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Analytically determining of the relative inaccuracy (error) of indirectly measurable variable and dimensionless scale characterising quality of the experiment *

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Abstract

In the following paper we present an easily applicable new method for analytical representation of the maximum relative inaccuracy (error) of an indirectly measurable variable $f = f(x_1, x_2, ..., x_n)$ as a function of the maximum relative inaccuracies (errors) of the directly measurable variables $x_1, x_2, ..., x_n$. Our new approach is more adequate for the objective reality. The gist of it is that in order to find the analytical form of the maximum relative inaccuracy of the variable f we take for being fixed variables statistical mean values $\overline{\left|\frac{x_1}{f}\cdot\frac{\partial f}{\partial x_1}\right|}, \overline{\left|\frac{x_2}{f}\cdot\frac{\partial f}{\partial x_2}\right|}, ..., \overline{\left|\frac{x_n}{f}\cdot\frac{\partial f}{\partial x_n}\right|}$ of the modules of the coefficients of influence of relative inaccuracies $\frac{\Delta x_1}{x_1}, \frac{\Delta x_2}{x_2}, ..., \frac{\Delta x_n}{x_n}$ in f. The numerical value of the maximum relative inaccuracy of the variable f is found using the statistical mean values of the absolute values of the relative inaccuracies $\overline{\left|\frac{\Delta x_1}{x_1}\right|}, \overline{\left|\frac{\Delta x_2}{x_2}\right|}, ..., \overline{\left|\frac{\Delta x_n}{x_n}\right|}$. Moreover, we look into functions which are continuous but are not differentiable in respect to certain arguments in some points. Having this in mind we develop the theory of errors, which we will call with what we feel is a more precise term – theory of inaccuracies. We introduce some new terms – space of the relative

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inaccuracy and plane of the relative inaccuracy of f. We also define a sample plane of the ideal absolutely accurate experiment and using it we define a universal numerical characteristic – a dimensionless scale for evaluation of the quality (accuracy) of the experiment.

Keywords: Indirectly measurable variable, maximum relative error, dimensionless scale.

1 Introduction

Many natural and social processes are described by indirectly measurable variables dependent on a finite number of directly measurable variables. As it is known, the measuring of the values of each directly measurable variable is accompanied by inaccuracies. With respect to their alteration characteristics, the inaccuracies are systematic or random [1].

The systematic inaccuracies are permanent or their alteration can be described by a law. They are the result of certain constant influences which cannot be foreseen. Towards the random accuracies we shall also account what we call *hidden inaccuracies* that are the result of influences of measured active objects [2] that appear in a time interval and change the natural progress of the observed process (such as objects that have their own sources of energy; living organisms with their own will, etc.).

Thus each measurable variable is determined with a total inaccuracy that is caused by both systematic and random inaccuracies. Therefore, it is of great importance to develop a reliable method for finding the total inaccuracy of given measurable variable.

In respect of the way a certain inaccuracy is represented it can be absolute or relative. An absolute inaccuracy is expressed in units of the measured variable. A relative inaccuracy is a dimensionless variable and is represented by the ratio of the absolute inaccuracy and the value of the measured experimental variable.

Usually the processes that are being studied are mathematically modelled with real functions that are differentiable in their domains.

(Even non-continuous functions can be viewed as differentiable in a certain sense.)

Here is a real differentiable function

$$f = f(x_1, x_2, ..., x_n) \neq 0 \tag{1}$$

of n real independent variables $x_1, x_2, ..., x_n$, which help one to be able to model directly measurable variables (with the help of measuring tools or methods). Then the function f models one indirectly measurable variable.

In order to calculate the absolute and the relative inaccuracies of indirectly measurable variable f which has continuous first partial derivatives in respect to all its variables there are two principles [3, 4, 5] given in scientific and academic literature.

The first principle gives the maximum absolute inaccuracy Δf of the function f. Initially we have to determine the full differential

$$df = \sum_{i=1}^{n} \frac{\partial f}{\partial x_i} dx_i \tag{2}$$

of the function (1). When the inaccuracies of the measurements are small enough in formula (2) the differential d can be replaced with the finite difference Δ, and in this substitution every minus is replaced with a plus to reach the maximum value of the inaccuracy. Thus one obtains

$$\Delta f = \sum_{i=1}^{n} \left| \frac{\partial f}{\partial x_i} \right| \cdot |\Delta x_i| \,, \tag{3}$$

where Δx_i is the maximum absolute inaccuracy of the directly measureable variable $x_i (i = 1, 2, ..., n)$. The maximum relative inaccuracy f_r is then determined by the expression

$$f_r = \frac{\Delta f}{|f|} = \frac{1}{|f|} \sum_{i=1}^{n} \left| \frac{\partial f}{\partial x_i} \right| \cdot |\Delta x_i| \,. \tag{4}$$

By the second principle, we initially find the maximum relative inaccuracy f_r of the function f. In order to do this, we find the logarithm of the function (1) and then we determine full differential of the result. Further we replace the differential d in the same way with the finite difference Δ and again replace every minus with a plus to reach the maximum value of the inaccuracy. Thus, we get the maximum relative inaccuracy f_r. Then the maximum absolute inaccuracy Δf is determined from the expression $\Delta f = f \cdot f_r$.

Since in the set of real numbers we can find only the logarithms of positive variables, the second principle limits the class of functions for which we can find the respective inaccuracies.

In [1], based solely on differentiation we define a simple method for representing the maximum absolute (total) inaccuracy of an indirectly measurable variable $f(x_1, x_2, ..., x_n)$ as a function of the maximum absolute (total) inaccuracies of the directly measurable variables $x_1, x_2, ..., x_n$. Based on this method we introduce a numerical characteristic – dimensionless scale for evaluation of the quality (accuracy) of the experiment.

The purpose of this paper is to apply this procedure for the maximum relative inaccuracy of $f(x_1, x_2, ..., x_n)$. Moreover, we extend the type of the function f by also looking into the case when it is not differentiable but is continuous in respect to some arguments in some points.

2 Analytic representation of the maximum relative inaccuracy of an indirectly measurable variable

Firstly, let the function f has continuous partial derivatives in respect to all its variables. Let us present formula (4) in this way

$$f_r = \sum_{i=1}^{n} \left| \frac{x_i}{f} \cdot \frac{\partial f}{\partial x_i} \right| \cdot \left| \frac{\Delta x_i}{x_i} \right| \quad (x_i \neq 0, i = 1, 2, ..., n). \tag{5}$$

This shows that evaluation of the relative inaccuracy of the indi-

rectly measurable variable $f(x_1, x_2, ..., x_n)$ is dependant not only on the relative inaccuracies with which the directly measureable variables $x_1, x_2, ..., x_n$ are determined, but also on the analytical form of the function f itself. For $i = 1, 2, ..., n$ every addend of the form $\left|\frac{x_i}{f} \cdot \frac{\partial f}{\partial x_i}\right| \cdot \left|\frac{\Delta x_i}{x_i}\right|$ is the partial relative inaccuracy of the result of the indirect measurement of the function f, caused by the inaccuracy $\frac{\Delta x_i}{x_i}$ with which the variable x_i is determined.

The variable $\left|\frac{x_i}{f} \cdot \frac{\partial f}{\partial x_i}\right|$ is in fact the coefficient of influence of the inaccuracy $\frac{\Delta x_i}{x_i}$ when determining the relative inaccuracy of $f(x_1, x_2, ..., x_n)$.

Let us now assume the function $f(x_1, x_2, ..., x_n)$ is not differentiable in respect to its argument x_k $(1 \leq k \leq n)$ in some points a_{kj}, but is continuous in respect to this argument in a_{kj}. The partial derivative $\frac{\partial f}{\partial x_k}$ does not exist in the points a_{kj}.

However, if for f there are right and left derivatives given $x_k \to a_{kj}$, then in order to compute the maximum relative inaccuracy in the formula (5) the variable $\frac{\partial f}{\partial x_k}$ is replaced by the one of the two limits in which the coefficient of influence $\left|\frac{x_k}{f} \cdot \frac{\partial f}{\partial x_k}\right|$ has greater value. (The values of the two limits are different, otherwise f would be differentiable in respect to x_k). An example for such function would be $f = A \arcsin(\sin x)$ representing the voltage of triangle signal [6]. In Figure 1 we have shown the graphics of the function when $A = 500$ V and $x = 2\pi v t \in \left[0, \frac{13}{4}\pi\right]$, where $v = 15$ kHz is the linear frequency and t is the time.

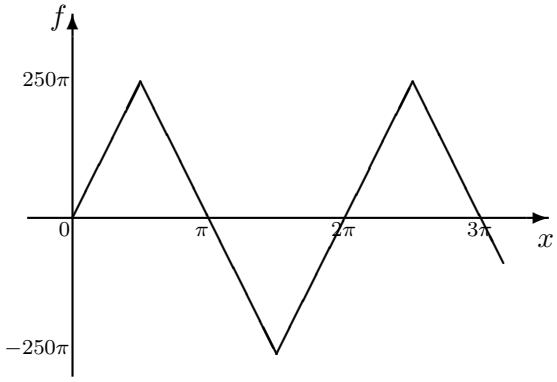

Figure 1. Graphics of the function $f = 500 \arcsin(\sin x),\ x \in \left[0, \frac{13}{4}\pi\right]$

We would like to point out that this procedure for indifferentiable but continuous functions in respect to its arguments in some points is proposed for the first time in the theory of inaccuracies.

Moreover, a novelty in our approach for determining the analytical form of the maximum relative (total) inaccuracy of the indirectly measureable variable $f(x_1, x_2, ..., x_n)$ is that in formula (5) we take for fixed variables the mean values $\overline{\left|\frac{x_1}{f} \cdot \frac{\partial f}{\partial x_1}\right|}, \overline{\left|\frac{x_2}{f} \cdot \frac{\partial f}{\partial x_2}\right|}, ..., \overline{\left|\frac{x_n}{f} \cdot \frac{\partial f}{\partial x_n}\right|}$ of the absolute values of the coefficients of influence of the relative inaccuracies $\frac{\Delta x_1}{x_1}, \frac{\Delta x_2}{x_2}, ..., \frac{\Delta x_n}{x_n}$ of the indirectly measurable variable f and the maximum relative inaccuracies $\frac{\Delta x_1}{x_1}, \frac{\Delta x_2}{x_2}, ..., \frac{\Delta x_n}{x_n}$ of the directly measurable variables $x_1, x_2, ..., x_n$ are considered to be variables.

Thus according to (5) *the maximum relative inaccuracy f_r of the indirectly measurable variable f is a linear function of the maximum relative inaccuracies $\frac{\Delta x_1}{x_1}, \frac{\Delta x_2}{x_2}, ..., \frac{\Delta x_n}{x_n}$ of the directly measurable variables $x_1, x_2, ..., x_n$.*

If we look at $\frac{\Delta x_1}{x_1}, \frac{\Delta x_2}{x_2}, ..., \frac{\Delta x_n}{x_n}, \pm f_r$ as a system of generalized orthogonal coordinates, for $n > 2$ we get an $n+1$-dimensional metric hyperspace F_r^{n+1}, where (5) is the equation of a hyperplane that passes through the origin of the coordinate system. The hyperspace F_r^{n+1} will

be called *space of the relative inaccuracy of* f, and f_r will be called *plane of the relative inaccuracy of* f.

For $n = 2$, according to formula (5) we have

$$f_r = \overline{\left|\frac{x_1}{f} \cdot \frac{\partial f}{\partial x_1}\right|} \cdot \left|\frac{\Delta x_1}{x_1}\right| + \overline{\left|\frac{x_2}{f} \cdot \frac{\partial f}{\partial x_2}\right|} \cdot \left|\frac{\Delta x_2}{x_2}\right|.$$

Thus the equation $f_r = f_r\left(\frac{\Delta x_1}{x_1}, \frac{\Delta x_2}{x_2}\right)$ is an equation of the plane of the relative inaccuracy in the three-dimensional metric space F_r^3 of the relative inaccuracy of f.

For $n = 1$ formula (5) becomes

$$f_r = \overline{\left|\frac{x}{f} \cdot \frac{\partial f}{\partial x}\right|} \cdot \left|\frac{\Delta x}{x}\right|.$$

Thus the equation $f_r = f_r\left(\frac{\Delta x}{x}\right)$ is an equation of the line of the relative inaccuracy in the two-dimensional metric space F_r^2 of the relative inaccuracy of f.

The term space of the relative inaccuracy is introduced for the first time in this paper. Moreover, as we pointed in [1], in a certain sense it is an analogy of the imaginary configurative space in the Hamiltonian reformulation of the classical mechanics [7]. In that same sense the system $\frac{\Delta x_1}{x_1}, \frac{\Delta x_2}{x_2}, ..., \frac{\Delta x_n}{x_n}, \pm f_r$ can be viewed as generalised orthogonal coordinates.

3 Determining the numerical value of the maximum relative inaccuracy of an indirectly measurable variable

Let us have an experiment where k measurements of the directly measureable variables $x_1, x_2, ..., x_n$ are made. On the m-th measurement ($m = 1, 2, ..., k$) the absolute values of the coefficients of influence

$\left|\frac{x_1}{f}\cdot\frac{\partial f}{\partial x_1}\right|_m, \left|\frac{x_2}{f}\cdot\frac{\partial f}{\partial x_2}\right|_m, ..., \left|\frac{x_n}{f}\cdot\frac{\partial f}{\partial x_n}\right|_m$ and of the relative inaccuracies $\left|\frac{\Delta x_1}{x_1}\right|_m, \left|\frac{\Delta x_2}{x_2}\right|_m, ..., \left|\frac{\Delta x_n}{x_n}\right|_m$ are calculated. After this the mean values $\overline{\left|\frac{x_j}{f}\cdot\frac{\partial f}{\partial x_j}\right|} = \frac{1}{k}\sum_{m=1}^{k}\left|\frac{x_j}{f}\cdot\frac{\partial f}{\partial x_j}\right|_m$ $(j = 1, 2, ..., n)$ are calculated and from formula (5) one can get the analytical representation (equation) of the plane of the inaccuracies.

Furthermore, if $\overline{\left|\frac{\Delta x_i}{x_i}\right|} = \frac{1}{k}\sum_{m=1}^{k}\left|\frac{\Delta x_i}{x_i}\right|_m$, then according to formula (5) the numerical value of the maximum relative inaccuracy

$$f_r = \sum_{i=1}^{n}\overline{\left|\frac{x_i}{f}\cdot\frac{\partial f}{\partial x_i}\right|}\cdot\overline{\left|\frac{\Delta x_i}{x_i}\right|}$$

is determined, as the point $\left(\overline{\left|\frac{\Delta x_1}{x_1}\right|}, \overline{\left|\frac{\Delta x_2}{x_2}\right|}, ..., \overline{\left|\frac{\Delta x_n}{x_n}\right|}, f_r\right)$ lies in the plane of the relative inaccuracy.

The function $f(x_1, x_2, ..., x_n)$ can be considered as a random variable of random independent variables. In that sense the suggested by us method for computing f_r is more adequate to the objective reality because the statistical mean value of a random variable is actually its most probable value. Again in that sense the plane of the relative inaccuracy of f is a *stochastic plane*.

The numerical value of the maximum absolute inaccuracy Δf of the experiment can be determined directly using [1] or using formula (4) and the known numerical value of f_r.

If the function $f(x_1, x_2, ..., x_n)$ is not differentiable in respect to an argument x_k $(1 \leq k \leq n)$ in some points a_{kj}, but is continuous in respect to this argument in a_{kj}, then the method is applied to the maximum absolute inaccuracy analogically to the already described way.

4 Scale characterising the quality of the experiment

It is very important and advantageous for every measuring method to have a numerical characteristic – a scale which is used to evaluate the quality of the experiment, i. e. its accuracy. For the first time in the theory of the inaccuracies we suggested this kind of scale in [1]. Here we suggest analogical scale which has the important property *dimensionless*, i. e. the quality of the experiment is expressed only with a number, not with the units of measurements.

Let us look at the stochastic plane α of the relative inaccuracy of f. According to (5) its general equation is of the following type

$$\alpha : \sum_{i=1}^{n} A_i \cdot \left| \frac{\Delta x_i}{x_i} \right| - f_r = 0,$$

where $A_i = \overline{\left| \frac{x_i}{f} \cdot \frac{\partial f}{\partial x_i} \right|} = const \geq 0$. As we have already emphasised, this is the equation of a hyperplane in the hyperspace F_r^{n+1} going through the beginning of the coordinate system.

Let us also take a look at the hyperplane

$$\varepsilon : f_r = 0.$$

It is obvious that the equation $f_r = 0$ is possible if and only if $\frac{x_1}{f} \cdot \frac{\partial f}{\partial x_1} = \frac{x_2}{f} \cdot \frac{\partial f}{\partial x_2} = ... = \frac{x_n}{f} \cdot \frac{\partial f}{\partial x_n} = 0$, i. e. if and only if $\frac{\partial f}{\partial x_1} = \frac{\partial f}{\partial x_2} = ... = \frac{\partial f}{\partial x_n} = 0$. Thus we take ε for *sample plan in the space of the relative inaccuracy* which represents an imaginary ideal perfectly accurate experiment even though this experiment is impossible and the sample plane ε is unreachable. However, by increasing the accuracy of the real experiment the plane α approximates ε. Thus the smaller the deviation of the plane α of the experiment from the sample plane ε of the ideal experiment is, i. e. the smaller the angle between these two planes is, the more accurate the experiment is. This angle

can be always calculated as it is equal to the angle between the normal vectors $\overrightarrow{n_\alpha}(A_1, A_2, ..., A_n, -1)$ of the plane α and $\overrightarrow{n_\varepsilon}(0, 0, ..., 0, -1)$ of the plane ε. Then the value of the cosine

$$k_\alpha = \cos\angle(\overrightarrow{n_\alpha}, \overrightarrow{n_\varepsilon}) = \frac{1}{\sqrt{A_1^2 + A_2^2 + ... + A_n^2 + 1}} \tag{6}$$

of this angle can be chosen for a *coefficient of accuracy* in a *dimensionless scale*, i. e. for a *numerical characteristic of the quality of the experiment.*

The scale for evaluating the quality of the experiment is the interval $[0, 1]$. An experiment is as accurate as the value of the coefficient of accuracy k_α is closer to 1 and is as inaccurate as the value of the coefficient of accuracy k_α is closer to 0. The value $k_\alpha = 1$ represents the ideal perfectly accurate experiment and the value $k_\alpha = 0$ – the ideal absolutely inaccurate experiment.

Then from formula (6) we get the following

Criterion for accuracy of an experiment – *An experiment is as accurate as possible if and only if the sum of the squares of the coefficients* $A_1^2 + A_2^2 + ... + A_n^2 = \left(\overline{\frac{x_1}{f} \cdot \frac{\partial f}{\partial x_1}}\right)^2 + \left(\overline{\frac{x_2}{f} \cdot \frac{\partial f}{\partial x_2}}\right)^2 + ... + \left(\overline{\frac{x_n}{f} \cdot \frac{\partial f}{\partial x_n}}\right)^2$ *is the least possible.*

The accuracy of the experiment can, of course, be interpreted using the angle between the stochastic plane α of the relative inaccuracy and the sample plane ε. Then the scale for evaluating the quality of the experiment is the interval $\left[0, \frac{\pi}{2}\right]$. An experiment is as accurate as the value of $\arccos k_\alpha$ is closer to 0, and is as inaccurate as the value of $\arccos k_\alpha$ is closer to $\frac{\pi}{2}$. The value 0 corresponds to the ideal perfectly accurate experiment and the value $\frac{\pi}{2}$ – to the ideal absolutely inaccurate experiment.

We take for basic scale the interval $[0, 1]$ and for basic measurement of the accuracy of the experiment the value of the coefficient of accuracy $k_\alpha = \cos\angle(\overrightarrow{n_\alpha}, \overrightarrow{n_\varepsilon})$ since both of them are dimensionless variables contrary to the second scale and measurement which are measured in radians.

5 An example and computations

We will present an example which illustrates our method and scale. It is known that the coefficient η of the viscosity of a liquid with density ρ can be determined using the Stokes' method when a sphere with radius r and density ρ_1 is put in a cylindrical container filled in with the examined liquid. In a given moment of time the sphere starts to descent steadily with a constant speed ν and its weight is the same as the buoyancy and the force of internal friction (viscosity) of the liquid as the following holds $\eta = \dfrac{2r^2 g}{9\nu}(\rho_1 - \rho)$, where $g = 9.8$ m·s^{-2} is the gravity of Earth.

We made an experiment for measuring the viscosity of glycerine (under temperature $t = 18^o$ C) using lead spheres having measured the density of the glycerine and the lead in advance. In Table 1 experimental data from the measurements and the corresponding calculated values of the viscosity with accuracy of four digits in the decimal part are given.

Table 1. Experimental data from the measurements of the viscosity of glycerine using the Stokes' method.

m-th measurement	r_m[m]	ν_m[m·s^{-1}]	ρ_m[kg·m^{-3}]	ρ_{1m}[kg·m^{-3}]	η_m[Pa·s]
1	5×10^{-4}	4.54×10^{-3}	1.262×10^3	1.1341×10^4	1.2087
2	5×10^{-4}	4.52×10^{-3}	1.262×10^3	1.1341×10^4	1.214
3	5×10^{-4}	4.5×10^{-3}	1.262×10^3	1.1341×10^4	1.2194
4	4.8×10^{-4}	4.22×10^{-3}	1.26×10^3	1.134×10^4	1.1944
5	4.8×10^{-4}	4.2×10^{-3}	1.26×10^3	1.134×10^4	1.2003

5.1 Computing the maximum relative inaccuracy η_r, using the classical method

According to (4) we have

$$\eta_r = \left|\frac{2}{r}\right| \cdot |\Delta r| + \left|\frac{1}{\nu}\right| \cdot |\Delta\nu| + \left|\frac{1}{\rho_1 - \rho}\right| \cdot |\Delta\rho| + \left|\frac{1}{\rho_1 - \rho}\right| \cdot |\Delta\rho_1|.$$

We firstly compute the mean values

$$\overline{r} = \frac{1}{5}\sum_{m=1}^{5} r_m = 4.92 \times 10^{-4} \text{ m},$$

$$\overline{\nu} = \frac{1}{5}\sum_{m=1}^{5} \nu_m = 4.396 \times 10^{-3} \text{ m} \cdot \text{s}^{-1},$$

$$\overline{\rho} = \frac{1}{5}\sum_{m=1}^{5} \rho_m = 1.2612 \times 10^{3} \text{ kg} \cdot \text{m}^{-3},$$

$$\overline{\rho_1} = \frac{1}{5}\sum_{m=1}^{5} \rho_{1m} = 1.13406 \times 10^{4} \text{ kg} \cdot \text{m}^{-3}.$$

Then $|\Delta r|_1 = |r_1 - \overline{r}| = 0.08 \times 10^{-4}$ m, $|\Delta\nu|_1 = |\nu_1 - \overline{\nu}| = 0.144 \times 10^{-3}$ m·s^{-1}, $|\Delta\rho|_1 = |\rho_1 - \overline{\rho}| = 0.0008 \times 10^{3}$ kg·m^{-3}, $|\Delta\rho_1|_1 = |\rho_{11} - \overline{\rho_1}| = 0.00004 \times 10^{4}$ kg·m^{-3} and thus we get the value of

$$\eta_{r,1} = \frac{2}{r_1}\cdot|\Delta r|_1 + \frac{1}{\nu_1}\cdot|\Delta\nu|_1 + \frac{1}{\rho_{11} - \rho_1}\cdot|\Delta\rho|_1 + \frac{1}{\rho_{11} - \rho_1}\cdot|\Delta\rho_1|_1 = 0.0779.$$

Analogically, we compute $\eta_{r,2} = 0.0737$, $\eta_{r,3} = 0.0694$, $\eta_{r,4} = 0.1172$, $\eta_{r,5} = 0.1224$. We finally get $\eta_r = \frac{1}{5}\sum_{m=1}^{5} \eta_{r,m} = 0.0921$.

5.2 Computing the maximum relative inaccuracy η_r, using the method introduced by us

According to Section 2 under the same measurements we compute the absolute values of the coefficients of influence $\left|\frac{r}{\eta}\cdot\frac{\partial\eta}{\partial r}\right| = 2$, $\left|\frac{\nu}{\eta}\cdot\frac{\partial\eta}{\partial\nu}\right| = 1$, $\left|\frac{\rho}{\eta}\cdot\frac{\partial\eta}{\partial\rho}\right|_m = \frac{\rho_m}{\rho_{1m}-\rho_m}$, $\left|\frac{\rho_1}{\eta}\cdot\frac{\partial\eta}{\partial\rho_1}\right|_m = \frac{\rho_{1m}}{\rho_{1m}-\rho_m}$ and the ones of the relative inaccuracies $\left|\frac{\Delta r}{r}\right|_m, \left|\frac{\Delta\nu}{\nu}\right|_m, \left|\frac{\Delta\rho}{\rho}\right|_m, \left|\frac{\Delta\rho_1}{\rho_1}\right|_m$. The results can be seen in Table 2.

Table 2. The values of the non-constant coefficients of influence and the relative inaccuracies of the variables in the experiment

m-th measurement	$\left\|\frac{\rho}{\eta}\cdot\frac{\partial\eta}{\partial\rho}\right\|_m$	$\left\|\frac{\rho_1}{\eta}\cdot\frac{\partial\eta}{\partial\rho_1}\right\|_m$	$\left\|\frac{\Delta r}{r}\right\|_m$	$\left\|\frac{\Delta\nu}{\nu}\right\|_m$	$\left\|\frac{\Delta\rho}{\rho}\right\|_m$	$\left\|\frac{\Delta\rho_1}{\rho_1}\right\|_m$
1	0.1252	1.1252	0.016	0.0317	6.339×10^{-4}	3.52×10^{-5}
2	0.1252	1.1252	0.016	0.0274	6.339×10^{-4}	3.52×10^{-5}
3	0.1252	1.1252	0.016	0.0231	6.339×10^{-4}	3.52×10^{-5}
4	0.125	1.125	0.025	0.0417	9.523×10^{-4}	5.29×10^{-5}
5	0.125	1.125	0.025	0.0467	9.523×10^{-4}	5.29×10^{-5}

We compute the statistical mean values

$$\overline{\left|\frac{\rho}{\eta}\cdot\frac{\partial\eta}{\partial\rho}\right|} = \frac{1}{5}\sum_{m=1}^{5}\left|\frac{\rho}{\eta}\cdot\frac{\partial\eta}{\partial\rho}\right|_m = 0.1251,$$

$$\overline{\left|\frac{\rho_1}{\eta}\cdot\frac{\partial\eta}{\partial\rho_1}\right|} = \frac{1}{5}\sum_{m=1}^{5}\left|\frac{\rho_1}{\eta}\cdot\frac{\partial\eta}{\partial\rho_1}\right|_m = 1.1251.$$

Then according to (5) the analytical form of η_r is

$$\eta_r = 2\left|\frac{\Delta r}{r}\right| + \left|\frac{\Delta\nu}{\nu}\right| + 0.1251\left|\frac{\Delta\rho}{\rho}\right| + 1.1251\left|\frac{\Delta\rho_1}{\rho_1}\right|.$$

Following Section 3 we compute the statistical mean values $\overline{\left|\frac{\Delta r}{r}\right|} = \frac{1}{5}\sum_{m=1}^{5}\left|\frac{\Delta r}{r}\right|_m = 0.0196$, $\overline{\left|\frac{\Delta \nu}{\nu}\right|} = \frac{1}{5}\sum_{m=1}^{5}\left|\frac{\Delta \nu}{\nu}\right|_m = 0.0341$, $\overline{\left|\frac{\Delta \rho}{\rho}\right|} = \frac{1}{5}\sum_{m=1}^{5}\left|\frac{\Delta \rho}{\rho}\right|_m = 7.6126 \times 10^{-4}$ and $\overline{\left|\frac{\Delta \rho_1}{\rho_1}\right|} = \frac{1}{5}\sum_{m=1}^{5}\left|\frac{\Delta \rho_1}{\rho_1}\right|_m = 4.228 \times 10^{-5}$. For numerical value of the maximum relative inaccuracy we get $\eta_r = 2\overline{\left|\frac{\Delta r}{r}\right|} + \overline{\left|\frac{\Delta \nu}{\nu}\right|} + 0.1251\overline{\left|\frac{\Delta \rho}{\rho}\right|} + 1.1251\overline{\left|\frac{\Delta \rho_1}{\rho_1}\right|} = 0.0734$.

We can see that there is a certain difference between the classic method and our method which gives more adequate to the reality results.

Furthermore, let $A_1 = \overline{\left|\frac{\partial \eta}{\partial r}\right|} = 2$, $A_2 = \overline{\left|\frac{\partial \eta}{\partial \nu}\right|} = 1$, $A_3 = \overline{\left|\frac{\partial \eta}{\partial \rho}\right|} = 0.1251$ and $A_4 = \overline{\left|\frac{\partial \eta}{\partial \rho_1}\right|} = 1.1251$. According to Section 4 the stochastic plane α of the relative inaccuracy of η in the space $\left(\frac{\Delta r}{r}, \frac{\Delta \nu}{\nu}, \frac{\Delta \rho}{\rho}, \frac{\Delta \rho_1}{\rho_1}, \pm\eta_r\right)$ of the relative inaccuracy has general equation $\alpha : A_1\left|\frac{\Delta r}{r}\right| + A_2\left|\frac{\Delta \nu}{\nu}\right| + A_3\left|\frac{\Delta \rho}{\rho}\right| + A_4\left|\frac{\Delta \rho_1}{\rho_1}\right| - \eta_r = 0$, i. e.

$$\alpha : 2\left|\frac{\Delta r}{r}\right| + \left|\frac{\Delta \nu}{\nu}\right| + 0.1251\left|\frac{\Delta \rho}{\rho}\right| + 1.1251\left|\frac{\Delta \rho_1}{\rho_1}\right| - \eta_r = 0.$$

According to (6) the value of the coefficient of accuracy of the experiment is

$$k_\alpha = \cos\angle(\overrightarrow{n_\alpha}, \overrightarrow{n_\varepsilon}) = \frac{1}{\sqrt{A_1^2 + A_2^2 + A_3^2 + A_4^2 + 1}} = 0.3706.$$

Conclusion 1. The experiment is not very accurate because the coefficient of accuracy k_α is closer to 0, not 1. It is important to point out that this does not necessarily mean that the experimental data for the directly measurable variables are very inaccurate. But when these conditions are met, small alterations of the values of the variables $\frac{\Delta \rho}{\rho}$

and $\dfrac{\Delta\rho_1}{\rho_1}$ lead to substantial alterations of the values of the function $\eta_r = \eta_r\left(\dfrac{\Delta r}{r}, \dfrac{\Delta\nu}{\nu}, \dfrac{\Delta\rho}{\rho}, \dfrac{\Delta\rho_1}{\rho_1}\right)$.

Conclusion 2. The Criterion for accuracy from Section 4 gives conditions under which the accuracy of the experiment can be increased, namely: to select such values of the directly measureable variables ρ and ρ_1, under which the values of the coefficients of influence $\dfrac{\rho}{\eta}\cdot\dfrac{\partial\eta}{\partial\rho} = \dfrac{\rho}{\rho_1-\rho}$ and $\dfrac{\rho_1}{\eta}\cdot\dfrac{\partial\eta}{\partial\rho_1} = \dfrac{\rho_1}{\rho_1-\rho}$ are smaller, i. e. the accuracy of the experiment can be increased if the sphere which is put in the glycerine is with higher density. It is important to point out that if that condition was met, this does not mean that the experimental data for ρ and ρ_1 were going to be more accurate, but that small alterations of the values of the variables $\dfrac{\Delta\rho}{\rho}$ and $\dfrac{\Delta\rho_1}{\rho_1}$ would lead to small alterations of the values of the function $\eta_r = \eta_r\left(\dfrac{\Delta r}{r}, \dfrac{\Delta\nu}{\nu}, \dfrac{\Delta\rho}{\rho}, \dfrac{\Delta\rho_1}{\rho_1}\right)$.

6 Discussion

The advantages of the presented in this paper method for analytically representing the maximum relative inaccuracy of an indirectly measurable variable and for computing its value can be summarised in the following basic directions.

(i) *More adequate* to the objective reality *quantity value* of the maximum relative inaccuracy of the indirectly measureable variable.

(ii) Using the Criterion of the accuracy of the experiment, the method shows *conditions under which the accuracy* of the experiment *is the greatest possible one*.

(iii) *Universality*, because this method can be applied in different scientific fields, in experiments held using various utensils and methods, in mathematical models described even with indifferential functions.

(iv) *Clarity* and *observability* of the results when $n = 1$ or $n = 2$ using a computer generated graphical representation, accordingly, using a line or a plane.

We have to point out that natural processes are described by continuous functions which are either differentiable or indifferentiable in some points, but there are left and right derivatives in these points. However, in mathematical models describing these processes this might not be the case. In these models the method is not applicable.

Each utensil for measurement measures a real and not mathematically modeled variable. The real variables are always finite and no matter how fast (or even explosive) their alteration is, it is never (strictly) jumping but rather smooth in a small enough interval of time. The representations of jumps in the behavior of a function and of an infinite value of a real variable are only theoretical (model). They are reasonable abstractions when describing the objective reality.

Moreover, the dimensionless scale of the quality of an experiment gives an opportunity for:

(i) *Quality evaluation* of the experiment;

(ii) *Comparison between the efficiency* of different experimental methods in one research can be compared. Moreover, the method makes it possible to even compare the efficiency of experimental methods from different scientific fields.

7 Conclusion

While in the classical method the mean arithmetic values of the indirectly measurable variable f_r are used, in our method we use the statistical mean values of the random variables that f is composed of. Thus we get the most probable value for f_r.

Moreover, in practice the maximum relative inaccuracy of a measurable variable finds a much wider application than the maximum absolute relative inaccuracy because it is a dimensionless variable and can be presented in percentages.

The suggested by us method is of great importance for every experimental science – physics, chemistry, biology, medicine, sociology, economics, etc. in which the studied processes are described by differentiable functions. Using it, not only more adequate to the reality numerical value of the maximum relative inaccuracy can be determined

given a certain experiment, but also using the dimensionless scale a quantity evaluation of the quality (accuracy) of the experiment can be given and the conditions, under which this accuracy can be increased, can be determined. The dimensionless scales used in the experimental science have certain advantages compared to the unit ones. An important one is that results from measurements of essentially different variables can be compared.

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