Proceedings of the Third Conference of Mathematical Society of Moldova
IMCS-50, August 19-23, 2014, Chisinau, Republic of Moldova

Explicit Thermal Stresses Within a Thermoelastic Half-Strip and Their Graphical Presentation Using Maple – 15 Soft

Victor Şeremet, Ion Creţu, Dumitru Şeremet

Abstract

This article presents a closed form of new solution of a particular boundary value problem (BVP) of thermoelasticity for a half-strip. The thermoelastic displacements and thermal stresses are created by an interior temperature gradient given within a rectangle of the thermoelastic half-strip. To solve this BVP we use Maysel's integral formula for thermoelastic displacements, Duhamel-Neumann law for thermal stresses and the obtained before influence functions for volume dilatation $\Theta^{(i)}(x,\xi); i=1,2$. Graphics of the derived in elementary functions thermal stresses are plotted using soft Maple 15.

Keywords: Green's functions, temperature gradient, thermal stresses, volume dilatation.

1 Calculation of the thermal stresses σ_{ij}

Suppose we want to determine the thermal stresses $\sigma_{ij}(\xi); i,j=1,2$ in the half-strip $V \equiv (0 \leq x_1 < \infty, 0 \leq x_2 \leq a_2)$, caused by the following interior temperature gradient $\Delta T(x)$ given within the rectangle $V^{\shortmid} \equiv [a \leq x_1 \leq b, c \leq x_2 \leq d] \in V$:

$$\Delta T(x) = \begin{cases} T_0 = const., & x \equiv (x_1, x_2) \in V^{\shortmid} \in V, \\ & a \geq 0, b \geq 0, c \geq 0, d \geq 0; \\ 0, & x \equiv (x_1, x_2) \in \Omega \equiv V \setminus V^{\shortmid}. \end{cases} \tag{1}$$

at the homogeneous mechanical boundary conditions:

$$\begin{aligned} &u_1 = \sigma_{12} = 0; \xi_1 = 0, 0 \leq \xi_2 \leq a_2; \\ &\sigma_{22} = u_1 = 0; \xi_2 = 0, 0 \leq \xi_1 < \infty; \\ &u_2 = \sigma_{21} = 0; \xi_2 = a_2, 0 \leq \xi_1 < \infty. \end{aligned} \tag{2}$$

To solve this BVP we have to use Maysel's formula [2]:

$$u_i(\xi) = \gamma \int\limits_V \Delta T(x) \Theta^{(i)}(x, \xi) dx_1 dx_2; i = 1, 2, \tag{3}$$

where $\Theta^{(i)}(x, \xi)$ are the influence functions of an inner unit point force on the volume dilatation and $\gamma = \alpha(2\mu + 3\lambda)$ is a thermoelastic constant; α is coefficient of linear thermal expansion; λ, μ are Lame's constants of elasticity. Functions $\Theta^{(i)}(x, \xi)$ were derived in the handbook [1] (see problem 12.L. 8 and the answer to it).

So, Maysel's formula (3) in our case can be rewritten as follows:

$$u_i(\xi) = \gamma T_0 \int\limits_a^b dx_1 \int\limits_c^d \Theta^{(i)}(x, \xi) dx_2 =$$

$$= -\frac{\gamma T_0}{4\pi(\lambda + 2\mu)} \frac{\partial}{\partial \xi_i} \int\limits_a^b dx_1 \int\limits_c^d \ln \frac{\bar{E}\bar{E}_1\tilde{E}_2\tilde{E}_{12}}{\tilde{E}\tilde{E}_1\bar{E}_2\bar{E}_{12}} dx_2, \tag{4}$$

where the functions $\bar{E}, \bar{E}_1, \tilde{E}_2, \tilde{E}_{12}, \tilde{E}, \tilde{E}_1, \bar{E}_2, \bar{E}_{12}$ are determined by the expressions:

$$\begin{aligned} &\bar{E} = 1 + 2e^{(\pi/2a_2)(x_1 - \xi_1)} \cos(\pi/2a_2)(x_2 - \xi_2) + e^{(\pi/a_2)(x_1 - \xi_1)}; \\ &\bar{E}_1 = \bar{E}(x; -\xi_1, \xi_2); \bar{E}_2 = \bar{E}(x; \xi_1, -\xi_2); \bar{E}_{12} = \bar{E}(x; -\xi_1, -\xi_2); \\ &\tilde{E} = 1 - 2e^{(\pi/2a_2)(x_1 - \xi_1)} \cos(\pi/2a_2)(x_2 - \xi_2) + e^{(\pi/a_2)(x_1 - \xi_1)}; \\ &\tilde{E}_1 = \tilde{E}(x; -\xi_1, \xi_2); \tilde{E}_2 = \tilde{E}(x; \xi_1, -\xi_2); \tilde{E}_{12} = \tilde{E}(x; -\xi_1, -\xi_2). \end{aligned}$$

Next, substituting in Duhamel-Neumann law [2]:

$$\sigma_{ij} = \mu(u_{i,j} + u_{j,i}) + \delta_{ij}(\lambda u_{k,k} - \gamma T); i, j = 1, 2, \tag{5}$$

Eq. (4) and taking the respective integrals, we obtain the final expressions for thermal stresses:

$$\sigma_{11}(\xi) = \frac{\mu\gamma T_0}{\pi(\lambda+2\mu)} F(\xi) + \begin{cases} -\gamma T_0; & \xi \in V' \\ 0, & \xi \in \Omega \end{cases}; \qquad (6)$$

$$\sigma_{22}(\xi) = -\frac{\mu\gamma T_0}{\pi(\lambda+2\mu)} F(\xi) + \begin{cases} -\gamma T_0; & \xi \in V' \\ 0, & \xi \in \Omega \end{cases}; \qquad (7)$$

$$\sigma_{12}(\xi) = -\frac{\mu\gamma T_0}{4\pi(\lambda+2\mu)} \ln \frac{\bar{E}\tilde{E}_1\bar{E}_2\tilde{E}_{12}}{\tilde{E}\bar{E}_1\tilde{E}_2\bar{E}_{12}} \Bigg|_{x_2=c}^{x_2=d} \Bigg|_{x_1=a}^{x_1=b}. \qquad (8)$$

In Eqs. (6) and (7) the function $F(\xi)$ is defined by the following expression:

$$F(\xi) = \left[-\bar{f} + \tilde{f}_1 + \bar{f}_2 - \tilde{f}_{12} - \tilde{f} + \bar{f}_1 + \tilde{f}_2 - \bar{f}_{12} \right]_{x_1=a; x_2=c}^{x_1=b; x_2=d}, \qquad (9)$$

where the functions $\bar{f}, \tilde{f}_1, \bar{f}_2, \tilde{f}_{12}, \tilde{f}, \bar{f}_1, \tilde{f}_2, \bar{f}_{12}$ are determined as follows:

$$\bar{f} = \arctan \frac{e^{(\pi/2a_2)(x_1-\xi_1)} + \cos(\pi/2a_2)(x_2-\xi_2)}{\sin(\pi/2a_2)(x_2-\xi_2)};$$
$$\bar{f}_1 = \bar{f}(x; -\xi_1, \xi_2); \bar{f}_2 = \bar{f}(x; \xi_1, -\xi_2); \bar{f}_{12} = \bar{f}(x; -\xi_1, -\xi_2);$$
$$\tilde{f} = \arctan \frac{e^{(\pi/2a_2)(x_1-\xi_1)} - \cos(\pi/2a_2)(x_2-\xi_2)}{\sin(\pi/2a_2)(x_2-\xi_2)};$$
$$\tilde{f}_1 = \tilde{f}(x; -\xi_1, \xi_2); \tilde{f}_2 = \tilde{f}(x; \xi_1, -\xi_2); \tilde{f}_{12} = \tilde{f}(x; -\xi_1, -\xi_2).$$

2 Graphical presentation of the thermal stresses σ_{ij}

Graphics of thermal stresses $\sigma_{11}, \sigma_{22}, \sigma_{12}$ caused by the following interior temperature gradient $T_0 = 50K$ given within the rectangle

$V^{\prime} \equiv [6 \leq x_1 \leq 10, 4 \leq x_2 \leq 6] \in V$ were constructed at the following values of elastic and thermal constants: Poisson ratio $\nu = 0.3$, the modulus of elasticity $E = 2.1 \cdot 10^5 MPa$ and $\alpha = 1.2 \cdot 10^{-5}(K)^{-1}$. The graphics constructed by using computer program Maple 15 are presented in the Fig. 1.

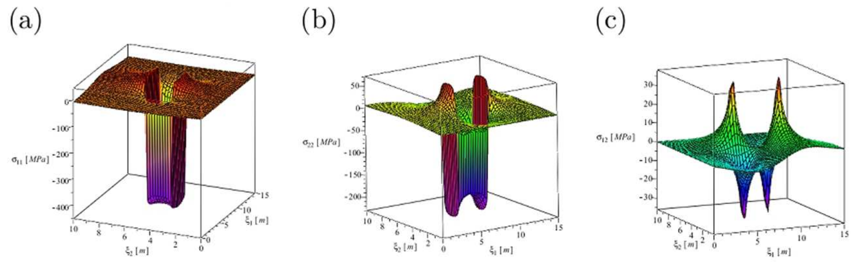

Figure 1. Graphics of normal σ_{11}, σ_{22} and tangential thermal stresses σ_{12} (figures 1(a), 1(b)) and (figure 1(c))

3 Conclusion

Analyzing the graphics of the thermal stresses (see Fig. 1), it should be noted that all boundary conditions are satisfied. Also, in the corner points of the inner rectangle the thermal stresses have some jumps (singularities).

References

[1] V.D. Şeremet. *Handbook on Green's Functions and Matrices.* WIT Press, Southampton, 2003.

[2] W. Nowacki. *The Theory of Elasticity.* Mir, Moscow, 1975.

Victor Şeremet [1], Ion Creţu [2], Dumitru Şeremet [1]

[1]Agrarian State University Of Moldova,
Mirceşti Street 44, 2049, Chişinău, Republic Of Moldova.
Email: v.seremet@uasm.md

[2]Technical University Of Moldova,
Dacia Avenue 41, 2060, Chişinău, Republic Of Moldova.
Email: ioncretu@mail.com