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Methodology of matrix representation of higher order elasticity constants. Six order tensor

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Abstract

The constitutive nonlinear equations of anisotropic materials are examined in reversible deformation area. The constitutive equations of the second order, in which the tensors of elastic constants of forth order listed, are analyzed in detail. The matrix representation of these tensors and analysis of independent constants of elasticity in function of material symmetry and type of atoms interactions is given.

Keywords: tensor, stress, strain, symmetry, constant elasticity.

1 A matrix reprezentation of a six order tensor

In the case of nonlinear relations between stress and strain the six and eight order tensors are intervened, and these tensors can be presented in the form of composite matrix. On basis of symmetric relations it is possible to pass from two indexes notations to one index after Viogt [1-3] convention $11 \sim 1, 22 \sim 2, 33 \sim 3, 23 \sim 4, 13 \sim 5, 12 \sim 6$. Adopting this convention, we will write $c_{ijnm} = c_{KM}$, $c_{ijmnrs} = c_{KMF}$, $c_{ijmnrskq} = c_{KMFL}$, where the small letters have the values 1,2,3, but big 1,2,...,6. Additionally we have

$$c_{KM} = c_{MK},\ \ c_{KMF} = c_{MKF} = c_{FMK} = c_{KFM}$$

$$c_{KMFL} = c_{MKFL} = c_{KMLF} = c_{MKLF} = c_{LFKM} = c_{LFMK} =$$

$$= c_{FLKM} = c_{FLMK} = c_{FKLM} = \dots$$

Matrixes $c_{KM}, c_{KMF}, c_{KMFL}$ don't represent the tensor in the obtained meaning. Therefore, in the rule of components transformation at rotation of reference system there is not directly given the rotation matrix r. It can be demonstrated, that for these matrixes the known rules of components transformation can be used, so

$$c'_{KM} = R_{KI}R_{MJ}c_{IJ}, c'_{KMF} = R_{KI}R_{MG}R_{FT}c_{IGT},$$

$$c'_{KMFL} = R_{KI}R_{MG}R_{FT}R_{LU}c_{IGTU}.$$

$(C_I)_{GT}$ matrix takes the form (1), i.e. $C_1 = \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}$.

$$C_1 := \begin{bmatrix} \begin{pmatrix} c_1 & c_2 & c_3 & c_4 & c_5 & c_6 \\ c_2 & c_7 & c_8 & c_9 & c_{10} & c_{11} \\ c_3 & c_8 & c_{12} & c_{13} & c_{14} & c_{15} \\ c_4 & c_9 & c_{13} & c_{16} & c_{17} & c_{18} \\ c_5 & c_{10} & c_{14} & c_{17} & c_{19} & c_{20} \\ c_6 & c_{11} & c_{15} & c_{18} & c_{20} & c_{21} \end{pmatrix} \\ \begin{pmatrix} c_2 & c_7 & c_8 & c_9 & c_{10} & c_{11} \\ c_7 & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ c_8 & c_{23} & c_{27} & c_{28} & c_{29} & c_{30} \\ c_9 & c_{27} & c_{28} & c_{31} & c_{32} & c_{33} \\ c_{10} & c_{25} & c_{29} & c_{32} & c_{34} & c_{35} \\ c_{11} & c_{16} & c_{30} & c_{33} & c_{35} & c_{36} \end{pmatrix} \\ \begin{pmatrix} c_3 & c_8 & c_{12} & c_{13} & c_{14} & c_{15} \\ c_8 & c_{23} & c_{27} & c_{28} & c_{29} & c_{30} \\ c_{12} & c_{13} & c_{37} & c_{38} & c_{39} & c_{40} \\ c_{13} & c_{28} & c_{38} & c_{41} & c_{42} & c_{43} \\ c_{14} & c_{29} & c_{39} & c_{42} & c_{44} & c_{45} \\ c_{15} & c_{30} & c_{40} & c_{43} & c_{45} & c_{46} \end{pmatrix} \end{bmatrix} \qquad C_2 := \begin{bmatrix} \begin{pmatrix} c_4 & c_9 & c_{13} & c_{16} & c_{17} & c_{18} \\ c_9 & c_{24} & c_{28} & c_{31} & c_{32} & c_{33} \\ c_{13} & c_{28} & c_{38} & c_{41} & c_{42} & c_{43} \\ c_{16} & c_{31} & c_{41} & c_{47} & c_{48} & c_{49} \\ c_{17} & c_{32} & c_{42} & c_{48} & c_{50} & c_{51} \\ c_{18} & c_{33} & c_{43} & c_{49} & c_{51} & c_{52} \end{pmatrix} \\ \begin{pmatrix} c_5 & c_{10} & c_{14} & c_{17} & c_{19} & c_{20} \\ c_{10} & c_{25} & c_{29} & c_{32} & c_{34} & c_{35} \\ c_{14} & c_{29} & c_{39} & c_{42} & c_{44} & c_{45} \\ c_{17} & c_{32} & c_{44} & c_{48} & c_{50} & c_{51} \\ c_{19} & c_{34} & c_{44} & c_{50} & c_{53} & c_{54} \\ c_{20} & c_{35} & c_{45} & c_{51} & c_{54} & c_{55} \end{pmatrix} \\ \begin{pmatrix} c_6 & c_{11} & c_{15} & c_{18} & c_{20} & c_{21} \\ c_{11} & c_{26} & c_{30} & c_{33} & c_{35} & c_{36} \\ c_{15} & c_{30} & c_{40} & c_{43} & c_{45} & c_{46} \\ c_{18} & c_{35} & c_{43} & c_{49} & c_{51} & c_{52} \\ c_{20} & c_{35} & c_{45} & c_{51} & c_{54} & c_{55} \\ c_{21} & c_{36} & c_{46} & c_{52} & c_{55} & c_{56} \end{pmatrix} \end{bmatrix} \qquad (1)$$

So, the tensor of elastic constants of fourth order is expressed by 21 independent components, but six order tensor form 56. These 56 components are presented in the form of column matrix with 56×1 dimensions.

If materials have other elements of symmetry too, the number of independent constants of elasticity is reduced. So, it can be proved, that for materials with cubic symmetry the number of elasticity constants of stress tensor of sixth order is decreased down to six.

The only non-zero constants of elasticity tensor of sixth order are

$$c_1 = C_{111} = c_{22} = C_{222} = c_{37} = C_{333},$$

$$c_2 = C_{112} = c_3 = C_{113} = c_7 = C_{122} = c_{23} =$$

$$= C_{223} = c_{12} = C_{133} = c_{27} = C_{233},$$

$$c_8 = C_{123}, \quad c_{51} = C_{456}, \quad c_{16} = C_{144} = c_{34} = C_{255} = c_{46} = C_{366},$$

$$c_{19} = C_{155} = c_{21} = C_{166} = c_{31} = C_{244} = c_{36} =$$

$$= C_{266} = c_{41} = C_{344} = c_{44} = C_{355}.$$

Therefore, the elastic behavior of material of cubic symmetry in approximation $t_{ij} = c_{ijnm} d_{nm} + c_{ijnmpq} d_{nm} d_{pq}$ is described by 9 independent constants; 3 components of forth order tensor a_1, a_4, a_7 and six independent components of sixth order tensor $c_1, c_2, c_8, c_{16}, c_{19}, c_{51}$. In the case of isotropic material, between independent constants of forth order tensor the relationship takes place $a_4 = \frac{a_1 - a_7}{2}$, but for elasticity constants of sixth order tensor tree more relations are obtained $c_{16} = \frac{1}{2}(c_2 - c_8)$, $c_{19} = \frac{1}{4}(c_1 - c_2)$, $c_{51} = \frac{1}{8}(c_1 - 3c_2 + 2c_8)$. Therefore, the governing equations of second order in case of isotropic materials are expressed from only 5 independent constants. If interaction between atoms is central, than the following relations exist $a_7 = a_4 = \frac{a_1}{3}$, $c_8 = c_{16} = c_{51} = \frac{7c_2 - c_1}{6}$, so, in case of one isotropic material with central interactions, the governing equations of the second order are expressed only by tree independent constants. In case of governing equations of tree order may interfere the eight order tensors. These tensors are expressed by square matrix of sixth order, each element is represented by the sixth order matrix.

2 Conclusions

For cubic symmetry materials the number of independent constants of elasticity is reduced down to 9. In the case of isotropic materials the number of independent constants of elasticity is reduced down to 5, if interaction between atoms is central, than the number of independent constants is reduced down to 3.

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