Computer Science Journal of Moldova, vol.17, no.2(50), 2009

Formation of the portfolio of projects for informatization programs

Ion Bolun

Abstract

Aspects referring to the formation of portfolio of projects for investments in informatization programs are approached: criteria of efficiency, general problem, aggregate problem in continuous form, general problem in discrete form and solving of problems. As criterion of informatization projects' economic efficiency, the total profit maximization due to investments is used. In preliminary calculations, the opportunity of considering continuous dependences of profit on the volume of investments by domain activities is grounded. Eleven classes of such dependences are investigated and analytical solutions and algorithms for solving formulated problems are described.

Keywords: project, informatization program, efficiency criteria, investments, optimization.

1 Introduction

Efficient informatization essentially contributes to economic development and society prosperity. Offered advantages impose the informatization of diverse activities at different levels of society: citizens, processes, workgroups, subdivisions, economic units (enterprises, organizations and institutions), economic activities (sectors, branches) and society as a whole. Informatics means are largely used in technical devices, machines and technological processes. For tens of years, in economically developed countries, but not only in such ones, an active government promotion policy in the domain is realized. A more recent example is the realisation of European Union's programs: eEurope 2002, eEurope 2005 and i2010.

Informatization in a large scale foresees a vast and complex set of scheduled in time works. For a concrete object and period of time, basic works are realized in form of projects, included in an Informatization program. Informatization programs can be for a short or mid period of time and strategic programs – for a long period of time. Each project of such a program is oriented to the complete or partial informatization of well-defined functions, aiming to accomplish certain objectives – objectives that will assure provided positive effects. In their turn, informatization effects are different for different projects and the realization of each project needs financial, material and labor resources spending. Therefore, it is important to determine priorities for the efficient use of available resources. Improving the efficiency of expenditures with informatization foresees the formation of portfolio of projects of each program in such a way that it will assure the best ratio between expected effects and the resources spending [1, 3, 4, 11].

The Informatization program can be influenced by such factors as: used efficiency criteria, volume of available resources, accomplished degree of informatization, etc. Criteria of economic efficiency, opportune for the estimation of investment projects, were discussed in many papers [1-4], inclusive, more recently for informatization apart, in [11-13]. A general problem for the formation of portfolio of informatization projects, aiming to maximize the total profit P due to investments I in these projects, is formulated in paper [14]. For the aggregate continuous form of this problem, nine dependences of the continuous rate of return r_i by the volume of investments I_i in domain i are proposed and investigated in paper [15]; these dependences are destined to concretize the general problem. Six particular problems in the aggregate continuous form, differing by dependences $r_i(I_i)$ only, are investigated in [16]. For the case of linear dependence $r_i(I_i)$, the optimal solution in analytical form is obtained. An algorithm for numerical solving of problems in aggregate continuous form, for which inverse dependences $I_i(r_i)$ is possible to determine in an explicit analytical form, was proposed, too. Some new aspects were investigated and an algorithm for numerical solving of the general aggregate problem in continuous form was described in article [17]. In this paper, a summary of results, obtained in the domain, is done and the general problem in discrete form is investigated.

2 Preliminary considerations

An informatization object (enterprise, institution, economic activity, society as a hole) can be characterized by activity domains. Domain i foresees the accomplishment of a set of functions. The informatization of a function results with a certain effect. At the same time, for the complete or partial informatization of a function, there are needed certain expenditures of resources, inclusive investments. Investments are done by projects, for each of which, when selecting for the Informatization program, indicators of economic efficiency are considered apart. Sometimes, a project can cover, completely or partially, many functions of an activity domain. When complete informatization of all functions referred to domain i is realized, the degree of informatization is $g_i = 1$.

The informatization program for n domains depends on many factors, including available financial resources. In hypothetical case of unlimited financial resources, it will be realized the subset of projects, that will assure the due extreme value (minimum, maximum) of the accepted optimization criteria, without taking into account the possibility of investment in projects of other domains; such projects will be investigated apart, because of enough resources for them, too. The limited character of available resources imposes the necessity of their rational distribution by certain projects, basing on grounded decisions.

Evidently, the state of domain i informatization (the subset of realized projects, referred to this domain) and, also, that of the entire object (the subset of realized projects, referred to all n domains) can influence, more or less, the indicators (technical, economical and so on) of unrealized informatization projects, yet. At the same time, with a sufficient degree of approximation, at a reasonable strategy of the object informatization, it can be considered that expenditures with an informatization project f_{ij} depend only on the degree g_i of the domain

i informatization, despite these can depend, sometimes significantly, on the degree g of the object informatization as a whole.

Let f_{ij}, $j = \overline{1, J_i}$ be the set of projects, needed for the complete informatization of the domain i, and $\cancel{I}_i$ – the total volume of investments needed for the complete informatization ($\cancel{g}_i = 1$) of this domain. At a linear dependence of the volume of investments I_i on the degree of informatization g_i of domain i, it takes place the relation

$$I_i = g_i \cancel{I}_i, \quad i = \overline{1, n}. \tag{1}$$

Let q_i be the weight of informatics resources' capital in domain i ($q_i \in [0; 1]$), then it takes place the relation

$$I_i = v_i q_i K_i, \quad i = \overline{1, n}. \tag{2}$$

In relation (2), K_i is the total capital referred to domain i and v_i is the capitalization rate of investments I_i.

The used optimization criteria is a primary factor when argumenting decisions. From the point of view of used criteria, two categories of problems are distinguished: monocriterion and multicriteria ones. Monocriterion problems are investigated in such papers as [1-5, 7, 10, 11]. Indicators proposed in this aim are, usually, composite indicators calculated on the base of many primary indicators such as: the probability of finishing in time the works on project, the probability of successful implementation of the project, the probability of achieving the outlined objectives, the volume of investment with the project, the outcomes from the project, etc. In the case of informatics projects, a part of these primary indicators became trivial, influencing insignificantly the solution.

For multicriteria problems, such methods as ELECTRE [6], ORESTE [8], PROMETHEE [9] and applications as SSD [11] etc can be used, depending on the case. In the frame of multicriteria problems, many factors are taken into consideration, but it not always leads to better results. Of great importance in such investigations are the plenitude and accuracy of information – requirements that are intensifying in the case of a large number of projects. Therefore, sometimes it can be opportune obtaining the solution in two stages: at the first stage, basing on a monocriterion approach, the set of potential projects is reduced, and at the second stage, using the multicriteria approach, the preferable alternative is selected. In the following, the monocriterion approach will be used.

For projects, which are characterized by the equivalence of performances of functionality and by the ascending dependence of production expenditures on the volume of investments, in papers [12, 13] it is proved that the maximization of the profit (annual or by the entire period of the use of informatics products), the maximization of the profit rate, the maximization of the economic effect (annual or by the entire period of the use of informatics product), the maximization of the rate of return on investments, the minimization of the time of return on investments, the maximization of the economic efficiency of investments, the minimization of the equivalent costs (annual or total), the maximization of the net present value, the maximization of the internal rate of return and the minimization of the total costs of ownership are reduced to the minimization of the volume of investments. The affirmation is valuable both, for ordinary values and for present values of the respective indicators. Thus, as optimization criteria for such projects, the minimization of the volume of investments can be used, assuring, at the same time, that the optimal solution, obtained conform this criterion, coincide with the optimal solution obtained using each of the others mentioned above criteria, considerably simplifying the problem.

Aiming to reduce the complexity of the problem of defining the Informatization program, the described above approach is opportune to use it at the pre-selection step, comparing one between another the projects that are equivalent by performances of functionality. At the next, final step, there will be compared, for including in the Informatization program, only projects that realize the informatization of some specific functions; the non including in the Program of one or more potential projects will result with the non informatization of the corresponding functions in the period, covered by the Program. By this particularity, the problem, investigated in this paper, differ significantly from problems discussed in papers [12, 13].

From the same considerations of reducing the complexity of the problem, it is opportune to investigate apart the informatization projects with social orientation, and the multitude of available resources will be appreciated by the volume of investment I. Thus, it is assumed that there are sufficient labor resources to cover the volume of needed works.

Usually, as optimization criteria for the evaluation of alternatives of goods production or offering the services the maximization of production volume or profit is used, using such typical production functions as: Cobb-Douglas, with complementary factors (Leontief) or with constant elasticity of substitution (CES) [18]. In case of informatization projects, it is more convenient to use, in this aim, the maximization of the profit. This fact is explained by the particularities of investments in informatization. The implementation of some informatization projects results with the reducing of production costs, but not with the increasing of production volume. Sure, informatization assures the increase of labor productivity, which permits, in certain conditions, the increase of production volume, too. But both these effects, and many others, are encompassed by indicator ,,profit". At the same time, just profit constitutes those resources that can be used both, for future developments, inclusive for informatization, and for consumption. Evidently, when needed, the criterion of profit maximization can be, with little adaptations, substituted by the maximization of production volume. Therefore, in this paper, the maximization of profit will be used as optimization criterion.

3 Formulation of the problem

At accepted in p. 2 suppositions, as optimization criterion for an Informatization program it is reasonable to use the maximization of the total profit P due to investments I in this program. Because of given volume of investments I, the use of total profit P maximization as optimization criterion is equivalent to the use, in this aim, of the criterion of maximizing the rate of return on investments R, between which the relation $R = P/I$ takes place.

Thus, the problem of Informatization program optimization can be formulated, roughly, as following. Let the volume I of investments for the partial or total informatization of an object is known. It is required to determine the optimal distribution $I^*_{ij} = \alpha^*_{ij} I_{ij}$, $j = \overline{1, J_i}$, $i = \overline{1, n}$ of investments I, for the informatization of n activity domains of the object, among projects f_{ij}, $j = \overline{1, J_i}$, $i = \overline{1, n}$, aiming to maximize the total profit P, that is

$$P = \sum_{i=1}^{n} \sum_{j=1}^{J_i} \alpha_{ij} P_{ij} \to \max, \tag{3}$$

$$\sum_{i=1}^{n} \sum_{j=1}^{J_i} \alpha_{ij} I_{ij} \leq I, \tag{4}$$

where α_{ij} is a Boolean variable that takes values 1, if project f_{ij} is included in the Informatization Program, and takes values 0, if it is not included in the program; P_{ij} is the profit due to investments I_{ij} in project f_{ij}. So, the rate of return R_{ij} on investments I_{ij} in project f_{ij} is determined as $R_{ij} = P_{ij}/I_{ij}$. The profit can be annual or present (actualized) – the respective concretization will be done, depending on the case.

The multitude of projects f_{ij}, $j = \overline{1, J_i}$, $i = \overline{1, n}$ leads to a problem of large dimension. Taking into consideration the relatively high error tolerance in estimation of projects' economic characteristics, in preliminary calculations it can be opportune to operate only with indicators by domains, without their differentiation by projects. Let P_i be the profit due to investments I_i in informatization of domain i. Then the problem (3)-(4) can be formulated, in an aggregate form, as following. It is known the volume I of investments, available for the partial or total informatization of n activity domains of the object. It is required to determine the optimal distribution I^*_i,$i = \overline{1, n}$ of investments I by n domains, aiming to maximize the total profit P, that is

$$P = \sum_{i=1}^{n} P_i \to \max, \tag{5}$$

$$\sum_{i=1}^{n} I_i \le I. \tag{6}$$

The profit P_i and investments I_i in informatization of domain i from relations (5), (6) are determined as:

$$P_i = \sum_{j=1}^{J_i} \alpha_{ij} P_{ij}, i = \overline{1,n}, \tag{7}$$

$$I_i = \sum_{j=1}^{J_i} \alpha_{ij} I_{ij}, i = \overline{1,n}. \tag{8}$$

Evidently, indicators P_{ij} and I_{ij} ($j = \overline{1, J_i}$, $i = \overline{1,n}$) are the discrete ones, so dependences $P_i(I_i)$ are the discrete ones, too, in function of the set of informatization projects. Therefore, both, problem (3)-(4) and the (5)-(6) one, refer to problems of Boolean linear programming.

At the same time, the delimitation of the cover area of a project is, frequently, not so strict. It can be reduced or extended, in function of available resources. Also, the estimation of resources, needed for the informatization projects, is an approximate one. More over, solving the problem (5)-(6) at discrete dependences $P_i(I_i)$, $i = \overline{1,n}$ is, usually, more complex, comparatively to the case with continuous ones. Therefore, the dependences $P_i(I_i)$, $i = \overline{1,n}$ can be considered, at least in preliminary calculations, continuous, their character being, usually, not linear and differing from one domain to another. In this case, the aggregate problem (5)-(6) becomes a problem of continuous mathematical programming.

4 Character and definition interval of functions $P_i(I_i)$, $R_{ij}(I_i)$ and $r_i(I_i)$

The complexity of the aggregate problem (5)-(8) depends on many factors, but most of all it depends on the character of dependences $P_i(I_i)$, $i = \overline{1,n}$. Basing on optimization criterion (5), at restriction (6) and taking into account relations (7) and (8), one has to give the priority for be included in Informatization Program, to projects with a higher rate of return on investments R_{ij}. So, all the J_i projects for each activity domain i have to be arranged in the decreasing order of R_{ij}. In the following, we will consider, without reducing the universality of investigations, that ordered projects correspond to their numeration $j = \overline{1,J_i}$, $i = \overline{1,n}$; from projects with the same value of R_{ij}, the priority is given to the project with the lowest I_{ij}.

Let us consider dependences $R_{ij}(I_i)$, $i = \overline{1,n}$, where R_{jj} is the rate of return on investments of the project f_{ij} for which the relation $j = \{k|\, I_i = \sum_{s=1}^{k} I_{is}\}$ takes place. Also, for the continuous form of the problem (5)-(6), it is useful the introduced in paper [9] notion of continuous rate of return r_i on investments I_i in activity domain i, determined as

$$r_i = r_i(I_i) = \frac{\partial P_i}{\partial I_i}, \quad i = \overline{1,n}. \tag{9}$$

Taking into account the defined above rule of including projects in Informatization Program, dependences $R_{ij}(I_i)$ and $r_i(I_i)$ are the non increasing ones, usually even the decreasing ones (considering that there are no two projects with the same rate of return on investments).

Evidently, dependences $R_{ij}(I_i)$ and $r_i(I_i)$ have both, an inferior limit and a superior limit values. The respective inferior limit is the value, at which the increase of volume of investments I_i does not lead to the increase of revenue V_i. That is the increase of investments in informatization results with negative profit only, equal by value to the increase of the volume of investments (informatization's elasticity becomes equal to zero). So, in the case of discrete dependence

$R_{ij}(I_i)$, for the respective marginal project f_{ij} and for all the following projects from the J_i ones, the relations $\Delta V_i = V_{ij} = 0$, $P_{ij} = -I_{ij}$ and $R_{ij} = P_{ij}/I_{ij} = -1$ take place. Thus, the inferior limit value $\breve{R}_{ij}$ for R_{ij} is equal to "-1" and to it corresponds the superior limit value $\widehat{I}_i$ of I_i determined as

$$\widehat{I}_i = \sum_{j=1}^{\widehat{j}_i} I_{ij}, \quad i = \overline{1,n}, \tag{10}$$

where $\widehat{j}_i = \min\limits_{k=\overline{1,J_i}} \{k |\, R_{i,k-1} > -1\}$.

Similarly, the condition for the inferior limit value $\breve{r}_i$ of dependence $r_i(I_i)$ is $\partial V_i/\partial I_i = 0$, beginning with $I_i = \widehat{I}_i$, and

$$\left.\frac{\partial P_i}{\partial I_i}\right|_{I_i=\widehat{I}_i} = \breve{r}_i = r_i(\widehat{I}_i) = -1, \quad i = \overline{1,n}, \tag{11}$$

where $\widehat{I}_i$ is the superior limit value for I_i that corresponds to the inferior limit value $\breve{r}_i$ for r_i.

The inferior limit value of the rate of return on investments equal to „-1" is specific, usually, to informatization programs. This leads to the fact that a real informatics system for a relatively complex object doesn't cover, as usual, all functions of the respective information system: it is not opportune, from economic point of view, the complete informatization of all objects' functions. Therefore, the informatization degree of an object doesn't achieve, as usual, the value 1.

The superior limit of the rate of return on investments $\widehat{R}_{ij}$ is determined by the rate of return of the project with the highest rate of return, that is by the rate of return of the project with the highest priority conform to criterion (3). This limit value will be noted as A_i, taking into account that $A_i < \infty$. So, the definition interval for the discrete function $R_{ij}(I_i)$ is

$$R_{ij} \in [\breve{R}_{ij};\ \widehat{R}_{ij}],\ \breve{R}_{ij} = \ -1;\ \widehat{R}_{ij} = R_{i1} = A_i, \quad i = \overline{1,n}. \tag{12}$$

In the same way, the definition interval for the continuous function $r_i(I_i)$ is

$$r_i \in [\breve{r}_i;\ \widehat{r}_i], \quad \breve{r}_i = r_i(\widehat{I}_i) = -1; \tag{13}$$
$$\widehat{r}_i = r_i(\breve{I}_i) = r_i(0) = A_i, \quad i = \overline{1,n},$$

where $\widehat{r}_i$ is the superior limit for r_i. Here $\breve{I}_i = 0$ is the inferior limit, corresponding to value $\widehat{r}_i$, and $\widehat{I}_i$ is the superior limit, corresponding to value $\breve{r}_i$ of investments I_i. So, knowing the definition interval $[-1; A_i]$ for $R_{ij}(I_i)$ and $r_i(I_i)$, one can determine the definition interval $[0; \widehat{I}_i]$ for I_i. Dependences $R_{ij}(I_i)$ and $r_i(I_i)$ are the decreasing ones in interval $[0; \widehat{I}_i]$, that is from $\widehat{R}_{ij} = R_{i1} = \widehat{r}_i = A_i$ to $\breve{R}_{ij} = \breve{r}_i = -1$, and $R_{ij}(I_i) = r_i(I_i) = -1$ for $I_i \geq \widehat{I}_i$, $i = \overline{1,n}$.

Basing on the rule of including in Informatization program the projects with a higher rate of return R_{ij}, from dependences $R_{ij}(I_i)$ (or $r_i(I_i)$ for the continuous case) and $P_i(I_i)$, a primary one is the $R_{ij}(I_i)$ (or $r_i(I_i)$). Knowing $R_{ij}(I_i)$ (or $r_i(I_i)$), one can determine the dependence $P_i(I_i)$, too. In the case of continuous function $r_i(I_i)$, the dependence $P_i(I_i)$ can be determined from the relation (9) that is

$$P_i = P_i(I_i) = \int r_i(I_i) dI_i, \quad P_i(0) = 0. \tag{14}$$

In the case of discrete function $R_{ij}(I_i)$, the dependence $P_i(I_i)$ is determined as

$$P_i = P_i(I_i) = \sum_{j=1}^{k} P_{ij}, \tag{15}$$

$$I_i = \sum_{j=1}^{k} I_{ij}. \tag{16}$$

As mentioned above, knowing the definition interval $[-1; A_i]$ for $R_{ij}(I_i)$ and $r_i(I_i)$, one can determine the definition interval $[0; \widehat{I}_i]$ for

I_i. In their turn, knowing the definition interval $[0; \widehat{I}_i]$, one can determine the corresponding definition interval $[0; \widehat{P}_i]$ for $P_i(I_i)$. More over, because of decreasing character of dependences $R_{ij}(I_i)$ and $r_i(I_i)$, the function $P_i(I_i)$ is a concave one in the interval $[0; \widehat{I}_i]$. Therefore, the function $P_i(I_i)$ is a mono extreme one in the interval $[0; \widehat{I}_i]$. In the case of continuity of reasonable behavior function $r_i(I_i)$ (without discontinuities of degree I or II), the maximum of function $P_i(I_i)$, the maximum profit $P_{\max i}$, can be determined from equation

$$r_i(I_{\max i}) = \left.\frac{\partial P_i}{\partial I_i}\right|_{I_i = I_{\max i}} = 0. \tag{17}$$

In the relation (17), $I_{\max i}$ is the volume of investments I_i in informatization of domain i, at which the maximum profit $P_{\max i}$ is achieved. For the aggregate problem (5)-(6) in continuous form the following relation takes place:

$$I_i^* \in [0; I_{\max i}], \quad i = \overline{1, n}. \tag{18}$$

Similarly, in the case of discrete function $R_{ij}(I_i)$, maximum of the function $P_i(I_i)$, the maximum profit $P_{\max i}$, can be determined as

$$P_{\max i} = \sum_{j=1}^{j_{\max i}} P_{ij}(I_i), j_{\max i} = \{\, k |\, R_{ik} = \min_{j=\overline{1,k}} \{R_{ij} \geq 0\}. \tag{19}$$

From practical use, most of the possible alternatives of the decreasing dependences $R_i(I_i)$ and $r_i(I_i)$ are covered by five categories: convex, linear, concave, hybrid concave-convex (hybrid1) and hybrid convex-concave (hybrid2) ones. In Figure 1 five examples (one for each category for $r_i(I_i)$) are illustrated.

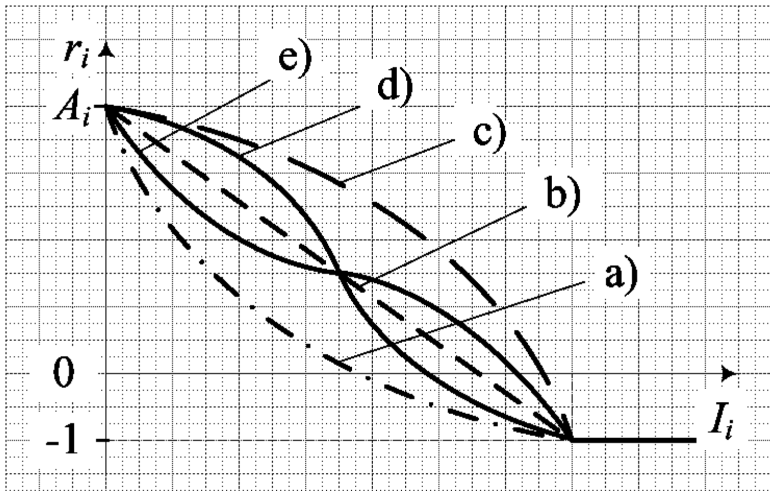

Figure 1. Dependences $r_i(I_i)$: a) convex; b) linear; c) concave; d) hybrid1; e) hybrid2.

5 Classes of continuous dependences $r_i(I_i)$ and $P_i(I_i)$

Ten typical representations of the five categories of dependences $r_i(I_i)$ from Figure 1 were proposed in papers [15, 17] and investigated in papers [16, 17]. These dependences are (constants a_i, b_i, c_i, s_i and h_i are nonnegative):

1) convex algebraic

$$r_i = a_i(I_i + c_i)^{h_i - 1} - b_i, \quad 0 < h_i < 1, \quad b_i > 1; \tag{20}$$

2) convex exponential

$$r_i = a_i e^{-s_i I_i} - b_i, \quad b_i > 1; \tag{21}$$

3) convex algebraic-exponential

$$r_i = a_i e^{-s_i I_i}(I_i + c_i)^{h_i - 1} - b_i, \quad 0 < h_i < 1; \tag{22}$$

4) linear

$$r_i = -a_i I_i + b_i; \tag{23}$$

5) concave algebraic

$$r_i = -a_i I_i^{h_i - 1} + b_i, \quad h_i > 2; \tag{24}$$

6) concave exponential

$$r_i = -a_i e^{s_i I_i} + b_i; \tag{25}$$

7) concave algebraic-exponential

$$r_i = -a_i e^{s_i I_i} I_i^{h_i - 1} + b_i, \quad h_i > 2; \tag{26}$$

8) concave parabolic

$$r_i = -a_i I_i^2 - b_i I_i + c_i; \tag{27}$$

9) concave-convex algebraic-exponential

$$r_i = a_i e^{-sI_i}(I_i + c_i)^{h_i - 1} - b_i, \quad h_i > 1, \tag{28}$$

for which nonnegative constants a_i, b_i, c_i, s_i and h_i take such values that dependence becomes concave on segment $I_i \leq \ddot{I}_i$ and convex on segment $I_i \geq \ddot{I}_i$, where $\ddot{I}_i$ corresponds to the inflexion point (at $I_i = \ddot{I}_i$ the equality $\partial^2 r_i / \partial I_i^2 = 0$ takes place);

10) convex-concave based on normal distribution

$$I_i(r_i) = \overleftrightarrow{I}_i \psi_i \int_{r_i}^{A_i} f(r) dr = \frac{\overleftrightarrow{I}_i \psi_i}{\sigma_i \sqrt{2\pi}} \int_{r_i}^{A_i} \exp\left[-\frac{(r - \bar{r}_i)^2}{2\sigma_i^2}\right] dr, \tag{29}$$

where $f(r)$ is the probability mass function of the rate of return r on investments in informatization, $\overleftrightarrow{I}_i$ is the total volume of investments for the complete informatization of domain i and ψ_i is the normalization coefficient, determined as following $\psi_i = 1 \Big/ \left(1 - 2 \int\limits_{-\infty}^{-1} f(r)dr\right)$. In the case of normal distribution, we have $f(r) = \frac{1}{\sigma\sqrt{3\pi}} \exp\left[-\frac{(r-\bar{r})^2}{2\sigma^2}\right]$, where $\bar{r}$ is the arithmetic mean and σ is the standard deviation of r.

To mention, that in conformity with definition interval (13), for all cases (20)-(29), the relations $r_i(I_i) =$ -1 at $I_i \geq \widehat{I}_i$, $i = \overline{1,n}$ take place. Condition $b_i > 1$ for dependences (20) and (21) is caused by definition interval (13).

Dependence $P_i(I_i)$ for (20) corresponds, roughly, to Cobb-Douglas one factor production function [18]. Exponential dependence, see (21), is frequently used in informatics, including the one for representation of repartition function of laboriousity of the data processing jobs. Dependence (22) is a generalization of dependences (20) and (21), changing, at $s_i = 0$, in dependence (20) and, at $h_i = 1$, in dependence (21). Dependences (24)-(26) are, practically, dual comparing to the (20)-(22) ones. Dependence (27) is, roughly, a particular case of the (24) one, differing by simplicity. Dependence (28) is determined by the same mathematical expression as in the (22) one, but the definition interval for constant h_i differs; this dependence has a more general behavior, being concave at relatively small volume of investments ($I_i \in (0; \ddot{I}_i)$) and convex at relatively high volume of investments ($I_i \in (\ddot{I}_i; \widehat{I}_i]$). Dependence (29), based on normal distribution, is more flexible than the (28) one.

The dependences $r_i(I_i)$ and $P_i(I_i)$, defined by expressions (20), (21), (23)-(25), (27) and (28) and normalized in such a way that for all of them $\breve{I}_i = 0$; $\widehat{I}_i = 50$ and $A_i = 3$, are shown in Figures 2, 3 from [15] by one example for each case.

Five examples of hybrid convex-concave category of dependences $r_i(I_i)$ based on normal distribution (29) from [17] are shown in Figure 4. Initial data for these examples are: $\widehat{r}_i = A_i = 3$, $\breve{r}_i = -1$, $\overleftrightarrow{I}_i = 50$, $\bar{r}_i = 1$, $i = \overline{1,n}$; $3\sigma_3 = (\widehat{r}_i + \breve{r}_i)/2 = 2$; $\sigma_1 = 0,25\sigma_3$; $\sigma_2 = 0,5\sigma_3$; $\sigma_4 = 2\sigma_3$; $\sigma_5 = 4\sigma_3$. So, the five variants differ only by the value of standard deviation σ_i. From Figure 4, one can see the large spec-

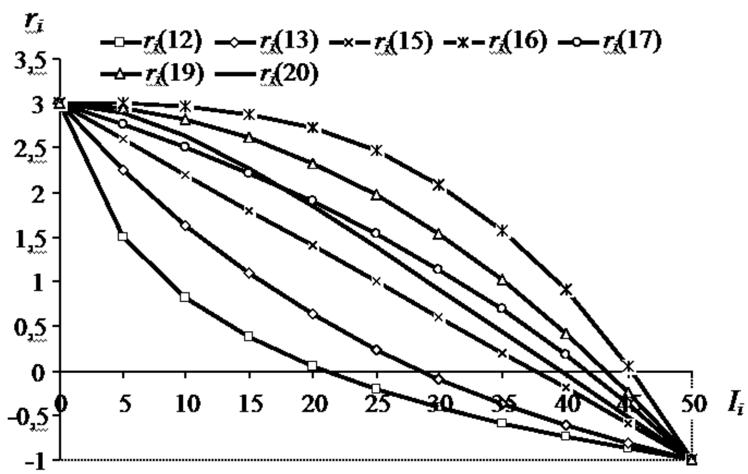

Figure 2. Examples of dependence $r_i(I_i)$.

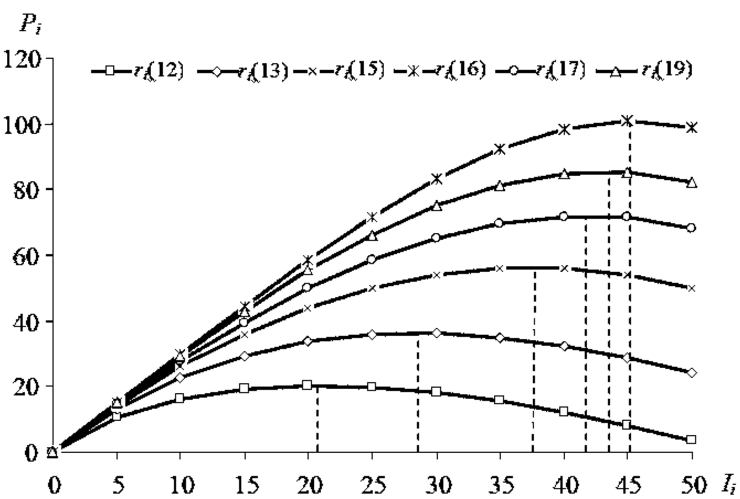

Figure 3. Examples of dependence $P_i(I_i)$.

trum of covered dependences $r_i(I_i)$ and, compared to concave-convex dependence (28), it is considerably more flexible. The high flexibility of normal distribution causes the interest of its use not only for the representation of the convex-concave dependence (29), but for the representation of a concave-convex one, too. In this aim it is sufficient to invert the use of variables in relation (29), that is

$$r_i(I_i) = \psi_i(A_i{+}1)\int\limits_{x_i}^{A_i} f(x)dx = \frac{\psi_i(A_i+1)}{\sigma_i\sqrt{2\pi}}\int\limits_{x_i}^{A_i} \exp\left[\frac{(x-\bar{x}_i)^2}{2\sigma_i^2}\right]dx, \quad (30)$$

where $x_i = \frac{I_i}{\overleftrightarrow{I}_i}(A_i+1)-1$, $\bar{x}_i = \frac{\bar{I}_i}{\overleftrightarrow{I}_i}(A_i+1)-1$. Five examples of hybrid concave-convex category of dependences $r_i(I_i)$ based on normal distribution (30) are shown in Figure 5. Initial data for examples in Figure 5 coincide with the ones in Figure 4.

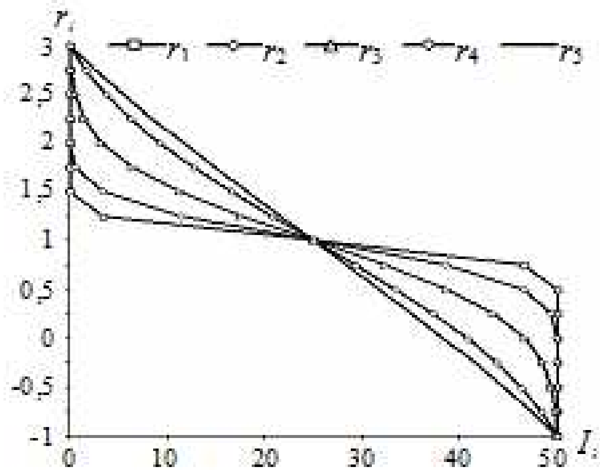

Figure 4. Convex-concave dependences $r_i(I_i)$ based on normal distribution (29).

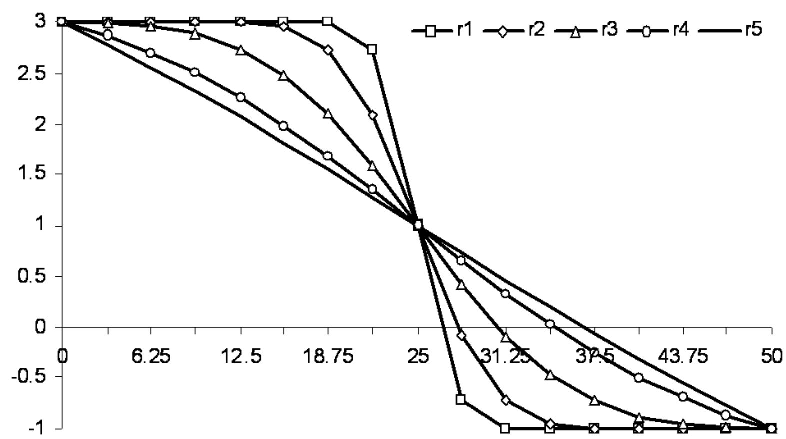

Figure 5. Concave-convex dependences $r_i(I_i)$ based on normal distribution (30).

From Figures 4, 5, one can see the large spectrum of variants of convex-concave (29) and concave-convex (30) dependences $r_i(I_i)$ based on normal distribution.

From alternatives (20)-(30), one can select the more relevant dependence for each concrete informatization domain.

6 Solving the aggregate problem in continuous form

With dependences $r_i(I_i)$, $i = \overline{1,n}$ of rational behavior (without discontinuities of degree I or II) at $I_i \in [0; I_{\max i})$, general aggregate problem (5)-(6) in continuous form can be solved using the Lagrange multiplies method. As it is proved in [16], the optimal solution of investigated problem is characterized by the equality of continuous rate of return for the all n domains, namely $r_i = r, i = \overline{1,n}$. Therefore, solving the problem (5)-(6) is reduced to solving equation

$$\sum_{i=1}^{n} I_i(r) - I = 0. \tag{31}$$

To obtain the equation (31), it is sufficient to find inverse dependences $I_i(r_i)$ for the respective classes of dependences $r_i(I_i)$. For six functions $r_i(I_i)$, determined by expressions (20), (21), (23)-(25) and (27), it is easy to obtain inverse dependences $I_i(r_i)$ and it is done in [16]. Moreover, dependence (29) is an inverse one, too. Thus, inverse dependences $I_i(r_i)$ are known for seven from eleven functions $r_i(I_i)$, determined by expressions (20)-(30).

By solving equation (31), one can obtain the optimal value r^* of rate r, and after that determine optimal values I_i^*, $P_i^*, i = \overline{1,n}$ and P^*, too. For the case of linear dependences $r_i(I_i)$ of form $r_i = -a_i I_i + b_i$ (see (23)), in paper [10] the following optimal solution is obtained

$$I_i^* = \frac{\sum\limits_{k=1}^{n} \frac{b_k}{a_k} - I}{a_i \sum\limits_{k=1}^{n} \frac{1}{a_k}} + \frac{b_i}{a_i}, P_i^* = -\frac{a_i}{2}(I_i^*)^2 + b_i I_i^*, i = \overline{1,n}. \tag{32}$$

If equation (31) cannot be solved in an analytical form, then it is relatively easy to solve it by numerical methods, taking into account that $I_i(r)$, $i = \overline{1,n}$ are decreasing functions at $I_i \in [0; I_{\max i}) \subseteq [0; \widehat{I}_i)$ and $0 \le r^* \le \max\limits_{i=\overline{1,n}} A_i$. An algorithm for solving the problem (5)-(6), for such cases of dependences, was proposed in paper [16].

A case study, which uses the optimal solution (32), is described in [16]. It is investigated an object with four distinct informatization domains ($n = 4$), all linear dependences $r_i(I_i)$ and constants' a_i, b_i, $i = \overline{1,4}$ values specified in Table 1. Informatization domains are sorted in the decreasing order of constants $b_i = A_i$, $i = \overline{1,4}$.

In Table 1, values of $B_i = \sum\limits_{j=1}^{i} I_j(A_{i+1})$, $i = \overline{1,4}$, where $I_j(A_{i+1})$ are calculated according to formula (25) which in this case takes the form

Table 1. Values a_i, b_i, B_i and $I_j(A_{i+1})$, $(i,j) = \overline{1,4}$.

	Domain i			
	1	2	3	4
a_i	0,1	0,05	0,025	0,01
b_i	4	3	2	1
$I_1(r_1 = A_{i+1})$	10	20	30	40
$I_2(r_2 = A_{i+1})$	0	20	40	60
$I_3(r_3 = A_{i+1})$	0	0	40	80
$I_4(r_4 = A_{i+1})$	0	0	0	100
B_i	10	40	110	280

$I_j(r_j = A_{i+1}) = (b_j - A_{i+1})/a_j$, are specified, too. One can see, from Table 1, that the definition domain for the volume I of investments in informatization is [0; 280] units. Charts of dependences $r_i(I_i)$, $i = \overline{1,4}$, taking into account the definition domain of rate r_i (6)-(8), are shown in Figure 6.

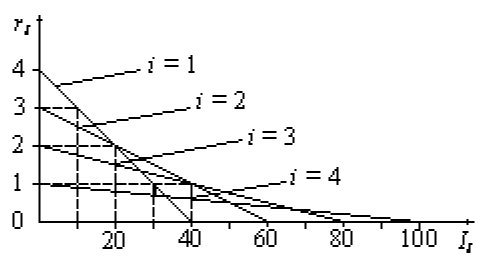

Figure 6. Linear dependences $r_i(I_i)$, $i = \overline{1,4}$.

The obtained optimal solutions $I_i^*(I)$, $i = \overline{1,4}$, depending on the value of total investments I, are shown in graphic form in Figure 7.

According to Figure 7, at $I \leq 10$ units all investments are distributed to domain 1, at $10 < I \leq 40$ the investments are distributed between the domains 1 and 2, at $40 < I \leq 110$ the investments are dis-

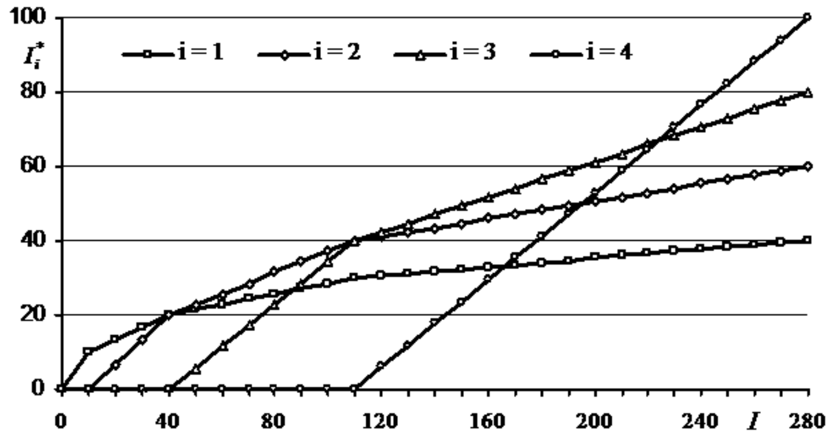

Figure 7. Dependences $I_i^*(I)$, $i = \overline{1,4}$.

tributed among the domains 1-3, and at $110 < I \leq 280$ the investments are distributed among the all four informatization domains. The distribution of investments among domains, in the frame of each of the four intervals for I ($I \leq 10$, $10 < I \leq 40$, $40 < I \leq 110$ and $110 < I \leq 280$), is of linear dependence by the value of total volume I of investments. Evidently, with each new domain implication in the use of investments, the quota of investments in anterior domains is reduced, although the absolute value of their volume is increased. Also, one can observe that the distribution of investments among informatization domains isn't a trivial one and depends on many factors.

The obtained results confirm the necessity, at works of large scale, of some special investigations with regard to the distribution of investments for informatization among different activity domains.

The most general form of the aggregate continuous problem (5)-(6) is when inverse dependences $I_i(r)$, $i = \overline{1,n}$ cannot be determined in an explicit analytical form. There are many real cases when we can meet such a situation. Four classes of such dependences $r_i(I_i)$ are the ones determined by expressions (22), (26), (28) and (30). An algorithm for solving the general aggregate problem (5)-(6) in continuous form is described in [17].

7 Solving the general problem in discrete form

Problem (5)-(8) can be solved using the known algorithms for problems of Boolean linear programming. At the same time, particularities of the problem, permit, in some cases, to simplify the procedure as it is described below.

1^o. Initialization.

2^o. Sorting the $m = \sum_{i=1}^{n} J_i$ projects in the decreasing order of R_{ij} values; projects with equal values of R_{ij} are sorted in the decreasing order of I_{ij} values. The obtained order of projects is $s = \overline{1,m}$ and $R_s = R_{i_s j_s}$, $i = \overline{1,n}, j = \overline{1,J_i}$, where R_s is the rate of return of project $f_{i_s j_s}$ on place s in the overall order of the m projects. Also, for simplicity, we note: $C_s = I_{i_s j_s}$, $D_s = P_{i_s j_s}$, $s = \overline{1,m}$.

3^o. Determining the value of k for which the relation $\sum_{s=1}^{k} C_s \leq I < \sum_{s=1}^{k+1} C_s$ takes place. If $I = \sum_{s=1}^{k} C_s$ or it is reasonably to limit to investments $\sum_{s=1}^{k} C_s$ only, then the solution includes only projects on places $s = \overline{1,k}$ of the overall order, $P^* := \sum_{s=1}^{k} D_s$. Stop.

4^o. If they have accepted investments $C'_{k+1} = I - \sum_{s=1}^{k} C_s$ in project $k+1$ from the overall order ($C'_{k+1} < C_{k+1}$), the profit due to investments C'_{k+1} is D'_{k+1} and $R'_{k+1} \geq R_{k+2}$, then the optimal solution includes only projects on places $s = \overline{1,k+1}$ of the overall order, $P^* := \sum_{s=1}^{k} D_s + D'_{k+1}$. Stop.

5^o. If $C_s = constant$, $s = \overline{1,m}$, then the optimal solution includes only projects on places $s = \overline{1,k}$ of the overall order, $P^* := \sum\limits_{s=1}^{k} D_s$. Investments $\Delta I = I - \sum\limits_{s=1}^{k} C_s$ are not used. Stop.

6^o. Solving the problem by an algorithm of Boolean linear programming (by example, Balash algorithm). Stop.

8 Conclusions

Informatization of diverse activity domains of enterprises, institutions and of the society as a whole has, as a rule, a different impact, that imposes to establish certain priorities. Some recommendations with refer to the assigning of such priorities, can be elaborated using the proposed and investigated models and procedures. The described eleven classes of dependences of informatization's economic effects by the volume of investments cover a large spectrum of variants; especially flexible are the dependences based on normal distribution. The obtained results confirm the necessity, at works of large scale, of some special investigations, depending on the case, with regard to the distribution of investments among different activity domains of the informatization object. In some cases, the described approach, with respective improvements, can be useful for the formation of investment programs in other domains, too.

References

[1] Românu I., Vasilescu I. *Economic Efficiency of Investments and of Fixed Capital.* – Bucharest: Didactic and Pedagogical Publishing House, 1993.

[2] Baker S.L. *Perils of the Internal Rate of Return*, 2000 (hspm.sph.sc.edu/COURSES/ECON/invest/invest.html).

[3] Vilensky P.L., Liphshitz V.N., Smoleak S.. *Estimation of the Efficiency of Investment Projects.* – M.: Delo, 2004.

[4] Albu S., Capsîzu V., Albu I. *Efficiency of Investment: Lecture Notes.* – Chisinau: CEP USM, 2005.

[5] Mieritz L., Kirwin B. *Total Cost of Ownership and the Operations Manager.* – Gartner Research, 13 November 2006.

[6] Roy B. *Classement en choix en presence de points de vue multiples (La methode Electre).* Revue francaise d'informatique et de Recherche Operationnelle, 1968, vol.6(8).

[7] Ansoff I. *Strategic Management.* - Palgrave Macmillan, 1979.

[8] Roubens M. *Preference relations on action and criteria in multicriteria decision making.* European Journal of Operational Research, vol.10, 1982.

[9] Brans I.P., Vincke Ph., Mareschal B. *How to select and how to rank projects: The PROMETHEE method.* European Journal of Operational Research, 1986, vol.24.

[10] Kyparisisa G.J., Gupta S.K., Ip Ch.-M. *Project selection with discounted returns and multiple constraints.* European Journal of Operational Research, vol.94, 1996.

[11] Gaindric C. *Luarea deciziilor.* - Chişinău: Ştiinţa, 1998.

[12] Bolun I., Andronatiev V. *With Regard to the Criteria of Economic Efficiency of Projects in Informatics.* Annals of the University of Economic Studies of Moldova, Vol. 6. – Chisinau: UESM Publishing House, 2008.

[13] Bolun I., Andronatiev V. *Criteria of the Economic Efficiency of Solutions for the Physical Configuration of Local Networks.* Increasing of Competitiveness and Economic Development Based on Knowledge. International Conference, 28-29 September 2007. Vol. II. Chisinau: UESM Publishing House, 2008.

[14] Bolun I. *An Analytical Approach to the Distribution of Investments for the Society Informatization.* Economica, nr.1(65)/2009. - Chisinau: UESM.

[15] Bolun I. *Character of Dependence of Economic Effects of Informatization on the Volume of Investments.* Annals of the University of Economic Studies from Moldova, Vol. 6. – Chisinau: UESM, 2009.

[16] Bolun I. *Distribution of Investments in Informatization Programs / Modernism and Progress in Arts and Sciences.* Proceedings of the 33^{rd} Annual Congress of the American Romanian Academy of Arts and Science, 2009.

[17] Bolun I. *Improving the efficiency of investments in informatization / Education, Research & Business Technologies.* Proceedings of the ninth International Conference on Informatics in Economy. – Bucharest: ASE Publishing House, 2009.

[18] Gametchi A., Solomon D. *Mathematical Modeling of Economic Processes.* – Chisinau: Evrica, 1998.

Ion Bolun, Received July 8, 2009

Ion Bolun,
The School of Cybernetics, Statistics, and Economic Informatics,
University of Economic Studies of Moldova,
Chisinau, Moldova
E–mail: *bolun@ase.md*