Application of Undular Analysis for Description of the Physical Phenomena

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Abstract. **— The spectral decomposition is one of the most fruitful methods applied in technique. However more often spectroscopic decomposition is applied to signals and functions, time-dependent, less often from spatial coordinates. In this article it is proposed to view formally real objects as the functions depending simultaneously both from spatial and time coordinates. Such function can be presented in the form of the linear superposition (i.e. series or integral) of standard set of functions having more simple properties, in particular a set of the harmonic waves. The offered approach release us from necessity to explore separately each function-object having dependence from time and coordinates. Any object appears presented in the form of superposition of harmonic functions - waves, and viewing of its evolution appears simplified up to a limit. In last part the featured approach is applied as example for substantiation of postulates of a special theory of relativity.**

Index Terms **– spectroscopic decomposition, hologram, undular model, a special theory of relativity, the carrier of waves.**

One of the most fruitful method applied in technique, is the spectral decompositions on the elementary functions. The domains where this approach is applied most often are the processing and transmission of information: broadcasting, telecasting etc. All information which we gain by phone, by radio or TV, what complex it would not be, actually represents the function (of voltages) from time. Such function can be presented as a sum (integral) of some elementary functions depending from time. The most convenient in this plan are harmonic functions. The representation of any function from time in the form of some, of harmonic functions, is termed as Fourier decomposition or Fourier analysis [1]. So, the function f periodic with the period T, can be presented in the form of series Fourier:

$$f(t)=\frac{a_0}{2}+\sum_{n=1}^{+\infty}A_n\cos(n\omega_0 t+\varphi_n),$$

where A_n is amplitude of k harmonic oscillations; $n\omega_0$ — circular frequency of k the harmonic oscillations; φ_n - the initial phase k oscillations. This series can be copied also as well in the complex mode

$$f(t)=\sum_{-\infty}^{+\infty}C_n e^{in\omega_0 t},$$

here C_n is the n complex amplitude.

Similar decomposition can be made and for the any function which depends on spatial coordinate. Thus a series (integral) Fourier will represent the sum of harmonic functions from coordinate. On this principle the holography is grounded.

The Fresnel rings represent an elementary hologram – the hologram of a point. If a laser beam passes through a hole having very small diameter, the system of rings of Fresnel (рис.1) will appear on a photographic plate placed behind a hole. This system of rings represents a plate, the transparence or reflection power of which changes under the harmonic law depending on spatial coordinates.

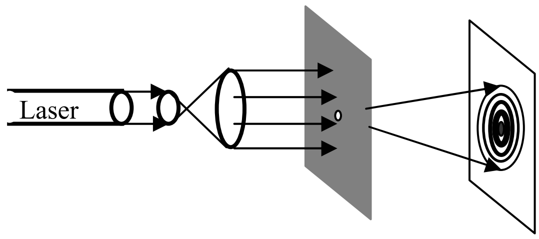

Fig.1. Record of point hologram

If now, a light beam pass through such plate a converging spherical wave is formed. This spherical wave will recover the image of luminous point (рис.2). That is, the spatial modulation of light by a sinusoidal function gives a luminous point.

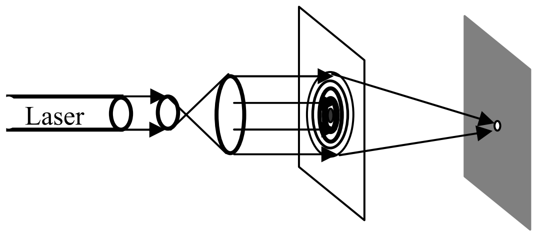

Fig.2. The reconstruction of image of point by means of the hologram

Having picked up a necessary number of Fresnel lenses it is possible to synthesize any image point by point. Thus, the hologram represents the system of the interference stripes containing the image in ciphered view.

Naturally the idea arises: to unite decomposition and synthesis on harmonic functions from time with decomposition (synthesis) on harmonic functions from spatial coordinates. As effect, we gain moving images, the image of which, in ideal case, will not differ from a reality. Thus, the image of real objects can be synthesized from sinusoidal functions depending simultaneously from temporary and spatial coordinates. Such functions represent waves. It is necessary to note, that till now the idea of spectroscopic decomposition on waves was applied only to wave functions especially in acoustics [2].

Formally it is possible to tell, that the harmonic functions-waves represent a set from which it is possible to synthesize any real function. Thus, the objects surrounding us can be presented as certain functions, from coordinates, and time. These functions will describe the change of dimensions and position of objects in dependence from time i.e. the evolution of object.

For the one-dimensional case the Fourier decomposition on the harmonic waves of periodic function p describing a wave with period $T=2\pi/\omega_0$ (at $k_0=\omega_0/c$) looks like:

$$p\left(t-\frac{x}{c}\right)=\sum_{n=-\infty}^{n=+\infty} p_n \exp\left(-in\omega_0 t+ink_0 r\right)$$

and have a view

$$p\left(t-\frac{x}{c}\right)=\int_{-\infty}^{+\infty} p_\omega \exp\left(-i\omega t+ikr\right)d\omega$$

(at $k=\omega/c$) for acyclic function-waves. We can see, the wave - object appears decomposed in each moment of time in the spectrum depending on coordinates, i.e. it is presented in the form of superposition of sinusoidal spatial functions. The amplitudes of spectral components of decomposition p_n (or p_ω) do not depend on coordinates or on time.

For the spectral description of observable objects which are fixed in space, it is required, that the waves form an interference figure in space. Therefore spectral components should consist from pairs of the waves which are propagating in direction opposite. Such pair forms a standing wave. The idea to consider real objects from the point of view of undular model become even more natural if to recollect that from the point of view of quantum mechanics, all processes, in the nature, have a character of wave, that is are, as a matter of fact, waves.

However when it is a question about waves, there is a question which give no rest to physicists already during many years, namely a question on the carrier of waves of a substance and fields – electromagnetic, gravitational etc. If to return on model described by us, this question is equivalent to a question the function p what represents for real objects which we wish to describe by means of spectral decompositions.

From such "rational" sciences as hydrodynamics and acoustics it is known that the waves represent the states of a certain medium and not independent entities. On the other hand attempts to construct such mechanical model of an ether which would give satisfactory interpretation even laws of electromagnetic field have appeared unsuccessful. Have not success and the direct attempts to find out the ether. Moreover, in the physics has taken roots the erroneous opinion, that the existence of medium-carrier of waves of a substance including electromagnetic waves, contradicts the special theory of relativity [3].

The cause of failures consists in that theoretical methods and experiments related to detection of the carrier of waves and a field has been transferred from hydrodynamics and acoustics. In hydrodynamics and acoustics are used tools, heterogeneous regarding to explored medium. But it is obvious, for working with medium which is the carrier of waves and fields is necessary to refuse the "solid" tools used in classical sciences. The essence of undular model consists in consecutive development of methods which allow using waves as tools for examination of waves. Now we shall show, that existence of the medium-carrier of waves of matter does not contradict the special theory of relativity.

As is known, the special theory of relativity is based on two postulates:

- the principle of relativity – the laws by which the states of physical systems undergo change are not affected, whether these changes of state be referred to the one or the other of two systems in uniform translatory motion relative to each other.
- the principle of invariant light speed – light in vacuum propagates with the speed c (a fixed constant) in terms of any system of inertial coordinates, regardless of the state of motion of the light source.

A frame of reference must contain scales for measuring time and length. Such scales consist of repeating intervals of time and length. A standing wave possesses the property of periodicity in both space and time, and is described by an equation of the form

$$a=A\cos(-kx)\cos(\omega t). \tag{1}$$

where A - amplitude of value describing a wave (pressure, density, velocity, etc.), k - the wave number, ω - circular (cyclic) frequency.

By choosing such a wave, we choose a metric where

- the direction of the x-axis coincides with the direction of propagation of the wave;
- the spatial scale is defined by the wavelength

$$\lambda=\frac{2\pi}{k};$$

- the temporal scale is defined by the period of the wave

$$T=\frac{2\pi}{\omega}.$$

In other words, a standing wave naturally plays the same role as rulers and clocks (or chronometers) in the special theory of relativity. Thus the term $\cos\omega t$ defines the instantaneous value of the amplitude a of the wave at a fixed point with coordinate x. This term defines a rhythm of time, or serves as a clock, at the point considered. The term $\cos kx$ specifies the dependence of the oscillation amplitude on the coordinate x hence defines the spatial scale.

If there is another wave-object described by the equation:

$$a_0=A\cos(-k_0x)\cos(\omega_0 t), \tag{2}$$

in a medium, a measurement of its length in the frame defined by equation (1) consists in the definition of a number equal to the ratio of the length of the wave-object to the wave scale:

$$n={\lambda_0}/{\lambda}. \tag{3}$$

Similarly, a measurement of the period of the wave-object consists in the definition of the ratio of the period of the wave-object to the wave scale:

$$n = T_0 / T, \tag{4}$$

The wave-object (2) can be decomposed into two waves running in opposite directions, of the following form:

$$a_{01} = \frac{A}{2}\cos(\omega_0 t - k_0 x), \tag{5}$$

$$a_{02} = \frac{A}{2}\cos(\omega_0 t + k_0 x). \tag{6}$$

In the general case, the frequencies and wave numbers in equations (5) and (6) can differ; in this case equations (5) and (6) can be rewritten as

$$a_{01} = \frac{A}{2}\cos(\omega_{01} t - k_{01} x), \text{ и } a_{02} = \frac{A}{2}\cos(\omega_0 t + k_0 x),$$

Here $\omega_{01} \neq \omega_0$ and $k_{01} \neq k_0$. In this case the wave-object can be described by the equation

$$\begin{aligned} a_0 &= a_{01} + a_{02} = \\ &A_0 \cos\left(\frac{\omega_{01} - \omega_0}{2} t - \frac{k_0 + k_{01}}{2} x\right) \times \\ &\cos\left(\frac{\omega_{01} + \omega_0}{2} t - \frac{k_0 - k_{01}}{2} x\right). \end{aligned} \tag{7}$$

This equation describes beats, or a standing wave in which the maxima move with time. We shall term such a wave 'quasi-standing'. Thus, the values

$$\omega' = \frac{\omega_{01} + \omega_0}{2}$$

and

$$k' = \frac{k_{01} + k_0}{2}$$

can be perceived as the frequency and wave number of the moving wave-object (7). The term

$$\alpha = \frac{\omega_{01} - \omega_0}{2} t$$

in the first factor defines the phase displacement of the wave-object along the spatial coordinate x. Similarly, the value

$$\theta = \frac{k_0 - k_{01}}{2} x$$

in the second factor of equation (7) defines a retardation or phase displacement along the temporal coordinate. The displacement of the wave-object at a point with coordinate x in an interval Δt will then be:

$$\Delta x = \frac{\omega_{01} - \omega_0}{k_0 + k_{01}} \Delta t.$$

Hence the velocity of displacement of the wave-object is equal to:

$$v_0 = \frac{\Delta x}{\Delta t} = \frac{\omega_{01} - \omega_0}{k_0 + k_{01}}. \tag{8}$$

Since

$$\omega = \frac{2\pi}{T}, \quad k = \frac{2\pi}{\lambda}$$

and

$$\frac{\lambda}{T} = \frac{\omega}{k} = c.$$

Equation (8) can be rewritten equivalently as:

$$\begin{aligned} v_0 &== \frac{\lambda_0 \lambda_{01}}{T_0 T_{01}} \frac{T_0 - T_{01}}{\lambda_{01} + \lambda_0} = c^2 \frac{T_0 - T_{01}}{\lambda_{01} + \lambda_0} = \\ &c^2 \frac{k_0 - k_{01}}{\omega_0 + \omega_{01}}. \end{aligned} \tag{9}$$

Recall that v_0 is the velocity at which points with a particular phase move, for example the maxima of a quasi-standing wave.

The velocity v_0 defined by equations (8) and (9) is not related to motion through a continuum, nor does it concern that continuum in any other way. But, *in the absence of any tools other than undular frames, only this velocity can characterize the motion of the wave-object* (7).

When $\omega_0 >> \omega_{01}$, the velocity tends to c, i.e. $v_0 \to c$, and when $\omega_0 << \omega_{01}$, $v_0 \to -c$, that is, the quasi-standing wave is transformed into a progressive wave.

When $\omega_0 = \omega_{01}$, $v_0 = 0$, this case corresponds to a standing wave. Thus, the absolute value of the velocity v_0 can vary from 0 to c. From this we can see that the velocity c of propagation of a perturbation serves as a natural limit on the velocity of the displacement of a body that has a wave nature.

If an observer moves with a velocity defined by equations (8) and (9), from his/her point of view the wave (7) will be a standing wave, i.e. immobile, and it will be described by an equation of the form (2), or, if $n = 1$, by an equation of the form (1). Hence, in the system of a moving observer, this wave can be used as the wave that defines the frame. Thus, we come to the conclusion that, within the limits of our model, there can be a set of frames of reference that move relative each other with different velocities, *but all of them are equivalent*.

Definition: an *undular frame of reference* is a frame of reference in which the period of a standing or quasi-standing wave at a fixed point serves as the scale of time, and the scale of length is the distance between two points that have identical phase.

As noted above, a preferred frame cannot exist. Consequently, the principle of relativity is valid for undular frames. However, there is one circumstance in which doubt might be cast on this statement. The velocity c of a travelling wave described by equations (5) and (6) is determined by the properties of the medium. Naturally, we might have the idea of using a standing wave as a tool for determining the velocity c of a travelling wave. Knowing the velocity c, it would be possible to determine the velocity of an undular frame relative to the medium. In fact, such an experiment would be similar to the experiment performed by Michelson and Morley in 1887 [5], in which an attempt was made to detect motion relative to the aether. In our case, if such an experiment were to give a positive result, then it would be possible to choose one 'true undular frame', in which the velocity of motion relative to the carrier medium was equal to zero. Such a system would be privileged in relation to other undular frames. In this case, the principle of relativity would not be fulfilled for undular systems. Let us prove that this is not true.

Theorem: the velocity c of a travelling wave has the same value in all undular frames.

We suppose that we have two undular frames, described by the following equations:

$$a = A\cos(-kx)\cos(\omega t). \tag{10}$$

and

$$a' = A\cos\left(\frac{\omega_1 - \omega}{2}t - \frac{k + k_1}{2}x\right)\times \cos\left(\frac{\omega_1 + \omega}{2}t - \frac{k - k_1}{2}x\right). \tag{11}$$

The velocity of relative motion of these systems is given by the equation

$$v = \frac{\omega_1 - \omega}{k + k_1} = c^2\frac{T - T_1}{\lambda_1 + \lambda}. \tag{12}$$

Let us suppose that some wave-object is at rest in the system defined by equation (11), and that in that system it is described by

$$a_0' = A_0\cos(-k_0'x')\cos(\omega_0't'). \tag{13}$$

The same wave-object will be described in the system defined by equation (10) by

$$a_0 = A_0\cos\left(\frac{\omega_{01} - \omega_0}{2}t - \frac{k_0 + k_{01}}{2}x\right)\times \cos\left(\frac{\omega_{01} + \omega_0}{2}t - \frac{k_0 - k_{01}}{2}x\right). \tag{14}$$

Let us rewrite equation (14) taking equation (12) into account:

$$a_0 = A_0\cos\left(\frac{k_0 + k_{01}}{2}(vt - x)\right)\times \cos\left(\frac{\omega_0 + \omega_{01}}{2}(t - \frac{v}{c^2}x)\right). \tag{15}$$

Equations (15) and (13) describe the same wave-object. In equation (15), the value of $(vt - x)$ represents the instantaneous coordinate of the wave-object, as well as x' in equation (13). Transformations of lengths of line segments parallel to this coordinate should take place according to the same law as transformations of this coordinate. Hence, the length of the moving wave-object (15) becomes

$$\lambda_0' = \lambda_0 - vT_0,$$

and its wave number becomes

$$k_0' = \frac{2\pi}{\lambda_0 - vT_0}. \tag{16}$$

Following similar reasoning for the frequency, we obtain

$$\omega_0' = \frac{2\pi}{T_0 - \frac{v}{c^2}\lambda_0}. \tag{17}$$

The ratio of the circular frequency ω to the wave number k is equal to the velocity of the travelling wave c. Thus, the proof of the theorem formulated above is reduced to the demonstration of the relation

$$c = \frac{\omega_0}{k_0} = \frac{\omega_0'}{k_0'} = c'.$$

By using equations(16) and (17), we obtain

$$c' = \frac{\omega_0'}{k_0'} = c^2\frac{\lambda_0 - vT_0}{c^2T_0 - v\lambda_0}.$$

With the help of equations (3), (4) and (12), we can write

$$c' = c.$$

We have now proved that in an undular frame, the velocity c of a travelling wave does not depend on the choice of frame. Hence, the velocity of a travelling wave cannot be used for the definition of a velocity relative to the carrier medium, and all undular reference frames are equivalent.

Thus, use of undular frames leads us naturally to statements which serve as postulates for a special theory of relativity. The applications of the undular analysis for description of mechanisms of electromagnetic and gravitational fields are given in the monograph [6]. Some data on this theme are published also on a site [7].

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