ISSN 0025-6544, Mechanics of Solids, 2023, Vol. 58, No. 3, pp. 727–747. © Allerton Press, Inc., 2023.
ISSN 0025-6544, Mechanics of Solids, 2023. © Allerton Press, Inc., 2023.
Russian Text © The Author(s), 2023, published in Izvestiya Rossiiskoi Akademii Nauk, Mekhanika Tverdogo Tela, 2023, No. 3, pp. 73–98.

Regularities of Changing the Limiting Values of Stress and Strain Invariants in Microinhomogeneous Media

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Received February 22, 2022; revised July 19, 2022; accepted July 25, 2022

Abstract—By using nonlinear equations of constraints between macro- and micro-states, the regularities of changes in the limiting values of stress and strain invariants in microinhomogeneous media are studied. It is shown that the extreme relative moduli of stress tensor deviators in polycrystals with a cubic lattice are invariant with respect to external conditions of reversible force and depend only on the crystal anisotropy factor. In the irreversible region of deformation, analytical relations are obtained for bulk and tensile normal stresses. The effect of cyclic change in bulk and tensile stresses in some subelements under external monotonic loading has been established. It is shown that, on the basis of nonlinear equations of constraints, a complex pattern of material failure can be described using the theory of maximum normal stresses at the local level.

Keywords: structure, stress, deformation, fracture, hardening, energy, microinhomogeneity, state, subelement

DOI: 10.3103/S0025654423700085

INTRODUCTION

The successes achieved in describing the complex phenomena occurring during the deformation of microinhomogeneous materials demonstrate that the most constructive fundamental concepts include the idea according to which the representative volume of a macroscopically homogeneous body is represented as an infinite number of interconnected subelements with different thermorheological properties. The models proposed by various authors differ from each other in the accepted relations between local stress $\overline{t}_{ij}$ and strain tensors $\overline{d}_{ij}$ and macroscopic stresses t_{ij} and strains d_{ij}. Most authors limit themselves to the assumption of a uniform deformed state of subelements $\overline{d}_{ij} = d_{ij}$ or a uniform stress state $\overline{t}_{ij} = t_{ij}$. The first multi-element model of the medium was developed by Mazing [1]. In this model, the process of deformation of a body element is simulated as the deformation of a finite number of rods of the same stiffness, which have the properties of ideal plasticity with different yield strengths and have the same deformation, which made it possible to describe the Bauschinger effect quite accurate. Assuming the constancy of deformation inside a polycrystalline body, Voigt [2] calculated macroscopic elastic constants based on the elastic constants of crystals. Reiss [3] obtained formulas for calculating the elasticity constant of a polycrystal based on the assumption of constant stresses. An extension of the model $\overline{d}_{ij} = d_{ij}$ to viscoplastic processes of deformation of an initially isotropic material, which reveals the anisotropy of strain hardening, aftereffects, and secondary creep, is contained in [4]. Usually, for this area of research, the structural models is used [5–9]. To analyze the behavior of materials under thermomechanical effects, models based on the maintenance of internal variables are also used [10]. The study of the plastic flow of polycrystals that is obtained from the behavior of single crystals, was carried out by Sachs (1928) and Taylor (1938). Both confirmed important predictions of the behavior of polycrystals. However, due to very simple underlying assumptions, most of their results are qualitative and allow only rather weak agreement with experimental data.

Kroner [11] opened a new way with the formulation of the so-called “self-consistent scheme”, referring to the problem of inclusion in an infinite matrix. According to his scheme, each grain of a polycrystal is sequentially considered as an inclusion in the “matrices” of all other grains. Then, the behavior of the polycrystal is calculated using some adequate averaging over all grains. As a result, he established the following linear law of interaction

$$\tilde{t}_{ij} - t_{ij} = B_{ijnm}(d_{nm} - \tilde{d}_{nm}), \quad B_{ijnm} = B_0\delta_{ij}\delta_{nm} + BI_{ijnm},$$
$$B = 2G\frac{7-5\nu}{8-10\nu}, \quad B_0 = 2G\frac{3-5\nu}{8-10\nu},$$

where G is the shear modulus, ν is the Poisson's ratio. The Kroner model satisfactorily agrees with the experimental data in the elastic region of deformation, however, in the irreversible region it leads to overestimated internal stresses.

To take into account the natural tendency of the material to reduce stress fluctuations inside a representative volume ΔV_0, Berveiller and Zaomi [12] introduced the so-called "plastic accommodation function" into the analysis. Such an approach is possible only if the isotropic elastoplastic interaction between the inclusion and the matrix is limited. In this model, the parameter B decreases with increasing plastic strain by almost two orders of magnitude in uniaxial tensile testing of single-phase polycrystalline materials. Further development of multi-element models in the framework of linear relations between fluctuations of stress and strain tensors is associated with the development of various methods for determining the parameters B and B_0 [12, 13, etc.]. A weak point of research in this area is the inconsistency of the linear equations of constraints of macro- and micro-states with the first law of thermodynamics[1]

$$\left\langle \int_0^t \tilde{t}_{ij}\dot{\tilde{d}}_{ij}dt \right\rangle < \int_0^t \langle\tilde{t}_{ij}\rangle\langle\dot{\tilde{d}}_{ij}\rangle dt$$

for any options for changing the parameters B and B_0, except for the limiting ones: $B = B_0 = 0$ or $B = B_0 = \infty$. It is problematic to use models based on linear constraint equation to describe the processes of destruction or the interaction of thermal and mechanical fields in the framework of a constrained theory. The absence of a relation between the deviatoric and spherical quantities leads, in particular, to the prediction of the impossibility of fracture under pure compression, which is inconsistent with experiment. Discussion of various theories based on linear constraint equations of macro- and micro-states is the scope of this article.

Since we cannot take into account in full measure the interactions of material particles in a representative volume, it is expedient to construct the constraint equations of macro- and micro-states, which would be consistent with the laws of thermodynamics, take into account the phenomenon of self-consistency of local processes of irreversible deformation, and meet the condition of uniqueness of the solution of the problem of representing the material in the model [8]. Due to the fact that the linearity of the relation between local and macroscopic parameters follows from the formulation of the problem of inclusion in an infinite matrix, in [7, 8, 14, 15] a different approach was proposed to construct the constraint equations between macro- and micro-states. As the primary element of the structure, a subelement, which is identified with a set of material particles inside a representative volume that have the same irreversible strain tensor, is chosen. Particles of the same subelement can have different orientations and positions in the space of the conglomerate. The number of particles in each subelement determines their weight and does not change during deformation. From this definition of the concept of a subelement, more complex interactions of material particles in a representative volume follow than the interaction of an inclusion with a matrix. Stresses and reversible strains in a subelement correspond to the average values of stresses and strains arising in a subset of material particles with the same irreversible strain tensors. Due to this circumstance, it is postulated that interactions between subelements are formed under the influence of only averaged connections. It is assumed that the medium continuity condition is ensured by the action of five independent slip systems. The experimentally established non-basic slip makes it possible to describe the scalar and tensor properties of materials in the irreversible region of deformation in terms common to Mechanics of a Deformable Solid. The model proposed in [7, 8, 14–16] is based on the following principles: averaged bonds, orthogonality of stress and strain fluctuation tensors, extremum of discrepancy between macroscopic measures and suitable average values of microscopic analogues. The constructed closed system of equations satisfies the laws of thermodynamics, takes into account the phenomenon of self-consistency of local processes of irreversible deformation, and meets the requirement for the uniqueness of the solution of the problem of representing a real material in a model. A detailed study of the system, which also contains the rule of discrete mechanical memory of the material about the set of characteristic moments of the prehistory of deformation and heating, in the case of cyclic proportional

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nonisothermal deformation of unstable materials, was carried out in [16]. The general patterns of behavior of materials sensitive to the strain rate were studied in [14–16, etc.].

In the listed articles [7–9, 14–16], only deformation processes are studied without taking into account the processes of initiation and accumulation of microcracks, which lead to the destruction of a body element. In this article, along with the yield criterion, we also consider the condition for the destruction of subelements. Irreversible deformations lead to an increase in crystal lattice defects, an increase in the level of stresses and prepare the metal for destruction, while normal tensile stresses lead to destruction. Therefore, for a joint consideration of the processes of deformation and destruction, it is necessary to study the pattern of change in the limiting values of the three stress/strain invariants in the set of subelements.

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Translated by A. Borimova