

REGARDING TO THERMOMECHANICS OF IRREVERIBLE PROCESSES

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INTRODUCTION

Deformable solid is a thermodynamic system, which can receive or yield to environment the mechanical work, electrical or magnetical tasks, heat, etc. The state of deformable solid is reflected with the help of thermodynamic variables, such as temperature and spherical tensor. Knowledge of thermodynamic variables equates to recognize the currents state of the deformable solid.

In this paper we study in detail the influence of thermodynamic variables: the stress tensor and temperature, on the thermodynamic processes which occur in the solid under action of exterior forces.

In this way we obtain passage from the formulation of principles of thermodynamics in global variables to local variables.

Through local variables may be described under a unitary form all inhomogeneous thermodynamic processes.

1. THE FIRST PRICIPLE OF THERMODYNAMICS

Mechanical Power of surface and volume forces which acted upon C' in current configuration (K) at t moment $t \in [t_0, t_n]$ is given by relationship

$$P = \int_{A'} \vec{t} \vec{u} dA + \int_{V'} \rho \vec{b} \vec{u} dv, \quad (1)$$

$$\vec{u} = \frac{d}{dt}(\vec{u}),$$

Integrals are taken in current configuration (K).

The caloric power obtained by C' can be written by form

$$Q = \int_{A'} c_i n_i dA + \int_{V'} \rho z dV, \quad (2)$$

where $c_i \in c_i(x_1, x_2, x_3, t)$ is amount of heat received by C' thought conduction per unit area of his boundaries, but $z \in z(x_1, x_2, x_3, t)$ is quantity of heat received by C' thought radiation per unite of mass and time.

The experience shows, that for all thermodynamic processes which bind the initial state at t_1 moment a final state at t_2 moment, where $t_1, t_2 \in [t_0, t_M]$, integral

$$\int_{t_1}^{t_2} (P + Q) dt \quad (3)$$

has the same value, than integrals $\int_{t_1}^{t_2} P dt$ and

$\int_{t_1}^{t_2} Q dt$ depend in general not by only states of

system at t_1 and t_2 moments but the thermodynamics processes which linking them. Plus, is found that (3) integral is proportional with system mass, if this system is homogeneous.

These experimental results involve the existence of some variable of extensive state, called the total system energy, which satisfies the equation

$$\dot{E} = P + Q. \quad (4)$$

The relationship (4) is called the energy balance equation, represents the math express of the first thermodynamic principles.

The difference

$$U = E - K \quad (5)$$

between total energy of system and his kinetic energy

$$K = \frac{1}{2} \int_V \rho \dot{u}_i \dot{u}_i dV \quad (6)$$

Is called the internal energy. As E and K are extensive variables, U should be the same extensive variables, and we can write

$$V = \int_V \rho e v dV, \quad (7)$$

where e is internal energy intensity per unit mass.

Introducing (1), (2) și (5) - (7) în (4) obtain

$$\begin{aligned} & \frac{d}{dt} \int_V \left(\frac{1}{2} \dot{u}_i \dot{u}_i + e \right) \rho dV = \\ & = \int_A \left(t_j^{(\bar{n})} u_j + c_i n_i \right) dA + \\ & + \int_V \rho (b_j \dot{u}_j + z) dV. \end{aligned} \quad (8)$$

One the other hand, from

$$t_i^{(\bar{n})} = t_{ij} n_j, \quad n_j = \cos(\bar{n}, \vec{e}_j)$$

we obtain

$$\bar{t} \vec{u} = t_i^{(\bar{n})} \dot{u}_i = t_{ij} n_j \dot{u}_i. \quad (9)$$

Substituting (9) in (8) and taking into account the equality

$$\begin{aligned} & \frac{d}{dt} \int_V \left(\frac{1}{2} \dot{u}_i \dot{u}_i + e \right) \rho dV = \\ & = \int_V \left(\frac{1}{2} \dot{u}_i \ddot{u}_i + \dot{e} \right) \rho dV \end{aligned}$$

obtain

$$\begin{aligned} \int_V \dot{e} \rho dV & = \int_A \left(t_{ij} n_j \dot{u}_i + c_i n_i \right) dA + \\ & + \int_V [\rho (b_i - \ddot{u}_i) \dot{u}_i + z] dV. \end{aligned} \quad (10)$$

Using in this relation the Gauss-Ostrogradski relation

$$\int_A t_{ij} n_j dA = \int_V \frac{\partial t_{ij}}{\partial x_j} dV = \int_V t_{ij,j} dV$$

will transform the surface integrals in volume integrals.

$$\begin{aligned} \int_A \left(t_{ij} \dot{u}_i \right) n_j dA & = \int_V \left(t_{ij} \dot{u}_i \right)_{,j} dV = \\ & = \int_V \left[t_{ij,j} \dot{u}_i + t_{ij} (\dot{u}_i)_{,j} \right] dV \end{aligned}$$

$$\int_A c_i n_i dA = \int_V c_{i,i} dV.$$

Consequently we obtain

$$\begin{aligned} \int_V \dot{e} \rho dV & = \int_V \left(t_{ij,j} + \rho b_i - \rho \ddot{u}_i \right) dV + \\ & + \int_V t_{ij,j} (\dot{u}_i)_{,j} dV + \int_V (c_{i,i} + z) dV. \end{aligned} \quad (11)$$

Taking into account in this relationship the equation of motion of continuum

$$t_{ij,j} + \rho b_i = \rho \ddot{u}_i$$

and that any tensor can be decomposed in symmetric and antisymmetric components

$$(\dot{u}_i)_{,j} = \dot{u}_{ij} + \dot{\omega}_{ij},$$

$$\dot{u}_{ij} = \frac{1}{2} \left[(\dot{u}_i)_{,j} + (\dot{u}_j)_{,i} \right],$$

$$\dot{\omega}_{ij} = \frac{1}{2} \left[(\dot{u}_i)_{,j} - (\dot{u}_j)_{,i} \right]$$

will find

$$\int_V t_{ij,j} (\dot{u}_i)_{,j} dV = \int_V t_{ij} \dot{u}_{ij} dV,$$

$$t_{ij} \dot{\omega}_{ij} = 0 \quad t_{ij} \dot{\omega}_{ij} = 0. \quad (12)$$

Replacing the past relationships in (11) and observing that sub domain is arbitrary, obtain

$$\begin{aligned}\dot{\epsilon}\rho &= t_{ij}u_{ij} - c_{i,i} + z, \\ q &= -c_{i,i} + r.\end{aligned}\quad (13)$$

The last relation expressed the local shape of balance equation of energy. We notice that the first thermodynamic principle allow establish of mechanic equivalent of heat, so, from dimension point of view, will have $[calorie] = ML^2T^{-2}$.

2. THE SECOND THERMODYNAMIC PRINCIPLES

The first principle of thermodynamic can be interpreted like a possibility of transformation of mechanical work in heat and mutual with a single condition, that the total energy of system remain the same. That way this principle does not require no one restriction on evaluation meaning of thermodynamic processes. Contrary, the second thermodynamic principle has introduced the discrimination between reversible and irreversible processes. The passing criterion of system from reversible state in irreversible is contained the second thermodynamic principle, which operates with two new definitions temperatures T and entropy S . Is supposed, that in each t moment exists one scalar field $T = T(x_1, x_2, x_3)$ determined in each point of deformable solid. Size T is called the absolute temperature of material particle M , which in t moment is in (x_1, x_2, x_3) point. The absolute temperature T has never had the negative values. The unite of thermodynamic temperature is taken by Kelvin. This new value can't be related in natural way with fundamental units of mass, length and time.

After Clausius for any process exists a state extensive variable S called entropy. dS variation of this state function will consist from dS_e variation thanks interaction with rest of the universe and from dS_i variation thanks to phenomena which have place in interior of system

$$dS = dS_i + dS_e \quad (14)$$

Allowed, that dS_e variation is given by

$$\rho dS_e = \frac{dq}{T}, \quad (15)$$

but dS_i of

$$\rho dS_i = \frac{d\omega}{T}, \quad d\omega \geq 0, \quad (17)$$

equality taking place in case of reversible processes, in case of inequality, the process is irreversible. From (16) – (18) results that

$$\rho T \dot{S} = q + \dot{\omega}, \quad \dot{\omega} \geq 0, \quad (18)$$

These relationships represent the second thermodynamic principles. An another form of this inequality, convenient for applications, can be obtained taking into account the local form (13) of the first principle. Than result

$$\dot{\epsilon} = T \dot{S} + \dot{\omega}_r, \quad (19)$$

where

$$\dot{\omega}_r = \frac{1}{\rho} (t_{ij} \dot{u}_{ij} - \dot{\omega}) \quad (20)$$

The second thermodynamic principle can be obtained starting from different assumptions of physic nature. So, Clausius reach to this result starting from another idea: is impossible that one consequently of one string of changes suffering to one system to decrease to complete transformation of one quantity of heat to mechanic work. Caratheodory used the more general hypothesis, he admitted that is impossible to reach to all thermodynamic states near the initial arbitrary state through one adiabatic process ($dQ = 0$). The thermodynamic processes for them $T = const.$, so $dT = 0$, is called isothermal, in case of one reversible isothermal process dQ is a total exact differential.

Both S entropy and U intern energy are sizes which can't be calculation than making an abstraction of additive constant S_0 , respective ϵ_0 , in practice this thing is sufficient, because these constants disappeared from calculus. Actually, S_0 constant can be determined by means of Nernst theorem, which stated that at zero temperature entropy S is equal with zero.

One thermodynamic process, for which entropy is constant, $dS = 0$ is called isothermal, the second thermodynamic principle is showed us

that reversible and adiabatic processes are isentropic.

3. CONCLUSIONS

Was obtained the local expresses of thermodynamic irreversible processes, expressed by thermodynamic variables the stress tensor and temperature.

Starting with general thermodynamic principles the structure of relation between strain tensor and gradient of displacement was defined, in function of way of define the stress tensor.

In this way was demonstrated, that the definition of strain tensor can't be defined in independent way.

So, synthesis of mechanical and thermodynamical principles is necessary not only at stage of constructing of governing equations and at stage of elaboration of geometric relations.

plastic continuum. In fundamental Aspects of Dislocation Theory, J.A. Simmons, R de Witt and R. Bullough eds, Nat.Bur.Stand, U.S., Spec.Publ, 317 II, 1070, p.837-876.

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Bibliography

1. *Şevcenco Iu. N., Marina V. Structurnaia modeli sredy pri neizotermicheskom protzesse nagruzheniya// Prikladnaya mexanika, 1976, no.12, s.19-27.*
2. *Marina V. Opredeleyushhie uravneniya pri chiklicheskom proporzional'nom deformirovanii nestabil'nyh materialov.// Prikladnaya mexanika, 1986, N6, s.92-99.*
3. *Marina V. Printzipy perekhoda ot mikro k makro napryazhenno deformirovannomu sostoyaniu. Izv. A.N. R.M., Matematika, 1988, N (27), s. 16-24.*
4. *Marina V. The Principles of the Transition from a Macroscopic to a Microscopic State//Science of SINTERING. The international journal of basic and applied sintering and related processes, 2000, N 3, p.155-124.*
5. *Hill R. On macroscopic measures of plastik work and deformation in mikroheterogeneous medium. L.: Matematikal Physiks. No16, 1975, p.214.*
6. *Trusdell K. Pervonachal'nyj kurs ratsional'noj mexaniki sploshnyh sred. Moskva, Mir, 1975, 592p.*
7. *Hill R. Kontinual'naya mikromexanika uprugoplasticheskikh polikristallov, Mexanica, 1967, Nr.3, p.1313-144.*
8. *Eringen A.S., Suhubi E.S., Nonlinear theory of simple microelastic solids, part I, Int.I.Eng.Sci., nr.2, p.189-203.*
9. *Teodosiu C.A., A dynamic theory of dislocations and its applications to the theory of the elastic-*

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