THE ALGORITHM OF TUNING CONTROLLERS TO THE MODEL OBJECT WITH INERTIA, ADVANCE AND TIME DELAY IN THE CASCADE CONTROL SYSTEM

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Abstract – *This paper proposes a tuning algorithm of linear controllers P, PI, PID in the multiple-loop feedback control systems. The control object consists of two subprocesses, which are described by the dynamical models with advance, inertia (first and third order) and time delay. The controllers in the internal contour and in the external contour tuning use the maximal stability degree method. P and PI controllers are used in the internal contour and P, PI, PID controllers are used in the external contour.*

Keywords *–multiple-loop feedback control system, internal contour, external contour, time delay, maximal stability degree method.*

1. INTRODUCTION

Many tuning methods of typical controllers are used at the projecting of multiple-loop control systems: frequency method, criteria (of modulus) method etc [1,2,4]. The procedure of tuning controllers in the multiple-loop feedback control system becomes difficult. In this paper, it is proposed to use the maximal stability degree method (M.S.D) for tuning of typical controllers P, PI, PID for a class of control objects' models with inertia, advance and time delay, which are connected in cascade, represented by two subprocesses and, as result with two regulating loops.

2. THE ITERATIVE ALGORITHM OF TUNING CONTROLLERS

The multiple-loop feedback control system is represented by two contours: internal contour with controller's transfer function $H_{R2}(s)$ and subprocess $H_{F2}(s)$, and external contour with controller's transfer function $H_{R1}(s)$ and subprocess $H_{F1}(s)$ (Fig. 1). It is recommended to carry out the tuning of controllers first in the internal contour then in the external contour.

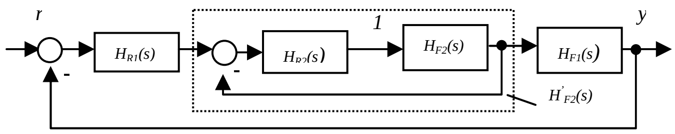

Fig.1 - The multiple-loop feedback control system

The control object consists of two inertial subprocesses with the transfer functions:

$$H_{F1}(s)=\frac{T_1 s+1}{(T_2 s+1)(T_3 s+1)(T_4 s+1)}=\frac{c_0 s+c_1}{d_0 s^3+d_1 s^2+d_2 s+d_3}, \quad (1)$$

where $c_0=T_1;\ c_1=1;\ d_0=T_2T_3T_4;$
$d_1=T_2T_3+T_2T_4+T_3T_4;\ d_2=T_2+T_3+T_4;\ d_3=1.$

$$H_{F2}(s)=\frac{ke^{-\tau s}}{T_5 s+1}, \text{ with } T_5 < T_2, T_3, T_4. \quad (2)$$

In expressions (1) and (2) we have the notations: k are transfer coefficients of the subprocesses; T_1, T_2, T_3, T_4, T_5 are time constants of respective subprocesses, τ - time delay.

3. THE TUNING CONTROLLERS IN THE INTERNAL CONTOUR

The tuning of controller with transfer function (t.f.) $H_{R2}(s)$ from internal contour to the subprocess with t.f. $H_{F2}(s)$ is implemented. We assume that P and PI controllers are used.

P controller is tuned to the object with transfer function (2), applied M.S.D. method and tuning parameters of controller are determined from relation:

$$k_{p2}=\frac{e^{-\tau J}}{k}(T_5 J-1)\cdot \quad (3)$$

In the relation (3) J is the maximal stability degree which is chosen from the following condition $J>0$.

To determinate the t.f. of the internal contour in case of tuning P and PI controllers it is proposed to approximate the value $e^{-\tau s}$ with Pade approximant [2]:

$$e^{-\tau s}=\frac{1}{\tau s+1}\cdot \quad (4)$$

The t.f. of internal contour with P controller tuning is:

$$H'_{F2}(s)=\frac{H_{R2}(s)\cdot H_{F2}(s)}{1+H_{R2}(s)\cdot H_{F2}(s)}=\frac{k_p k}{(T_5 s+1)\cdot(\tau s+1)+k_p k}=\frac{k'}{l_0 s^2+l_1 s+l_2}, \quad (5)$$

where $k'=\dfrac{k_{p2}k}{1+k_{p2}k};\ l_0=\dfrac{\tau T_5}{1+k_{p2}k};\ l_1=\dfrac{\tau+T_5}{1+k_{p2}k};\ l_2=1.\cdot$

PI controller is tuned to the object with the transfer function (2), applied the M.S.D. method and tuning parameters of controller are determined from relations [3,4]

$$k_{p2}=\frac{e^{-\tau J}}{k}(-\tau T_5 J^2+(\tau+2T_5)J-1); \quad (6)$$

$$k_{i2} = \frac{e^{-\tau J}}{k_2} J^2(-\tau T_5 J + \tau + T_5) \cdot \quad (7)$$

We can obtain the values of parameters k_{p2}, k_{i2}, changing the $J>0$ value, for that the performances of control system are sated.

The t.f. of internal contour with PI controller tuning, using expression (4) is:

$$H'_{F2}(s) = \frac{H_{R2}(s)H_{F2}(s)}{1+H_{R2}(s)H_{F2}(s)} = \frac{k(k_{p2}s+k_{i2})}{s(T_5 s+1)\cdot(\tau s+1)+k(k_{p2}s+k_{i2})} =$$

$$= \frac{l_0 s + l_1}{r_0 s^3 + r_1 s^2 + r_2 s + r_3}, \quad (8)$$

where $l_0 = \frac{k_{p2}}{k_{i2}};\ l_1 = 1;\ r_0 = \frac{\tau T_5}{k_{i2}k};\ r_1 = \frac{(\tau + T_5)}{k_{i2}k};$

$r_2 = \frac{1+k_{p2}k_2}{k_{i2}k_2};\ r_3 = 1.$

4. THE TUNING CONTROLLERS IN THE EXTERNAL CONTOUR

The structure block scheme of external contour is represented in the Fig. 2 *a, b*.

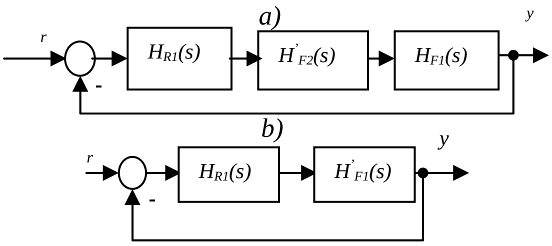

Fig. 2 - Structure block schema of external contour

For the tuning of P, PI, PID controllers in the external contour it is necessary to determine the equivalent transfer function of object (5) with P controller tuning in the internal contour with the t.f. of subprocess (1)

$$H'_{F1}(s) = H'_{F2}(s)H_{F1}(s) = \frac{k'}{(l_0 s^2 + l_1 s + l_2)} \cdot \frac{c_0 s + c_1}{(d_0 s^3 + d_1 s^2 + d_2 s + d_3)} =$$

$$= \frac{b_0 s + b_1}{a_0 s^5 + a_1 s^4 + a_2 s^3 + a_3 s^2 + a_4 s + a_5}, \quad (9)$$

where $b_0 = \frac{k_{p2}kT_1}{1+k_{p2}k_2};\ b_1 = \frac{k_{p2}k}{1+k_{p2}k_2};\ a_0 = \frac{\tau T_2 T_3 T_4 T_5}{1+k_{p2}k};$

$a_1 = \frac{(\tau+T_5)T_2T_3T_4}{1+k_{p2}k} + \frac{\tau T_5(T_2T_3+T_3T_4+T_2T_4)}{1+k_{p2}k};\ a_2 = T_2T_3T_4 +$

$+ \frac{(\tau+T_5)(T_2T_3+T_3T_4+T_2T_4)}{1+k_{p2}k} + \frac{\tau T_5(T_2+T_3+T_4)}{1+k_{p2}k}; a_3 = (T_2T_3+T_3T_4+T_2T_4)+$

$+ \frac{(\tau+T_5)(T_2+T_3+T_4)}{1+k_{p2}k} + \frac{\tau T_5}{1+k_{p2}k};\ a_4 = T_2+T_3+T_4+\frac{\tau+T_5}{1+k_{p2}k};\ a_5 = 1.$

For object with t.f. (9) P, PI, PID controllers can be tuned by applying the M.S.D. method using the relation from [3,4].

Control system with P controller:

$$k_p = \frac{a_0 J^5 - a_1 J^4 + a_2 J^3 - a_3 J^2 + a_4 J - a_5}{b_1 - b_0 J}. \quad (10)$$

Control system with PI controller:

$$k_p = \frac{-d_0 J^6 + d_1 J^5 - d_2 J^4 + d_3 J^3 - d_4 J^2 + d_5 J - d_6}{(b_1 - b_0 J)^2}, \quad (11)$$

where $d_0 = 5a_0b_0;\ d_1 = 6a_0b_1 + 4a_1b_0;$

$$d_2 = 5a_1b_1 + 3a_2b_0;\ d_3 = 4a_2b_1 + 2a_3b_0; \quad (12)$$

$d_4 = 3a_3b_1 + a_4b_0;\ d_5 = 2a_4b_1,\ d_6 = a_5b_1.$

$$k_i = \frac{-a_0J^6 + a_1J^5 - a_2J^4 + a_3J^3 - a_4J^2 + a_5J}{b_1 - b_0J} + k_pJ. \quad (13)$$

Control system with PID controller:

$$k_d = \frac{d_0J^7 - d_1J^6 + d_2J^5 - d_3J^4 + d_4J^3 - d_5J^2 + d_6J - d_7}{2(b_1 - b_0J)^4}, \quad (14)$$

where $d_0 = 5a_0b_0^3;\ d_1 = 68a_0b_0^2b_1 + 12a_1b_0^3;$

$d_2 = 78a_0b_0b_1^2 + 42a_1b_0^2b_1 + 6a_2b_0^3;$

$d_3 = 30a_0b_1^3 + 50a_1b_0b_1^2 + 22a_2b_0^2b_1 + 2a_3b_0^3;$

$d_4 = 20a_1b_1^3 + 24a_2b_0b_1^2 + 8a_3b_0^2b_1;\ d_5 = 12a_2b_1^3 + 12a_3b_0b_1^2;$

$d_6 = 6a_3b_1^3 + 2a_4b_0b_1^2 - 2a_5b_0^2b_1;\ d_7 = 2a_4b_1^3 - 2a_5b_0b_1^2,$

$$k_p = \frac{-d_0J^6 + d_1J^5 - d_2J^4 + d_3J^3 - d_4J^2 + d_5J - d_6}{(b_0 - b_1J)^2} + 2k_dJ, \quad (15)$$

where $d_0, d_1, d_2, d_3, d_4, d_5, d_6$ are determined from relations (12).

$$k_i = \frac{-a_0J^6 + a_1J^5 - a_2J^4 + a_3J^3 - a_4J^2 + a_5J}{b_1 - b_0J} - k_dJ^2 + k_pJ. \quad (16)$$

For the tuning of P, PI, PID controllers in the external contour it is necessary to determine the equivalent t.f of object (8) with PI controller in the internal contour with the t.f. of subprocess (1)

$$H''_{F1}(s) = H'_{F2}(s)H_{F1}(s) = \frac{l_0 s + l_1}{(r_0 s^3 + r_1 s^2 + r_2 s + r_3)} \cdot \frac{c_0 s + c_1}{(d_0 s^3 + d_1 s^2 + d_2 s + d_3)} =$$

$$= \frac{b_0 s^2 + b_1 s + b_2}{a_0 s^6 + a_1 s^5 + a_2 s^4 + a_3 s^3 + a_4 s^2 + a_5 s + a_6}, \quad (17)$$

where $b_0 = \frac{k_{p2}T_1}{k_{i2}};\ b_1 = \frac{k_{p2}}{k_{i2}} + T_1;\ b_2 = 1;\ a_0 = \frac{\tau T_2T_3T_4T_5}{k_{i2}k};$

$a_1 = \frac{(\tau+T_5)T_2T_3T_4}{k_{i2}k} + \frac{\tau T_5(T_2T_3+T_2T_4+T_3T_4)}{k_{i2}k};$

$a_2 = \frac{(1+k_{p2}k_2)T_2T_3T_4}{k_{i2}k_2} + \frac{(\tau+T_5)(T_2T_3+T_2T_4+T_3T_4)}{k_{i2}k} + \frac{\tau T_5(T_2+T_3+T_4)}{k_{i2}k};$

$a_3 = T_2T_3T_4 + \frac{(1+k_{p2}k_2)(T_2T_3+T_2T_4+T_3T_4)}{k_{i2}k_2} + \frac{(\tau+T_5)(T_2+T_3+T_4)}{k_{i2}k} + \frac{\tau T_5}{k_{i2}k};$

$a_4 = T_2T_3 + T_2T_4 + \frac{(1+k_{p2}k_2)(T_2+T_3+T_4)}{k_{i2}k_2} + \frac{(\tau+T_5)}{k_{i2}k};$

$a_5 = T_2 + T_3 + T_4 + \frac{1+k_{p2}k_2}{k_{i2}k_2};\ a_6 = 1.$

For object with t.f. (17) P, PI, PID controllers can be tuned by applying the M.S.D. method using the relations from [3,4].

Control system with P controller

$$k_{p1} = \frac{-a_0J^6 + a_1J^5 - a_2J^4 + a_3J^3 - a_4J^2 + a_5J - a_6}{b_0J^2 - b_1J + b_2}. \quad (18)$$

Control system with PI controller

$$k_{p1} = \frac{-d_0J^8 + d_1J^7 - d_2J^6 + d_3J^5 - d_4J^4 + d_5J^3 - d_6J^2 + d_7J - d_8}{(b_0J^2 - b_1J + b_2)^2}, \quad (19)$$

where $d_0 = 5a_0b_0;\ d_1 = 6a_0b_1 + 4a_1b_0;$

$$d_2 = 5a_1b_1 + 3a_2b_0 + 7a_0b_2;\ d_3 = 4a_2b_1 + 2a_3b_0 + 6a_1b_2; \quad (20)$$

$d_4 = 3a_3b_1 + a_4b_0 + 5a_2b_2;\ d_5 = 2a_4b_1 + 4a_3b_2;$

$d_6 = a_5b_1 + 3a_4b_2 - a_6b_0;\ d_7 = 2a_5b_2;\ d_8 = a_6b_2,$

$$k_{i1} = \frac{a_0J^7 - a_1J^6 + a_2J^5 - a_3J^4 + a_4J^3 - a_5J^2 + a_6J}{b_0J^2 - b_1J + b_2} + k_pJ. \quad (21)$$

Control system with PID controller

$$k_d = \frac{c_0 J^{11} + c_1 J^{10} - c_2 J^9 + c_3 J^8 - c_4 J^7 - c_5 J^6 + c_6 J^5 - c_7 J^4 + c_8 J^3 - c_9 J^2 + c_{10} J - c_{11}}{2(b_0 J^2 - b_1 J + b_2)^4}, \quad (22)$$

where $c_0 = 4d_0 b_0^2$; $c_1 = 3d_1 b_0^2 + 10 d_0 b_0 b_1$;

$c_2 = 2d_2 b_0^2 + 8d_1 b_0 b_1 + 12 d_0 b_0 b_2 + 6 d_0 b_1^2$;

$c_3 = d_3 b_0^2 + 6 d_2 b_0 b_1 + 10 d_1 b_0 b_2 + 5 d_1 b_1^2 + 14 d_0 b_1 b_2$;

$c_4 = 4 d_3 b_0 b_1 + 8 d_2 b_0 b_2 + 4 d_2 b_1^2 + 12 d_1 b_1 b_2 + 8 d_0 b_2^2$;

$c_5 = 2 d_4 b_0 b_1 - d_5 b_0^2 + 6 d_3 b_0 b_2 + 3 d_3 b_1^2 + 10 d_2 b_1 b_2 + 7 d_1 b_2^2$;

$c_6 = 4 d_4 b_0 b_2 - 2 d_6 b_0^2 + 2 d_4 b_1^2 + 8 d_3 b_1 b_2 + 6 d_2 b_2^2$;

$c_7 = 2 d_5 b_1 b_2 - 3 d_7 b_0^2 - 2 d_6 b_0 b_1 + d_5 b_1^2 + 6 d_4 b_1 b_2 + 5 d_3 b_2^2$;

$c_8 = 4 d_5 b_1 b_2 + 4 d_4 b_2^2 - 4 d_8 b_0^2 - 4 d_7 b_0 b_1$;

$c_9 = 2 d_6 b_1 b_2 + 3 d_5 b_2^2 - 6 d_8 b_0 b_1 - 2 d_7 b_0 b_2 - d_7 b_1^2$;

$c_{10} = 2 d_6 b_2^2 - 2 d_8 b_1^2 - 4 d_8 b_0 b_2$; $c_{11} = d_7 b_2^2 - 2 d_8 b_1 b_2$.

$$k_{p1} = \frac{-d_0 J^8 + d_1 J^7 - d_2 J^6 + d_3 J^5 - d_4 J^4 + d_5 J^3 - d_6 J^2 + d_7 J - d_8}{(b_0 J^2 - b_1 J + b_2)^2} + 2 k_{d1} J, \quad (23)$$

where $d_0, d_1, d_2, d_3, d_4, d_5, d_6, d_7, d_8$ are determined from relations (20).

$$k_{i1} = \frac{a_0 J^7 - a_1 J^6 + a_2 J^5 - a_3 J^4 + a_4 J^3 - a_5 J^2 + a_6 J}{b_0 J^2 - b_1 J + b_2} - k_{d1} J^2 + k_{p1} J. \quad (24)$$

Values of k_{p1}, k_{i1}, k_{d1} parameters are obtained, varied the *J* value, for that the performances of control system are sated. The following procedure is proposed to determine the optimal values of parameters k_p, k_i, k_d from relations (3), (6), (7), (10), (11), (13)-(16), (18), (19), (21)-(24) which represent the dependences of the maximal stability degree *J*. The variable *J* is changing and the curves $k_p = f(J)$, $k_i = f(J)$, $k_d = f(J)$ for the respectively controller and object were obtained. After, the sets of values of the k_p, k_i, k_d parameters get the optimal and quasioptimal value of *J*.

5. APLICATIONS AND COMPUTER SIMULATION

To show the efficiency of the proposed algorithm of tuning the typical controllers in the multiple-loop feedback control system with inertia (fourth order) using the presented relations an example with the object model which has the following parameters: $H_{F2}(s)$- $k_2=2$, $T_5=4$, $\tau = 2$ and $H_{F1}(s)$ – $T_1=5$, $T_2=15$, $T_3=12$, $T_4=8$ was examined.

The *P, PI* controllers were tuning in internal contour and *P, PI* and *PID* controllers were tuned in external contour using the maximal stability degree method, which permitted to obtain the high performance, varied values of *J* and choosing the respectively values of the P, PI, PID controllers.

The computer simulation has been made in MATLAB and the simulation diagram of multiple-loop feedback control system is presented in the Fig. 3.

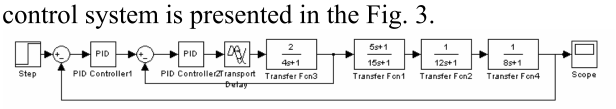

Fig. 3 - Simulation diagrams of the control system

The transition processes of the multiple-loop feedback control system for external contour is presented in the Fig. 4: *a)* – transition process in the external contour, with P controller tuning in the internal contour and P, PI, PID controller tuning in the external contour; *b)* – transition process in the external contour with PI controller tuning in the internal contour and P, PI, PID controllers tuning in the external contour. The obtained values of respectively controllers are presented in Table 1, where the number of rows correspond with number of curves from Fig. 4 *a, b*. The tuning values for curves *1 – 5* were obtained using the maximal stability degree method and curve 6 was obtained for the case of optimization the tuning value of the PID controller in the external contour using MATLAB.

Table 1 – The values of tuning controllers by maximal stability degree method

Nr. curves	k_{p2}=0.0185	J_{opt}=0.19 k_{p2}=0.156 k_{i2}= 0.038
1	k_{p1}=0.118	kp1=0.00237
2	J_{opt}=0.041 k_{p1}=7.124 k_{i1}= 0.427	Jopt=0.036 kp1=0.229 ki1= 0.0131
3	J_{opt}=0.067 k_{p1opt}=21.202 k_{i1opt}= 0.780 k_{dopt}=142.353	Jopt=0.058 kp1=0.5849 ki1= 0.021 kd1=4.0425
4	J=0.05 k_{p1}=14.714 k_{i1}= 0.60 k_{d1}=83.813	J=0.45 kp1=0.4391 ki1= 0.0174 kd1=2.5625
5	J=0.08 k_{p1}=16.06 k_{i1}= 0.586 k_{d1}= 108.29	J=0.7 kp1=0.4789 ki1= 0.0175 kd1=3.238
6	k_{p1}=39.032 k_{i1}= 1.387 k_d=142.412	kp1=3.0899 ki1= 0.0726 kd=40.177

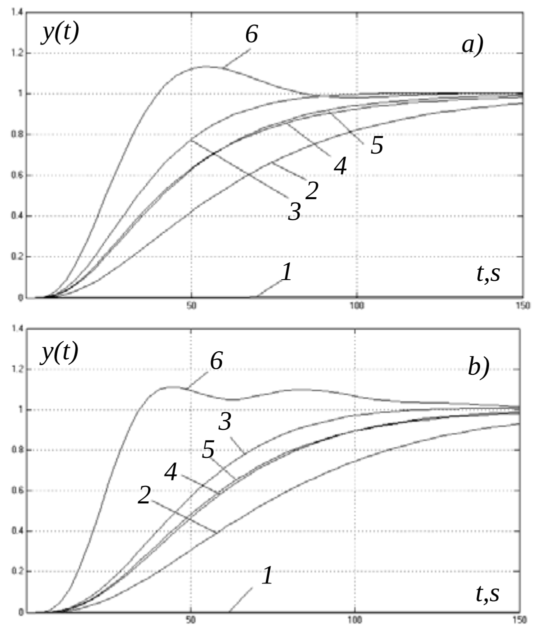

Fig. 4 - Transition processes of the multiple-loop feedback control system

To compare the obtained results the tuning of the typical controller using Ziegler-Nichols method was made. The Fig. 5 presents the transition processes of the multiple-loop feedback control system for external contour in the case of using Ziegler-Nichols method: *a)* – transition process in the external contour, with P controller tuning in the internal contour and P, PI, PID controllers tuning in the external contour; *b)* – transition process in the external contour with PI controller tuning in the internal contour and P, PI, PID controller tuning in the external contour.
The obtained values of respectively controllers are presented in Table 2, where the number of rows correspond with number of curves from Fig. 5 *a, b*.

Table 2 – The values of tuning controllers by Ziegler – Nichols method

Nr. curves	$k_{p2}=1.025$	$k_{p2}=0.92$ $k_{i2}=0.17$
1	$k_{p1}=9$	$k_{p1}=5.1$
2	$k_{p1}=8.1$ $k_{i1}=0.048$	$k_{p1}=4.59$ $k_{i1}=0.039$
3	$k_{p1}=13.60$ $k_{i1}=0.064$ $k_d=2.6$	$k_{p1}=7.711$ $k_{i1}=0.05$ $k_{d1}=3.2$

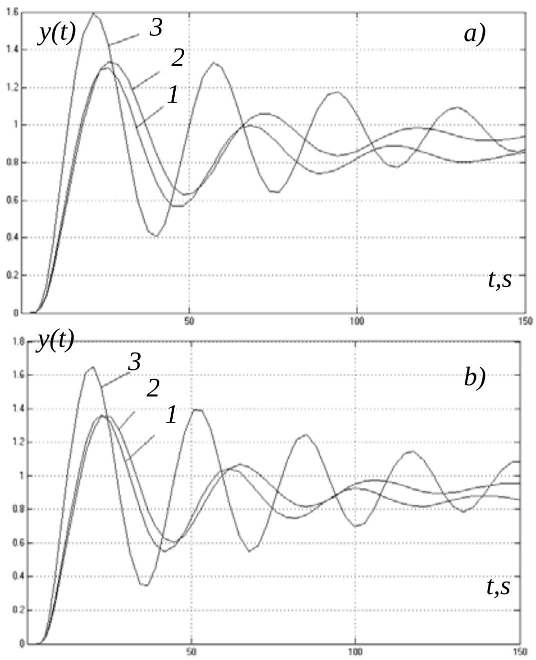

Fig. 5 - Transition processes of the multiple-loop feedback control system

The Fig. 6 *a)* presents the transition processes of the multiple-loop feedback control system, the Fig. 6 *b)* presents the repartition of the poles for the following cases: *1*- control system with P controller tuning in the internal contour and PID in the external contour using the maximal stability degree method, *2* - control system with PI controller tuning in the internal contour and PID in the external contour using the maximal stability degree method, *3*- control system with P controller tuning in the internal contour and PID in the external contour using the Ziegler – Nichols method, *4*- control system with PI controller tuning in the internal contour and PID in the external contour using the Ziegler – Nichols method.

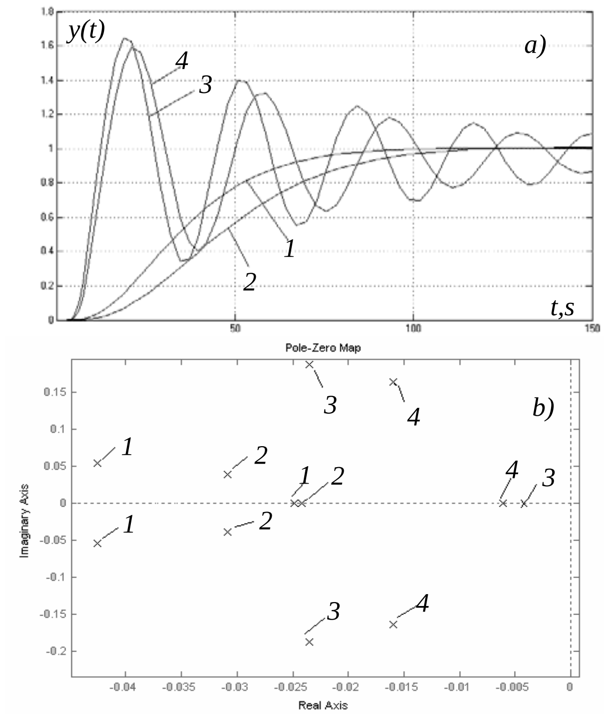

Fig. 6 - Transition processes of the multiple-loop feedback control system *(a)* and presentation of distribution of the systems' poles *(b)*

6. CONCLUSIONS

As a result, after tuning the P, PI, PID controller to the multiple-loop feedback control system with object's models (1), (2) with known parameters, the following conclusions can be made:

1. The tuning of P, PI controller in the internal contour in conformity with the maximal stability degree method permitted to obtain the high results varying the value of the $J>0$ and choosing the parameters of the respectively controllers for obtaining the sated performance of the internal contour.
2. The tuning of P, PI, PID controllers in the external contour using the maximal stability degree method permitted to obtain the high results varying the value of the $J>0$ and obtained the optimal value and the suboptimal values of the J, and choosing the set of the values of controllers' parameters for obtaining the sated performances.

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