Multi-Objective Optimal Solution Search based on Genetic Algorithms

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Abstract—The paper presents the results of research carried out to solve complex problems aimed at the efficient use of natural and energy resources. The objectives of the paper are achieved by identifying the control process based on a Multi-Agent system with distributed data processing that implements a Multi-objective optimal solution search model based on the application of a Genetic Algorithm with Collective Computation. The set of Agents presents a computational architecture that forms a structured network topology based on a P-Systems model presented in the form of a Venn diagram. The Object diagram and the Venn diagram of the P-Systems model are presented in the paper. The correctness of the developed models was verified on the basis of a control system of the artificial lighting process that provides for the minimization of energy consumption, while ensuring the quality of the lighting process.

Keywords—Multi-Objective Optimization; Genetic Algorithms; P-Systems; Multi-Agent System; Sensor Network; Distributed Computing; Collective Computing.

I. INTRODUCTION

The application of optimization methods has expanded in recent decades in all fields of science, engineering, and technologies. New generalized and specialized theoretical and algorithmic methods have been developed for application in certain fields. At the same time, a trend has been observed in the development of optimization methods oriented towards the interdisciplinary nature of the scope. Optimization has become a very useful tool for all fields of applied mathematics, engineering, medicine, and economics. Depending on the field of application and the solved problem are used approaches non-linear optimization (convex and non-convex), stochastic optimization, optimal control, discrete optimization, Multi-Objective optimization and heuristic [1].

Most real-world problems can be defined in terms of multi-objective functions, which are usually in the process of competition. In order to optimize these functions, it is necessary to adjust the functional description variables while also respecting the spatial constraints. This approach is considered efficient because computationally it requires a relatively simple implementation of the process of solving the individual objective functions [2]. However, the process of adjusting the functional description variables of the Multi-Objective model can be difficult because conflicts of interest may arise where an optimal solution for one objective function is unacceptable for the other functions. Solving the conflict of interest problem was worked it up with the application of genetic algorithms [3, 4] in combination with the weighted sum approach for solving Multi-Objective problems [5].

The Multi-Objective Decision-Making process is a very efficient modeling and methodological tool for solving complex problems in various fields [6, 7]. The Multi-Objective approach is also characteristic for cases when it is necessary to apply evolutionary algorithms in the modeling of knowledge-based systems, or where heuristic evaluations are required [8].

Solving complex problems defined on the basis of Multi-Objective models requires applying new methods of structural modeling and formal description. Analysis of structural modeling and formal description methods highlights membrane calculus (P-Systems). Membrane computing is a new paradigm in nature-inspired (biology) data interpretation and processing methodology in which distributed and parallel computing are modeled. At the same time, other computing models inspired by nature can
be mentioned such as: evolutionary computing, molecular computing and neural computing, which can supplement the functionality of membrane computing [9, 10].

For the first time the idea of Membrane Computing was proposed by Gh. Paun [11, 12], who was inspired by the structure and behavior of the living cell. In Membrane Computing systems, the membrane is a separator that structures the evolutionary architecture of the rule-based computing system.

This paper proposes the design and research of a distributed decision-making system defined Multi-Objective oriented towards the search for the optimal solution based on the genetic algorithm. The stated objectives are achieved by identifying the decision-making system based on a Multi-Agent system [13] defined in a Multi-Objective optimization space [3, 5, 6]. The Multi-Agent system integrates a network of sensors and actuators [14] that act on the basis of collective intelligence models [15].

II. MULTI-OBJECTIVE PROBLEM FOR OPTIMAL SOLUTION SEARCH

Let be defined the Multi-Objective problem which describes the process dynamics \( P(X) \in R^S \). The search for the optimal (minimal or maximal) Multi-Objective solution is characteristic for practically all fields of human activity (science, technology and economy) [8].

In this case, the Multi-Objective optimization problem can be defined based on the mathematical model (1):

\[
F(X) \rightarrow extr \quad \forall X \in Q,
\]

where: \( F(X) = \{ f_1(X), f_2(X), ..., f_s(X) \} \) - are the set of target functions defined for each dimension \( S \) of the process \( P(X) \);

\( extr \in \{\text{min, max}\} \) - the set of solutions of the target functions defined in the space \( Q \);

\( F(X) = F_{\text{min}}(X) \cup F_{\text{max}}(X) \), - mode of formation the set of target functions, where:

\( F_{\text{min}}(X) \rightarrow \min_{X \in Q} \) - the set of target functions that ensure the condition of searching the minimum solution;

\( F_{\text{max}}(X) \rightarrow \max_{X \in Q} \) - the set of target functions that ensure the condition of searching the maximum solution.

The minimum/maximum value search condition will be considered to be met if the values have been identified for \( X_{\text{min}} \in Q \) which ensures \( F_{\text{min}}(X_{\text{min}}) \leq F(X), \forall X \in Q \), or if the values have been identified for \( X_{\text{max}} \in Q \) which ensures \( F_{\text{max}}(X_{\text{max}}) \geq F(X), \forall X \in Q \).

Thus the Multi-Objective search process will be defined based on the model (2):

\[
\begin{align*}
F_{\text{min}}(X_{\text{min}}^*) &= \min_{X \in Q} (F(X)), \\
F_{\text{max}}(X_{\text{max}}^*) &= \max_{X \in Q} (F(X)).
\end{align*}
\]

III. SYNTHESIS OF THE P-SYSTEMS MODEL

Multi-objective models have been observed to be specific for complex systems that consist of a spatially distributed structure of objects, mobile objects, or a combination of these. In the present paper, solving the problem of searching the optimal solution is carried out based on a set of Agents that respect the structure of the complex system. Figure 1 shows the diagram of an Object that includes: an Agent which activates in the environment Environment, environment perception system Perceiver, and the action system on the environment Actuator.

![Object diagram](image)

The Venn diagram of the P-Systems model synthesis result is shown in Figure 2.

![Venn diagram of the P-Systems](image)
- The multitude of complex membranes \( Membrane_i, \forall i = 1, \bar{N} \) with the rule set \( R_i, \forall i = 1, \bar{N} \);
- The multitude of elementary membranes \( M_{ij}, \forall i = 1, \bar{N}, \forall j = 1, \bar{K} \) with the rule set \( R_{ij}, \forall i = 1, \bar{N}, \forall j = 1, \bar{K} \).

Functionally, each complex membrane and elementary membrane presents an Agent that implements the set of rules defined for it. In turn, each complex membrane \( Membrane_i, \forall i = 1, \bar{N} \) groups the multitude of membranes \( M_{ij}, \forall i = 1, \bar{N}, \forall j = 1, \bar{K} \) in computational clusters, respectively, consolidating computing power in order to solve complex problems. This consolidation leads to the implementation of collective computing models [18, 19] made by the set of Agents involved in the process.

IV. EXAMPLE FOR MULTI-OBJECTIVE OPTIMIZATION

As an example of the application of the Multi-Objective optimal solution searching method, we will analyze a Multi-Agent system for controlling electricity consumption in the artificial lighting process [22].

Let be defined in the space \( \mathcal{Q} \subset \mathbb{R}^4 \) artificial lighting process \( P = F(t, X^M(t), X^S(t), U(t)) \), where: \( t \) - the evolution of the process over time; \( X^M(t) = (x^M(t), \forall i = 1, \bar{S}) \) - the set of motion sensors that identify the presence of the person in the space controlled by that Agent; \( S \) - the number of Agents to control the lighting process; \( X^S(t) = (x^S(t), \forall i = 1, \bar{S}) \) - the set of sensors for identifying the intensity of the light flow in the space controlled by that Agent; \( U(t) = (u_i(t), \forall i = 1, \bar{S}) \) - the multitude of command signals with the artificial lighting process; \( F = (f_i, \forall i = 1, \bar{S}) \) - the set of functions to ensure the condition defined in the model (1). The multitude of the functions \( F \) ensures control signals calculation \( U(t) \):

\[
F : X(t) \rightarrow U(t),
\]

where \( X(t) = X^M(t) \cup X^S(t) \) and \( X^M(t) \cap X^S(t) = \emptyset \).

The energy consumed by an artificial lighting device \( W_i(t) \) in the time interval \( t \in [0; T] \) is calculated based on the formula (4):

\[
W_i(t) = \int_{t=0}^{T} (w_i(t), u_i(t), \kappa_i) dt,
\]

where: \( w_i(t) \) - the control signal obtained as a result of the calculations in the model; \( u_i(t) \) - the control signal obtained as a result of the calculations in the model (3); \( \kappa_i \) - the coordination coefficient.

To optimize the energy consumption, the Multi-Objective optimization problem is defined (5):

\[
\begin{align*}
F_{\min} (X^T_{\min}) &= \min_{x \in \Omega} \{ F(X^M(t)) \}, \\
F_{\max} (X^T_{max}) &= \max_{x \in \Omega} \{ F(X^S(t)) \},
\end{align*}
\]

where: \( X^T_{\min} \) - are the values that ensure the minimum electricity consumption obtained as a result of the solution \( \min_{x \in \Omega} \{ F(X^M(t)) \} \); \( X^T_{\max} \) - are the values that ensure the artificial lighting process of a satisfactory quality obtained as a result of the solution \( \max_{x \in \Omega} \{ F(X^S(t)) \} \).

Functional verification of the Multi-Agent system for Multi-Objective optimization of energy consumption used for artificial lighting was carried out based on ESP-12E devices (Figure 3). For data acquisition was used sensors: PIR Sensor – identification of the presence of people in the control area and a light sensor. The action element is made in the form of a relay that connects the 220V power source to the artificial lighting Lamp.

![Figure 3. Functional diagram of the Agent.](image)

The topology of the collective computing network is developed based on the P-Systems model and is presented in Figure 4. The collective computing network consists of wired LAN, and N Collective computing clusters consisting of Agents that are in the control space of the artificial lighting process. Communication between Agents is realized on the basis of Wi-Fi Router which are connected to the network LAN.
V. GENETIC ALGORITHM FOR MULTI-OBJECTIVE OPTIMIZATION

The process of searching for the optimal solution is based on the application of a Multi-Objective Genetic Algorithm. For this purpose will be created the population $A$ consisting of the multitude of Agents $A_{i,j}$, $\forall i \in N \& j = 1, K$ in which each Agent will be assigned a Chromosome $G_{i,j}$, $\forall i \in N \& j = 1, K$ made of $K$ Genes $G_{i,j} = \{ g_{i,j,k}, \forall i \in N \& j = 1, K \& k = 1, K \}$. [21].

Each gene $g_{i,j,k}$ each gene can receive one of the values $\{00, 01, 10, 11\}$, where:

- 00 - passive state of the Agent $A_{i,j}$ in the Multi-Agent system architecture;
- 01 - identifies the position of the Agent $A_{i,j}$ in the Multi-Agent system architecture;
- 10 - identifies the direction of movement of the person in the space controlled by the Multi-Agent system;
- 11 - determines the active state of the Agent $A_{i,j}$ which identified the person’s presence in the controlled space.

Examples of Chromosomes for the Agent $A_{i,j}$ from the architecture topology shown in Figure 4:

- $A_{i,j} = [00 00 01 00 00 00 00 00]$ - identifies position 3 of the Agent in the Multi-Agent system architecture;
- $A_{i,j} = [00 00 11 10 00 00 00 00]$ - identifies the direction of movement of the person in the space controlled by the Agent $A_{i,j}$ in the space controlled by the Agent $A_{i,j}$;
- $A_{i,j} = [00 10 11 00 00 00 00 00]$ - identifies the direction of movement of the person in the space controlled by the Agent $A_{i,j}$ in the space controlled by the Agent $A_{i,j}$.

The sequence of operations of the process of searching for the optimal solution is shown in Figure 5.

![Functional diagram of the Multi-Agent collective computing.](image)

![The sequence of operations of the optimal solution search process.](image)

The sequence of operations includes:

Model-defined population initiation (5):

$$
A^0 = A_i^{0,0} = \begin{bmatrix}
\tilde{g}_{i,1,1,1} & \tilde{g}_{i,1,1,2} & \cdots & \tilde{g}_{i,1,1,K}
\end{bmatrix}, \quad A_i^{0,0} = \begin{bmatrix}
\tilde{g}_{i,2,1} & \tilde{g}_{i,2,2} & \cdots & \tilde{g}_{i,2,K}
\end{bmatrix}, \\
\vdots & \vdots & \ddots & \vdots \\
A_i^{0,K} = \begin{bmatrix}
\tilde{g}_{i,K,1} & \tilde{g}_{i,K,2} & \cdots & \tilde{g}_{i,K,K}
\end{bmatrix},
\forall i \in \overline{1,N}
$$
**Fitness Assignment** - The fitness function determines how fit an individual is. It gives a fitness score to each individual. The probability that an individual will be selected for control decision;

**Selection** - The idea of selection phase is to select the fittest individuals and let them pass their genes to the next generation. Two pairs of individuals (parents) are selected based on their fitness scores. Individuals with high fitness have more chance to be selected for control operation;

**Crossover** - Crossover is the most significant phase in a genetic algorithm. For each pair of parents to be mated, a crossover point is chosen at random from within the genes;

**Mutation** - In certain new offspring formed, some of their genes can be subjected to a mutation with a low random probability. This implies that some of the bits in the bit string can be flipped;

**Optimal Solution Computing** - Application of the mathematical model for the calculation of the optimization condition.

**Opt?** - The iteration terminates if the population has converged.

**Control** - Calculation and action on the controlled object.

**VI. CONCLUSION**

Optimization in Multi-Objective processes presents an important direction in the research and development of complex systems. These models are extensively described in numerous scientific papers in which their advantages and disadvantages are highlighted. Along with the advantages offered by the Multi-Objective optimization methods comes the main disadvantage which is expressed by the algorithmically and technological complexity of implementation. These disadvantages impose the need to develop new implementation methods and models based on the use of Genetic Algorithms, new methods of formal description based on P-Systems, the presentation of the system in the form of a Multi-Agent System with Distributed Data Processing and Collective Computing.

In the present work, the research results of a system for searching for the optimal Multi-Objective solution by using Genetic Algorithms are presented. The general problem of the Multi-Objective optimal solution search process was formulated. The synthesis of the topology of the Multi-Objective optimal solution search system was realized in the form of Venn diagram and P-Systems models. For the functional demonstration, an example of controlling the artificial lighting process was analyzed, which provides for the search for the minimum solutions for the consumption of electricity and the search for the maximum solutions for the quality of the lighting process. The process of searching for optimal solutions was carried out based on the Genetic Algorithm that is described in the paper.

Future research is planned in developing a set of operators to perform Fitness, Selection and Crossover functions to reduce algorithmic complexity and implementation costs.

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