Analysis of the correlation properties of direct and inverse composite Walsh functions

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Abstract: *The article discusses the correlation properties of pseudo-random sequences (PRS) used in the formation of noise-like signals in code division coupled communication systems. The direct and inverse composite Walsh functions are considered. Comparative analysis of the correlation properties of the PRS is carried out in the Matlab medium. It is shown that the correlation properties of certain ensembles of composite Walsh functions are independent of the time shift. The possibility of using these signals in the development of systems with channels code separation in order to reduce the level of interference of multiple access is justified. Directions for further research have been determined.*

Keywords*: PSEUDO-RANDOM SEQUENCES, NOISE-LIKE SIGNALS, AUTOCORRELATION FUNCTION, CROSS CORRELATION FUNCTION, DIRECT COMPOSITE WALSH FUNCTION, INVERSE COMPOSITE WALSH FUNCTION*

1. Introduction

Information transmission systems (ITS) are currently one of the most developing areas in radio communication engineering. The flows of information and the speed of its transmission are increasing, the requirements for the accuracy and reliability of the transmitted information are increasing, the requirements for electromagnetic compatibility with other devices and the use of the frequency resource are becoming more stringent. Among a large number of various ITSs, communication systems using noise-like signals (NLS) are of the greatest interest [1-5].

The use of broadband methods makes it possible to obtain such results in the field of communication systems that cannot be obtained using conventional signals, namely: the energy, structural and informational secrecy of the ITS. In addition, systems with NLS are capable of providing code addressing of a large number of subscribers and their code division when operating in a common frequency band, and have much better electromagnetic compatibility compared to narrowband radio communication systems.

Pseudo-random sequences (PRS) are widely used to generate noise-like signals (NLS) in communication systems with direct-sequence spread spectrum (DSSS) or frequency hopping spread spectrum (FHSS). Examples of such systems are DS-CDMA, GPS / Navstar, Glonass, and IEEE 802.11b wireless networks.

2. Preconditions and means for resolving the problem

2.1. Theoretical Data

As noted above, PRS is widely used in various systems and devices of data transmission systems. When using noise-like signals for data transmission, correlation methods of processing the received signals are used. In this case, the degree and quality of using the PRS largely depends on the correlation characteristics of the sequences used. In general, the following requirements are imposed on pseudo-random sequences used to spread the spectrum of signals [1,2,5]:

a large volume of an ensemble of sequences formed using a single algorithm;

- "good" auto- and cross-correlation properties of the sequences included in the ensemble;
- balanced structure;
- the maximum period for a given length of the shift register that forms the sequence;
- unpredictability of the structure of a sequence by its undistorted segment of limited length.

The simplest method of receiving NLS is correlation. The main subject of comparison is the autocorrelation function (ACF) of the sequence. ACF is a measure of the similarity of a signal and its time-shifted copy. When the signal coincides with its copy, we get the maximum (main) ACF overshoot, thanks to which the detection equipment can detect and extract a useful signal from the noise. The characteristic and comparison of broadband signals is based on the value of the maximum lateral ejection (lobe) of the ACF. Sometimes it is provided as a percentage of the value of the main emission.

The autocorrelation function of discrete signals is calculated by the formula:

$$R_u(n) = \sum_{j=-\infty}^{\infty} u_j\, u_{j-n}, \qquad (1)$$

where n is an integer, positive, negative, or zero.

The cross-correlation function (CCF) between two discrete signals is calculated using a similar formula:

$$R_{uv}(n) = \sum_{j=-\infty}^{\infty} u_j\, v_{j-n}, \qquad (2)$$

In order to improve the correlation characteristics of the PSP, various methods are proposed. In [6-9], various modifications of Barker codes are considered - composite Barker codes, Barker-Volynskaya codes. However, the use of such modifications of Barker codes does not allow obtaining large PSP ensembles, which is a necessary condition for their use in multichannel code division multiplexing systems. In addition, to reduce the level of multiple access interference, the applied PRSs should be orthogonal, since in this case the CCF between any other PRSs will be equal to zero. Codes with such properties include Haar, Rademacher, Walsh codes. However, a feature of orthogonal codes is that the orthogonality of these codes is performed only at the "point", i.e. in the absence of time shifts. In real conditions, such conditions are not met, orthogonality is violated, which in turn leads to an increase in the level of multiple access interference and the appearance of errors in the processing of input data. Therefore, various methods are used to eliminate these disadvantages [10-12].

Let us consider the application of the so-called composite Walsh codes from this point of view. The method for obtaining such codes is as follows - this code is formed by multiplying two standard sequences. The first of them (for example, the Walsh function) is called a generator, and the second (for example, M is a sequence) is called elementary. Each character in the generating sequence is replaced by a direct or inverse elementary sequence, depending on what value the character has (+1 or -1) in the generating sequence. As a result of multiplying the first sequence by the second, sequences with a large number of digits are obtained. We call such a sequence the direct composite Walsh function. If M - a sequence is taken as the first, and the Walsh function is taken as an elementary one, then such a sequence will be called the inverse composite Walsh function.

As an M - sequence, we take a primitive, irreducible polynomial of the 3rd order - $h(x) = x^3 + x + 1$, with the help of which we can obtain several PRSs depending on the initial state of the shift register. The most important feature of M - sequences is that their periodic autocorrelation function is optimal in the class of possible autocorrelation functions of binary sequences of length $L = 2^n - 1$. It is the good autocorrelation properties of M - sequences and the simplicity of their formation that have led to their widespread use in communication systems [1,2,6,12].

As the Walsh function, we choose the 4th order Walsh function, which can take the following values: {(+1, +1, +1, +1), (+1, +1, -1, -1), (+1, -1, -1, +1), (+1, -1, 1, -1}. Given the values of the PRS generated by the shift register corresponding to the polynomial $h(x) = x^3 + x + 1$, the following direct composite Walsh functions can be obtained:

(1,-1, 1,-1) · (1,-1,-1, 1, 1, 1,-1) = 1,-1,-1, 1, 1, 1,-1, -1, 1, 1,-1,-1,-1, 1, 1,-1,-1, 1, 1, 1,-1, -1, 1, 1,-1,-1,-1, 1;

(1,-1, -1,1) · (1,-1,-1, 1, 1, 1,-1) = 1,-1,-1, 1, 1, 1,-1, -1, 1, 1,-1,-1,-1, 1, -1, 1, 1,-1,-1,-1, 1, 1,-1,-1, 1, 1, 1,-1.

And also, the inverse composite Walsh functions:

(1,-1,-1, 1, 1, 1,-1) · (1,-1, 1,-1) = 1,-1, 1,-1,-1, 1,-1, 1,-1, 1,-1, 1, 1,-1, 1,-1, 1,-1, 1,-1, 1,-1, 1,-1,-1, 1,-1, 1;

(1,-1,-1, 1, 1, 1,-1) · (1,-1,-1, 1) = 1,-1,-1, 1,-1, 1, 1,-1,-1, 1, 1,-1, 1,-1,-1, 1, 1,-1,-1, 1, 1,-1,-1, 1, -1, 1, 1,-1.

2.2. Experimental Data

The analysis of the correlation characteristics of direct composite Walsh functions, which are presented above, performed in the Matlab environment, gave the following results (Fig. 1, .., Fig. 4).

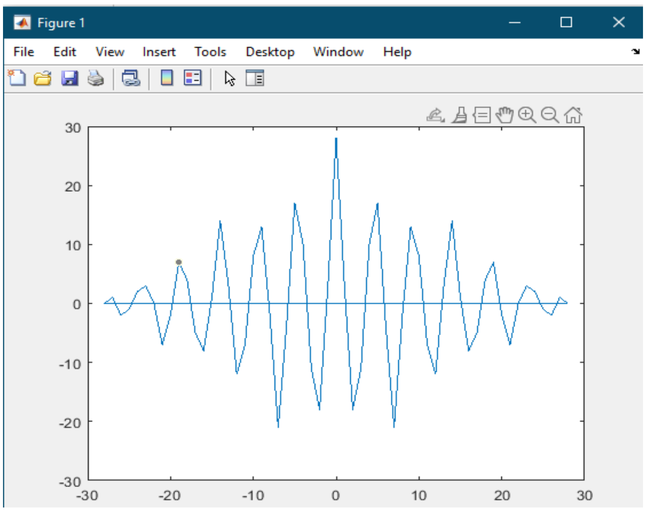

Fig. 1 *ACF of direct composite Walsh functions and M – sequences:* (1, -1, 1, -1) ·(1, -1, -1, 1, 1, 1, -1)

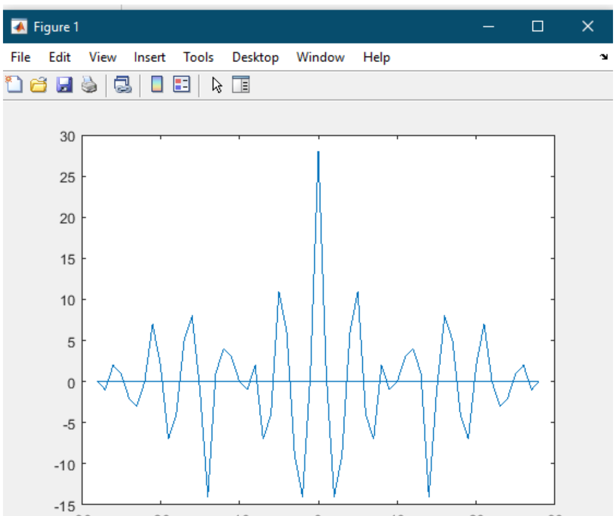

Fig. 2 *ACF of direct composite Walsh functions and M – sequences*: (1, -1, -1, 1) · (1, -1, -1, 1, 1, 1, -1)

In the Fig. 3 the cross-correlation characteristic of direct composite Walsh functions is given.

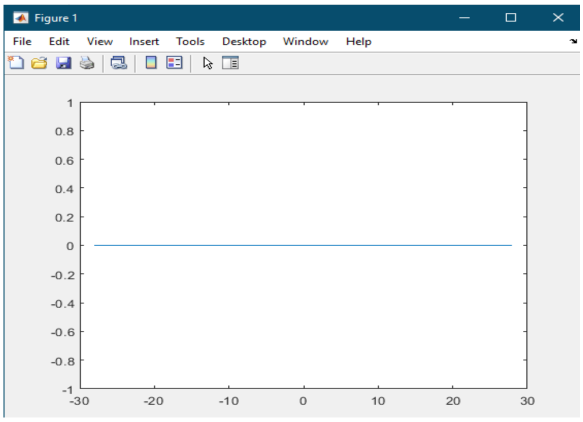

Fig. 3 *CCF of direct composite Walsh functions.*

From Fig. 1, ..., Fig. 3 it follows that direct composite Walsh functions based on M - sequences have poor ACFs - there is no pronounced central lobe of the characteristic and large levels of side lobes, but very good cross correlation characteristics. Thus, direct composite Walsh functions cannot provide reliable synchronization on reception, correct decoding of input data, but provide a minimum level of multiple access interference, since the CCF is zero at any time offsets.

Fig. 4, … , Fig.6 shows the correlation characteristics of the inverse composite Walsh functions.

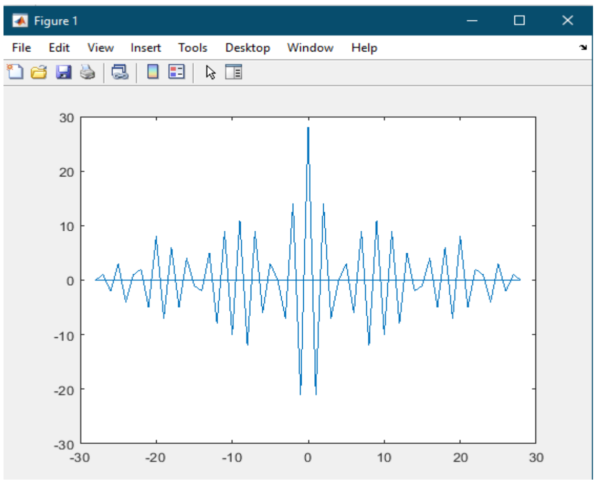

Fig. 4 *ACF of inverse composite Walsh functions and M -sequences:* (1, -1, -1, 1, 1, 1, -1) · (1, -1, 1, -1)

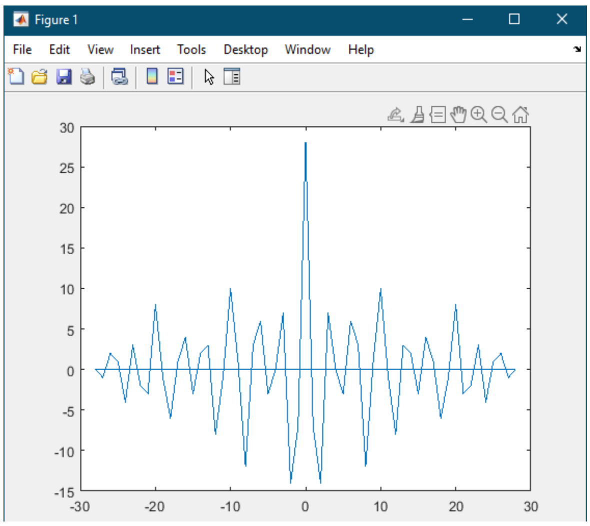

Fig. 5 *ACF of inverse composite Walsh functions and M -sequences:* (1, -1, -1, 1, 1, 1, -1) · (1, -1, -1, 1)

Fig. 6. the cross-correlation characteristic of the inverse composite Walsh functions is given.

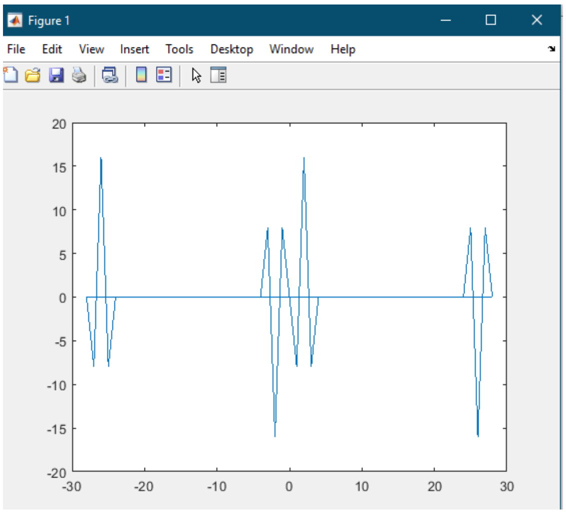

Fig. 6 *CCF of inverse composite Walsh functions.*

From Fig. 4, …, Fig. 6 it follows that the inverse composite Walsh functions have better ACFs compared to the direct ones - there is a pronounced central lobe of the characteristic and lower levels of side lobes, but not very good CCF.

Thus, inverse composite Walsh functions can provide reliable timing on reception, correct decoding of input data, but slightly increase the level of multiple access interference at small time offsets, and significantly less at large time offsets.

Consider the case when the de Bruijn sequences are used as a standard sequence - (1, -1, 1, 1, 1, -1, -1, -1) and (1, 1, 1, -1, 1, -1, -1, -1). In this case, the following direct composite Walsh functions can be obtained:

(1,-1, 1,-1) · (1,-1, 1, 1, 1,-1,-1,-1) = 1,-1, 1, 1, 1,-1,-1,-1,-1, 1,-1,-1,-1, 1, 1, 1, 1, 1,-1, 1, 1, 1,-1,-1,-1,-1, 1,-1,-1,-1, 1, 1,1;
(1,-1, -1, 1) · (1, 1, 1,-1, 1,-1,-1,-1) = 1, 1, 1,-1, 1,-1,-1,-1,-1,-1,-1, 1,-1, 1, 1, 1,-1,-1,-1, 1,-1, 1, 1, 1, 1, 1, 1,-1, 1,-1,-1,-1,

as well as the inverse composite Walsh functions:

(1,-1, 1, 1, 1,-1,-1,-1)·(1,-1, 1,-1) = 1,-1, 1,-1,-1, 1,-1, 1, 1, -1, 1,-1, 1,-1, 1,-1, 1,-1, 1,-1,-1, 1,-1,-1,-1, 1,-1, 1,-1, 1,-1,1;
(1, 1, 1,-1, 1,-1,-1,-1) · (1,-1,-1, 1) = 1,-1,-1, 1, 1,-1,-1, 1, 1, -1,-1, 1 ,1,-1, 1, 1,-1, 1,-1,-1, 1,-1, 1, 1,-1,-1, 1, 1,-1,-1, 1, 1,-1.

The analysis of the correlation characteristics of the composite Walsh functions, which are presented above, carried out in the Matlab environment, gave the following results (Fig. 7, …., Fig. 9).

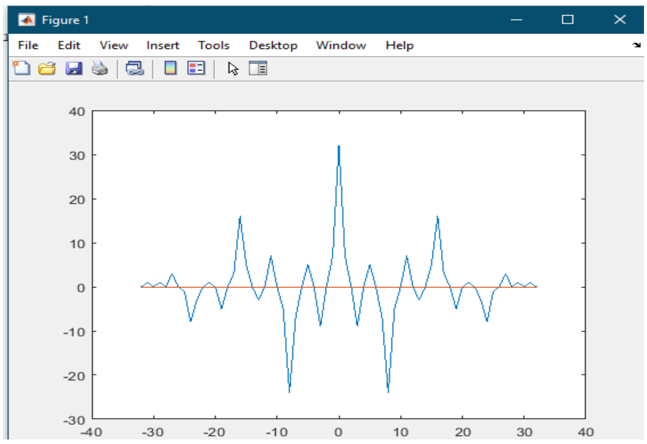

Fig. 7 *ACF of direct composite Walsh functions and de Bruijn sequences: (1, -1,1, -1) ·(1, -1,1,1,1, -1, -1, -1)*

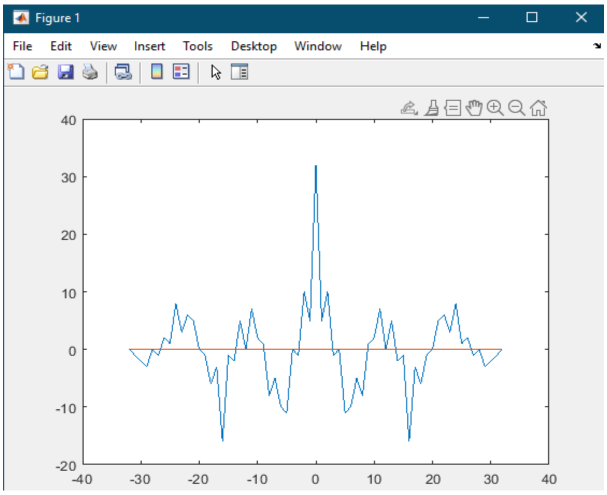

Fig. 8 *ACF of direct composite Walsh functions and de Bruijn sequences: (1, -1, -1,1) ·(1,1,1, -1,1, -1, -1, -1)*

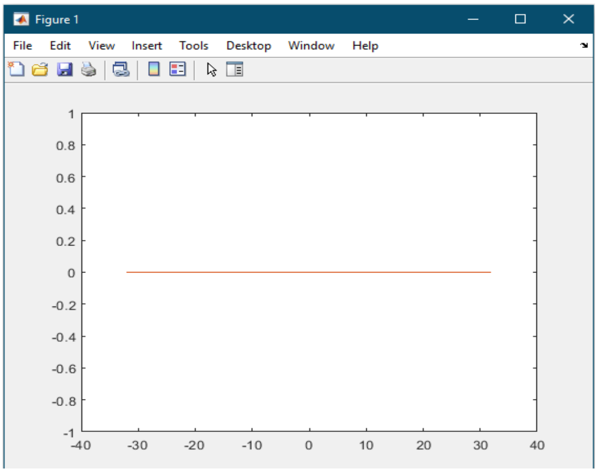

Fig. 9 *CCF of direct composite Walsh functions.*

From Fig. 7, …, Fig. 9 it follows that direct composite Walsh functions based on de Bruijn sequences have relatively good ACFs - there are pronounced central lobes of characteristics, the suppression coefficient - the ratio of the amplitude of the central lobe to the maximum side lobe is equal to two, and very good CCF. Thus, direct composite Walsh functions based on de Bruijn sequences can provide reliable synchronization on reception, correct decoding of input data, and provide a minimum level of multiple access interference, since the cross-correlation function is zero at any time offsets.

Fig. 10, …, Fig. 12 show the correlation functions of the inverse composite Walsh functions.

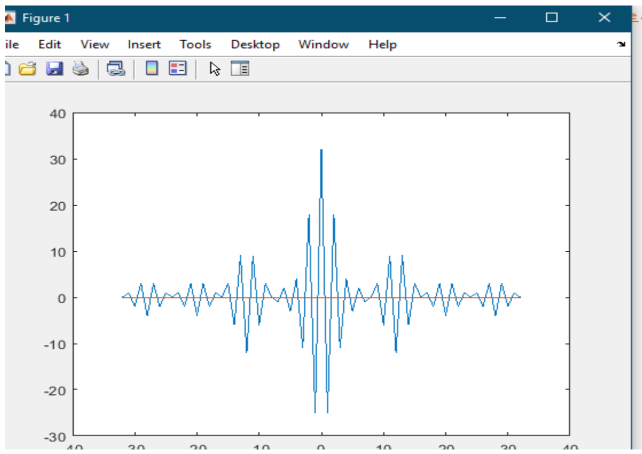

Fig. 10 *ACF of inverse composite Walsh functions and de Bruijn sequences*: (1, -1, 1, 1, 1, -1, -1, -1) · (1, -1, 1, -1)

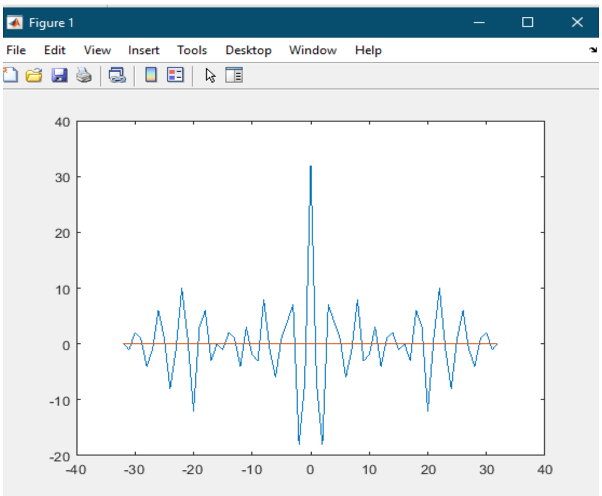

Fig. 11 *ACF of inverse composite Walsh functions and de Bruijn sequences*: (1, -1, 1, 1, 1, -1, -1, -1) · (1, -1, 1, -1)

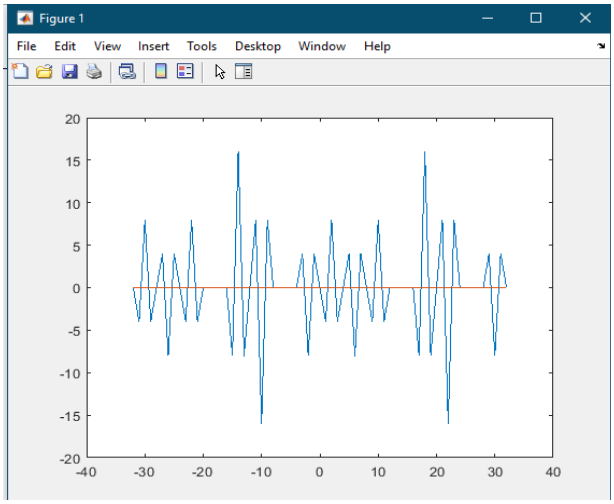

Fig. 12 *CCF of inverse composite Walsh functions.*

From Fig. 10, …, Fig. 12 follows that the inverse composite Walsh functions based on de Bruijn sequences have approximately the same autocorrelation functions as the direct ones - there are pronounced central lobes of characteristics, the suppression coefficient is the ratio of the amplitude of the central lobe to the maximum side lobe is two, but poor cross-correlation characteristics. Thus, inverse composite Walsh functions based on de Bruijn sequences can provide reliable synchronization on reception, correct decoding of input data, but have a large level of multiple access interference.

Consider the case when the following arbitrary sequences are used as a standard sequence:

(1, 1, 1, -1, -1, -1, -1, 1) and (-1, -1, -1, 1, 1, 1, -1, 1).

In this case, the following direct composite Walsh functions can be obtained:

(1,-1,1,-1) · (1,1,1,-1,-1,-1,-1,1) = 1,1,1,-1,-1,-1,-1,1,-1,-1, -1,1,1,1,-1,1, 1,1,1,-1,-1,-1,-1,1,-1,-1,-1,1,1,1,-1,1;
(1,-1, -1,1) ·(-1,-1,-1,1, 1,1,-1,1) = -1,-1,-1,1,1,1,-1,1,1,1,1, - 1,-1,-1,1,-1,1,1,1,-1,-1,-1,1,-1, -1,-1,-1,1, 1,1,-1,1,

as well as the inverse composite Walsh functions:

(1,1,1,-1,-1,-1,-1,1) ·(1,-1, 1,-1) = 1,-1,1,-1,1,-1,1,-1,1,-1,1, -1, -1,1,-1,1,-1,1,-1,1,-1,1,-1,1,-1,1,-1,1,1,-1,1,-1;

(-1,-1,-1,1,1,1,-1,1) · (1,-1,-1, 1) = -1,1,1,-1,-1,1,1,-1,-1,1,1, -1,1, -1,1,1,-1,-1,1,1,-1,-1,1,-1,1,1,-1,1,-1,1,-1,1.

The analysis of the correlation characteristics of the composite Walsh functions, which are presented above, carried out in the Matlab environment, gave the following results (Fig. 1.13, …, Fig. 1.15).

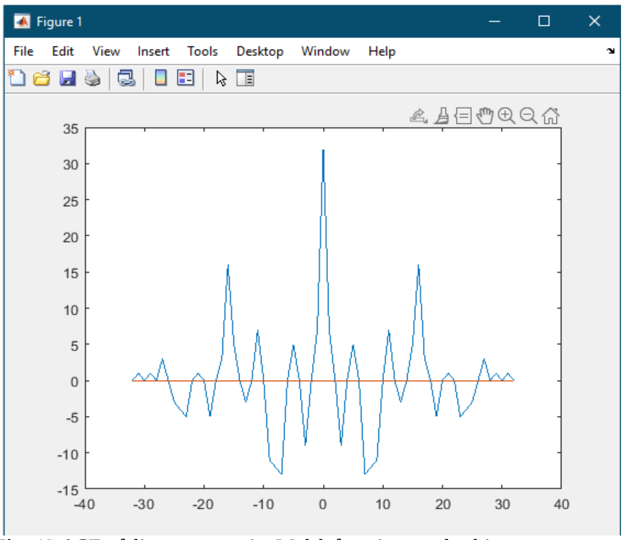

Fig. 13 *ACF of direct composite Walsh functions and arbitrary sequences: (1, -1,1, -1) ·(1,1,1, -1, -1, -1, -1,1).*

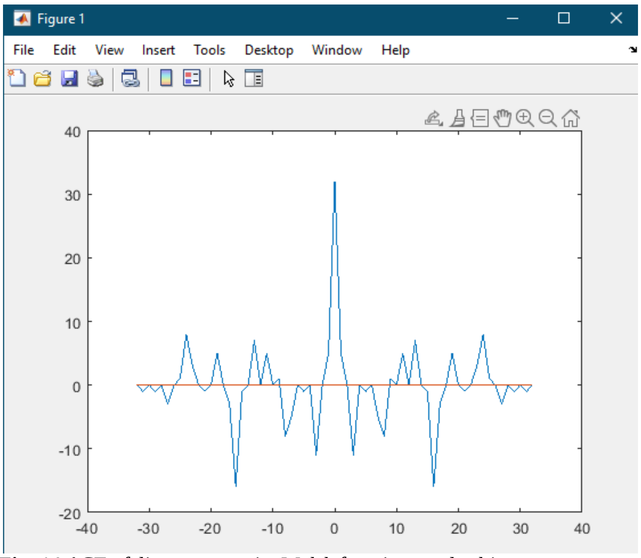

Fig. 14 *ACF of direct composite Walsh functions and arbitrary sequences: (1, -1, -1,1) ·(-1, -1, -1,1,1,1, -1,1).*

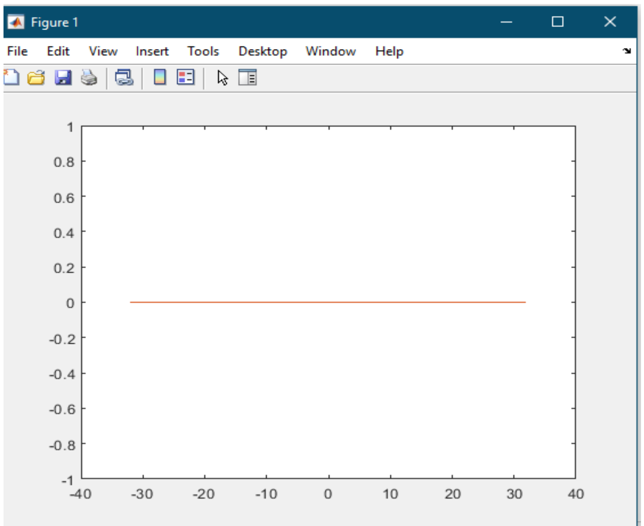

Fig. 15 *CCF of direct composite Walsh functions and arbitrary sequences.*

Fig. 16, …, Fig. 18 show the correlation characteristics of the inverse composite Walsh functions and arbitrary sequences.

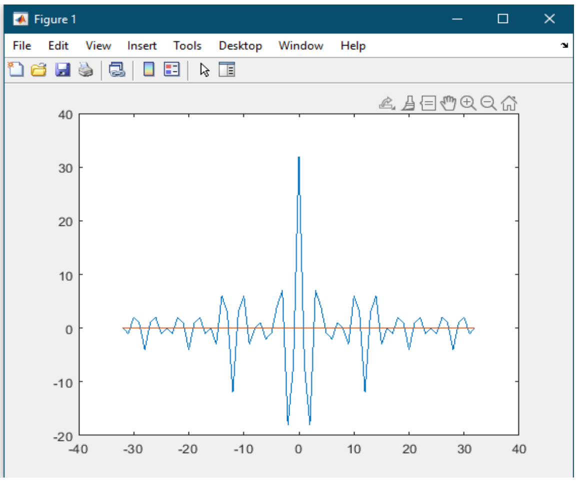

***Fig. 16** ACF of inverse composite Walsh functions and arbitrary sequences: (-1, -1, -1,1,1,1, -1,1) ·(1, -1, -1,1).*

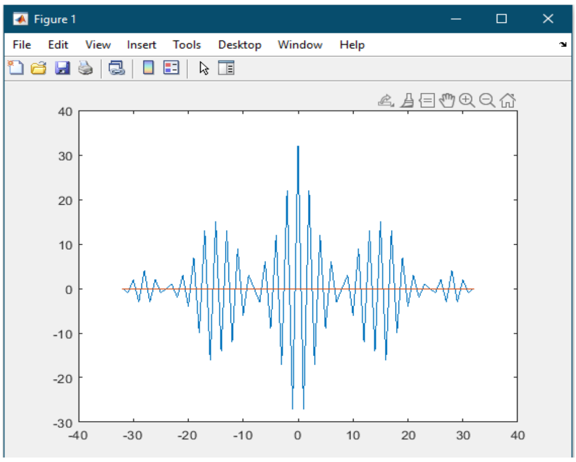

***Fig. 17** ACF of inverse composite Walsh functions and arbitrary sequences: (1,1,1, -1, -1, -1, -1,1) ·(1, -1,1, -1).*

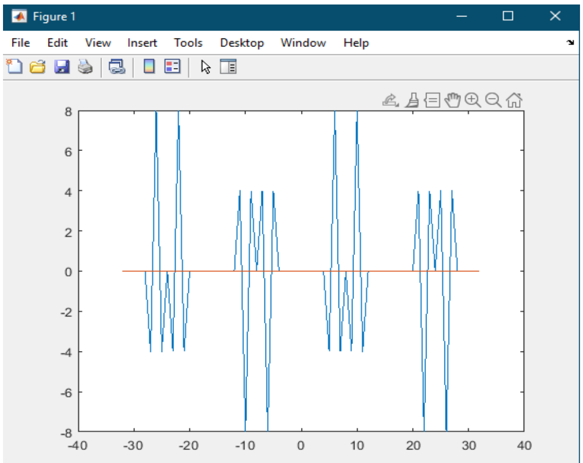

***Fig. 18** CCF of inverse composite Walsh functions and arbitrary sequences.*

From Fig. 13, …, Fig. 15 it follows that direct composite Walsh functions based on arbitrary sequences have relatively good autocorrelation functions – there are pronounced central lobes of characteristics, the suppression coefficient-the ratio of the amplitude of the central lobe to the maximum lateral is equal to two, and very good cross-correlation characteristics. Thus, direct composite Walsh functions based on arbitrary sequences can provide reliable synchronization at reception, correct decoding of input data, and provide a minimum level of multiple access interference, since the cross-correlation function is zero at any time shifts.

From Fig. 16, …, Fig. 18 follows that inverse composite Walsh functions based on arbitrary sequences have worse auto - and cross-correlation functions than direct ones. Thus, inverse composite Walsh functions based on arbitrary sequences cannot provide reliable synchronization at reception, correct decoding of input data, have a high level of multiple access interference, and cannot be used in data transmission systems.

3. Conclusion

The analysis of the correlation characteristics of the composite Walsh functions allows to draw the following conclusions:

- Direct composite Walsh functions have slightly better correlation characteristics than inverse composite Walsh functions.
- Direct composite Walsh functions are time-shift invariant, i.e., the cross-correlation of such functions is independent of the time shift and is orthogonal.
- The type of correlation characteristics of composite Walsh functions largely depends on the type of generators and elementary sequences that are conjugated with the corresponding Walsh functions.
- Pseudo-random sequences of composite Walsh functions are well balanced.

The area of application of PSP, which are based on various approaches, is huge and varied. By selecting the appropriate properties of the generators or elementary sequences, a satisfactory result can be achieved in most cases of operation of broadband systems. Generation of PSP ensembles of arbitrary length with given correlation properties is an urgent practical problem.

Consequently, further careful study of the properties of derivatives and elementary sequences is required for the formation of various composite functions for solving the corresponding applied problems.

4. References

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