, λ_{ci}, V_i, d_i, $i = \overline{1, k_c}$; c_{ri}, λ_{ri}, μ_{ri}, $i = \overline{1, k_r}$;, c_{si}, λ_{si}, μ_{si}, $i = \overline{1, k_s}$.
2. $h := \max\{D_{ci}, i = \overline{1, n_c};\ D_{ri}, i = \overline{1, n_r};\ D_{si}, i = \overline{1, n_s}\} + 1$.
3. $D_{c,n_c+1} := h$; $D_{r,n_r+1} := h$; $D_{s,n_s+1} := h$.
4. Taking into account conditions of InEq. (4), are determined:

$$u_{ci} := z | \min_{z=\overline{1,n_{c+1}}} \{D_{cz} \geq \lambda_{ci}\}, i = \overline{1, k_c};$$

$$u_{ri} := z | \min_{z=\overline{1,n_r+1}} \{D_{rz} \geq \lambda_{ri}\}, i = \overline{1, k_r};$$

$$u_{si} := z | \min_{z=\overline{1,n_{s+1}}} \{D_{sz} \geq \lambda_{si}\}, i = \overline{1, k_s}.$$

5. $u_c := \max_{i=\overline{1,k_c}} \{u_{ci}\}, i = \overline{1, k_c}$; $u_r := \max_{i=\overline{1,k_r}} \{u_{ri}\}, i = \overline{1, k_r}$; $u_s := \max_{i=\overline{1,k_s}} \{u_{si}\}, i = \overline{1, k_s}$;
6. If $u_c = n_c + 1$ or $u_r = n_r + 1$ or $u_s = n_s + 1$, then the problem has no solutions, because the conditions of InEq (4) of network steady-state operation are not met.
7. The performances allowed for common-use components are determined according to the solution of Problem 1:

$$\check{d}_i := D_{cz} | \{D_{cz} \leq d_i < D_{c,z+1}\}, z_{ci} := z, i = \overline{1, k_c};$$

$$\check{\mu}_{ri} := D_{rz} | \{D_{rz} \leq d_i < D_{r,z+1}\}, z_{ri} := z, i = \overline{1, k_r};$$

$$\check{\mu}_{si} := D_{sz} | \{D_{sz} \leq d_i < D_{s,z+1}\}, z_{si} := z, i = \overline{1, k_s}.$$

8. The minimum reasonable performances for common-use entities are determined:

$$\bar{d}_i := \max\{D_{cu_{ci}}; \check{d}_i\}, i = \overline{1, k_c};$$

$$\bar{\mu}_{ri} := \max\{D_{ru_{ri}}; \check{\mu}_{ri}\}, i = \overline{1, k_r};$$

$$\bar{\mu}_{si} := \max\{D_{su_{si}}; \check{\mu}_{si}\}, i = \overline{1, k_s}.$$

9. The summary costs $\overline{C}$ are determined at the minimum reasonable performances for common-use entities:

$$\overline{C} = \sum_{i=1}^{k_c} c_{ci}\overline{d}_{ci} + \sum_{i=1}^{k_r} c_{ri}\overline{\mu}_{ri} + \sum_{i=1}^{k_s} c_{si}\overline{\mu}_{si}.$$

10. The reserve $\Delta\bar{C}$ of available financial resources is determined:

$$\Delta\bar{C} \coloneqq C^* - \bar{C}.$$

11. For each of common-use component *i* of the network, the additional cost is determined when replacing its current performance with the next one from the allowed series of performances:
    a) if $\bar{d}_i \le d_i$, then $\Delta C_{ci} \coloneqq c_{ci}(D_{c,z_{ci}+1} - D_{cz_{ci}})$, otherwise $\Delta C_{ci} := 0$, $i = \overline{1, k_c}$;
    b) if $\bar{\mu}_{ri} \le \mu_{ri}$, then $\Delta C_{ri} \coloneqq c_{ri}(D_{r,z_{ri}+1} - D_{rz_{ri}})$, otherwise $\Delta C_{ri} := 0$, $i = \overline{1, k_r}$;
    c) if $\bar{\mu}_{si} \le \mu_{si}$, then $\Delta C_{si} \coloneqq c_{si}(D_{s,z_{si}+1} - D_{sz_{si}})$, otherwise $\Delta C_{si} := 0$, $i = \overline{1, k_s}$.
12. An algorithm for solving the backpack problem is applied: among sizes ΔC_{ci} ($i = \overline{1, k_c}$), ΔC_{ri} ($i = \overline{1, k_r}$) and ΔC_{si} ($i = \overline{1, k_s}$) to select the ones which would minimize $\Delta C = C^* - C^{**}$. ■

Solving the problem using the resource concentration rule. Obviously, if $C^{**} \neq C$, then the time *T* must be calculated according to Eq. (10) and not to Eq. (22) or, equivalently, to Eq. (23). Therefore, in such cases, taking into account the opportunity to concentrate the resources [2], when increasing the performance it is appropriate to base on the rule: component *i* is preferable to component *j*, if $\Delta C_i/\Delta T(i) < \Delta C_j/\Delta T(j)$, where $\Delta T(i)$ is the decrease in time *T* caused by the increase ΔC_i in the cost of component *j*. Following this rule is all the more successful the higher the values of the quantities n_c, n_r and n_s. Another solves the problem based on the resource concentration rule

This rule is the basis of **Algorithm 2** for the heuristic solution of Problem 2:

1-11. Steps 1-11 coincide with Steps 1-11 of Algorithm 1.

12. $g_{ci}:=\Delta C_{ci}/\Delta T(c,i), i = \overline{1, k_c}$; $g_{ri}:=\Delta C_{ri}/\Delta T(r,i), i = \overline{1, k_r}$; $g_{si}:= \Delta C_{ci}/\Delta T(s,i)$, $i = \overline{1, k_s}$.
13. Arrangement of quantities g_{ci}, $i = \overline{1, k_c}$; g_{ri}, $i = \overline{1, k_r}$ and g_{si}, $i = \overline{1, k_s}$ in ascending order forming two arrays: a_i, b_i, $i = \overline{1, k_c + k_r + k_s}$ where a_i specifies the type (*c*, *r* or *s*), and b_i – the order number of the capacity of the common-use network component in the respective performances' string ($\overline{1, k_c}$, $\overline{1, k_r}$ or $\overline{1, k_s}$).
14. Increase to the next performance in the performances' series in ascending order of b_i up to $b_i|\min\{\Delta C_i \geqslant 0\}$. The adjusted solution is obtained, ■

4. Minimizing the Total Cost of Network Components

Problem 3. Let, for the model {Eq. (8), Eq. (9)} and conditions of InEq. (4), the parameters are known: T^*; β; γ, k_c; k_r; k_s; λ_{ci}, V_i, c_{ci}, $i = \overline{1, k_c}$; λ_{ri}, c_{ri}, $i = \overline{1, k_r}$; λ_{si}, c_{si}, $i = \overline{1, k_s}$, where T^* is the maximum allowed average response time to data processing requests. It is required to determine the necessary capacities d_i, $i = \overline{1, k_c}$ of channels, those μ_{ri}, $i = \overline{1, k_r}$ of routers and those μ_{si}, $i = \overline{1, k_s}$ of network servers that would ensure a minimum summary cost *C* of channels, routers and servers of the network at the average response time *T* to data processing requests in the network that would not exceed the given value T^*, i.e.:

$$C = \sum_{i=1}^{k_c} c_{ci}d_{ci} + \sum_{i=1}^{k_r} c_{ri}\mu_{ri} + \sum_{i=1}^{k_s} c_{si}\mu_{si} \to \min \tag{26}$$

upon compliance with the restriction

$$T=\frac{1}{\gamma}\left(\sum_{i=1}^{k_c}\frac{\lambda_{ci}V_i}{d_{ci}-\lambda_{ci}V_i}+\sum_{i=1}^{k_r}\frac{\lambda_{ri}}{\mu_{ri}-\lambda_{ri}}+g\sum_{i=1}^{k_s}\frac{\lambda_{si}}{\mu_{si}-\lambda_{si}}\right)\leq T^* \tag{27}$$

in conditions of InEq. (4). ■

Solving. From the essence of the problem, it is obvious that the solution corresponds to the limit value in Eq. (27), that is, $T = T^*$. Then the problem {Eq. (26), Eq. (27)} can be solved using the method of Lagrange multipliers. The respective Lagrangian L is

$$L=\sum_{i=1}^{k_c}c_{ci}d_i+\sum_{i=1}^{k_r}c_{ri}\mu_{ri}+\sum_{i=1}^{k_s}c_{si}\mu_{si}+\chi(\frac{1}{\gamma}\left(\sum_{i=1}^{k_c}\frac{\lambda_{ci}V_i}{d_i-\lambda_{ci}V_i}+\sum_{i=1}^{k_r}\frac{\lambda_{ri}}{\mu_{ri}-\lambda_{ri}}+g\sum_{i=1}^{k_s}\frac{\lambda_{si}}{\mu_{si}-\lambda_{si}}\right)-T^*). \tag{28}$$

The Lagrangian L contains $k_c + k_r + k_s + 1$ unknowns: χ; d_i, $i=\overline{1,k_c}$; μ_{ri}, $i=\overline{1,k_r}$ and μ_{si}, $i=\overline{1,k_s}$. The partial derivatives of L with respect to unknowns equalized to 0 are:

$$\begin{cases}\dfrac{\partial L}{\partial d_{ci}}=c_{ci}+\dfrac{\chi}{\gamma}\cdot\dfrac{-\lambda_{ci}V_i}{\left(d_i-\lambda_{ci}V_i\right)^2}=0,\ i=\overline{1,k_c}\\ \dfrac{\partial L}{\partial \mu_{ri}}=c_{ri}+\dfrac{\chi}{\gamma}\cdot\dfrac{-\lambda_{ri}}{\left(\mu_{ri}-\lambda_{ri}\right)^2}=0,\ i=\overline{1,k_r}\\ \dfrac{\partial L}{\partial \mu_{si}}=c_{ri}+\dfrac{g\chi}{\gamma}\cdot\dfrac{-\lambda_{si}}{\left(\mu_{si}-\lambda_{si}\right)^2}=0,\ i=\overline{1,k_r}\\ \dfrac{\partial L}{\partial \chi}=\dfrac{1}{\gamma}\left(\begin{array}{l}\displaystyle\sum_{i=1}^{k_c}\frac{\lambda_{ci}V_i}{d_i-\lambda_{ci}V_i}+\\ \displaystyle+\sum_{i=1}^{k_r}\frac{\lambda_{ri}}{\mu_{ri}-\lambda_{ri}}+g\sum_{i=1}^{k_s}\frac{\lambda_{si}}{\mu_{si}-\lambda_{si}}\end{array}\right)-T^*=0.\end{cases} \tag{29}$$

Similarly to the determination of expressions of Eq. (15)-(17), from the first three lines of Eq. (29) and taking into account the observance of conditions of InEq. (4) of operation in the steady state, it is obtained:

$$d_i=\lambda_{ci}V_i+\sqrt{\frac{\chi\lambda_{ci}V_i}{\gamma c_{ci}}},\ i=\overline{1,k_c}; \tag{30}$$

$$\mu_{ri}=\lambda_{ri}+\sqrt{\frac{\chi\lambda_{ri}}{\gamma c_{ri}}},\ i=\overline{1,k_r}; \tag{31}$$

$$\mu_{si}=\lambda_{si}+\sqrt{\frac{g\chi\lambda_{si}}{\gamma c_{ri}}},\ i=\overline{1,k_r}. \tag{32}$$

Substituting d_i, $i=\overline{1,k_c}$; μ_{ri}, $i=\overline{1,k_r}$ and μ_{si}, $i=\overline{1,k_s}$, from the last line of Eq. (29) with expressions of Eq. (30)-(32), respectively, as a result of some simple transformations, one obtains

$$\sqrt{\chi} = \frac{\sum_{i=1}^{k_c} \sqrt{c_{ci}\lambda_{ci}V_i} + \sum_{i=1}^{k_r} \sqrt{c_{ri}\lambda_{ri}} + \sum_{i=1}^{k_s} \sqrt{gc_{si}\lambda_{si}}}{T^*\sqrt{\gamma}}. \tag{33}$$

Substituting in Eq. (30)-(32) $1/\sqrt{\chi}$ by the expression of Eq. (33), one has:

$$d_i = \lambda_{ci}V_i + \frac{G\sqrt{c_{ci}\lambda_{ci}V_i}}{T^*\gamma c_{ci}}, i = \overline{1,k_c}, \tag{34}$$

$$\mu_{ri} = \lambda_{ri} + \frac{G\sqrt{c_{ri}\lambda_{ri}}}{T^*\gamma c_{ri}}, i = \overline{1,k_r}, \tag{35}$$

$$\mu_{si} = \lambda_{si} + \frac{G\sqrt{gc_{si}\lambda_{si}}}{T^*\gamma c_{si}}, i = \overline{1,k_s}, \tag{36}$$

where

$$G = \sum_{i=1}^{k_c} c_{ci}\lambda_{ci}V_i + \sum_{i=1}^{k_r} c_{ri}\lambda_{ri} + \sum_{i=1}^{k_s} c_{si}\lambda_{si}.$$

Substituting expressions of Eq. (34), Eq. (35) and Eq. (36) for d_i, ($i = \overline{1,k_c}$), μ_{ri} ($i = \overline{1,k_r}$) and μ_{si} ($i = \overline{1,k_s}$) in Eq. (26), the expression for the optimal value C^* of quantity C, one has

$$C^* = \sum_{i=1}^{k_c} c_{ci}\lambda_{ci}V_i + \sum_{i=1}^{k_r} c_{ri}\lambda_{ri} + \sum_{i=1}^{k_s} gc_{si}\lambda_{si} + \frac{1}{\gamma T^*}\left(\sum_{i=1}^{k_c} \sqrt{c_{ci}\lambda_{ci}V_i} + \sum_{j=1}^{k_r} \sqrt{c_{rj}\lambda_{rj}} + \sum_{i=1}^{k_s} \sqrt{gc_{si}\lambda_{si}}\right)^2. \tag{37}$$

The analytical solution of the problem is constituted by Eq. (34)-(37). It should be noted that the value of cost C^* can also be determined according to Eq. (26), i.e. the relations {Eq. (26), Eq. (34)-(36)} can also be used as a solution. The expected values of the quantities C^*, d_i ($i = \overline{1,k_c}$), μ_{ri} ($i = \overline{1,k_r}$) and μ_{si} ($i = \overline{1,k_s}$) can be calculated relatively easily, including in MS Excel.

Remark 2. Of course, the values of quantities d_i, $i = \overline{1,k_c}$; μ_{ri}, $i = \overline{1,k_r}$ and μ_{si}, $i = \overline{1,k_s}$ obtained according to Eq. (34)-(36), are positive real numbers, and the performances for channels, routers and servers that could be used in the network are discrete ones, belonging to a finite set. Therefore, an additional adjustment of the solution may be necessary. ■

Eq. (33) can be presented in the form

$$C^* = \frac{A}{\gamma T^*} + B. \tag{38}$$

Comparing Eq. (23) and Eq. (38), it can be seen that they coincide, differing only in their form. Thus, the relationships between the quantities T^* and C^* for Problems 1 and 3 are the same.

From Eq. (24) and Eq. (25), it can be seen that if conditions of InEq. (4) are not taken into account, then the expressions for determining the values of quantities A and B do not depend on the capacities of channels, routers and servers. Also, the value of the summary cost C^* of core components of the network is inversely proportional to the average response time T^* to data processing requests. So the smaller the value of difference $\Delta T = T^{**} - T^*$ will be, the smaller the value of the difference $\Delta C = C^* - C^{**}$, where C^{**} and T^{**} are the values of quantities C and T at allowed values of capacities of core components of the network. From here, the essence of the problem of necessary adjustment of the solution of Problem 3 mentioned in Remark 2.

Problem 4 - adjusting the solution of Problem 3. Let the initial data of Problem 3 and also the increasing strings of capacities of components that can be used in the network are known: D_{ci}, $i = \overline{1, n_c}$ of data transfer rates of channels, D_{ri}, $i = \overline{1, n_r}$ of packet processing rates by routers and D_{si}, $i = \overline{1, n_s}$ of request processing rates by servers. It is required to adjust the solution of problem {Eq. (22), Eq. (23)} obtained according to Eq. (30)-(32) so that, at allowed values of capacities of core entities of the network and respecting the conditions of InEq. (4), to minimize the value $\Delta T = T^{**} - T^*$.

Solving. The adjustment of the solution of Problem 3, so that the allowed capacities of channels, routers and servers are used, is performed in a similar way to that described in Algorithms 1 and 2 of the adjustment of the solution of Problem 1, but reversing the roles of T and C.

5. Conclusions

Because of considerable expenses with the creation and development of computer networks, it is important their minimization. Moreover, sometimes it is useful the quick preliminary estimation of basic characteristics of the network. In this aim, taking into account the Jackson's partitioning theorem [1] and considering the linear dependence of the costs of channels, routers, and servers on their performance, a simplified analytical model of the backbone subnet and server set of wide area computer networks is defined. Based on this model, two optimization problems are formulated: minimizing the average response time to user requests of data processing and minimizing the summary cost of servers, channels and routers of the computer network. For these problems, simple analytical solutions regarding the necessary performances of channels, routers and servers are obtained. In the analytical solutions, the equations for the optimization criteria of the two problems differ only in their form. Knowing the initial data, the performances of common-use components of computer networks can be calculated, for example, in MS Excel.

Obtained in this way performances are positive real numbers. Usually they do not coincide with the allowed performances of channels, routers and servers, which take values from a series of discrete numbers. This is why, if necessary, the problem of adjusting the obtained values of performances of analytical solutions to the allowed discrete performances of the common-use network components, is formulated. To solve it, two algorithms are proposed: by reducing the initial problem to that of backpack and by using the resource concentration rule. The first of them is exact, but relatively complex, and the second one is simple, but not exact.

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