The Tuning Algorithm of Controllers in Multiple-Loop Feedback Control System with Three Contours

Irina COJUHARI
Technical University of Moldova, Automatica and Informational Technologies Department,
Ştefan cel Mare av., 168, MD 2004, Chişinău, Republic of Moldova
Phone: (+373-22) 509911, fax: (+373-22) 509905
cojuhari_irina@yahoo.com

***Abstract*— A tuning algorithm of linear controllers *P, PI, PID* in multiple-loop feedback control systems with three contours is proposed in this paper. The control objects consists from three subprocesses, which are described by dynamical models with inertia (third order) and time delay. The controllers *P, PI, PID* in the inertial contours 1 and 2 and in the external contour are tuning using the maximal stability degree method. The tuning process of linear controllers *P, PI, PID* consists from three stage: in the first stage it was made the tuning of *P, PI* controllers in the first internal contour, at the second stage it was made the tuning of *P, PI* controllers in the second internal contour, at the third stage it was made the identification of the transfer process of first and second internal contour, after identification it was obtained the equivalent transfer function, and for this equivalent transfer function it was tuning the *P, PI* and *PID* controllers using the maximal stability method in the external contour. The obtained results were optimized in MATLAB using the NCD Outport block and compared with the results obtained for the case of tuning *P, PI, PID* controllers using Ziegler Nichols method.**

***Index Terms* — maximal stability degree method, multiple-loop feedback control system, tuning of controllers.**

I. INTRODUCTION

At the projecting of multiple-loop control systems are used many tuning methods of typical controllers: frequency method, criteria (of modulus) method etc. Frequency method is accompanied with difficulties of calculating [1,2,3]. Criteria (of modulus) method becomes unacceptable when the control processes are slowly, and they have big time constants and this reduce the performances of entire system [1,2,3]. To bypass these above-cited inconveniences in the paper is proposed to use of the maximal stability degree (MSD) method for tuning of typical controllers *P, PI, PID* for a class of control objects' models with inertia, which are connected in cascade, represented by three subprocesses and, as result with three regulating loops.

The multiple-loop feedback control system is represented by three contours: internal contour 1 with controller's transfer function $H_{R3}(s)$ and subprocess $H_{F3}(s)$, internal contour 2 with controller's transfer function $H_{R2}(s)$ and equivalent subprocess $H'_{F2}(s)$, and external contour with controller's transfer function $H_{R1}(s)$ and equivalent subprocess $H'_{F1}(s)$.

The tuning of controllers is recommended to realize first in the inertial contour 3, after in the second inertial contour, then in the external contour. The control object consists from three inertial subprocesses with the transfer functions (t.f):

$$H_{F1}(s)=\frac{k_1}{T_1 s+1}, \quad (1)$$

$$H_{F2}(s)=\frac{k_2}{T_2 s+1}, \text{ cu } T_1>T_2. \quad (2)$$

$$H_{F3}(s)=\frac{k_3 e^{-\tau s}}{T_3 s+1}, \text{ cu } T_1>T_2>T_3. \quad (3)$$

In expressions (1), (2) and (3) we have the notations: k_1, k_2, k_3 are transfer coefficients of subprocesses; T_1, T_2, T_3 are time constants of respective subprocesses, τ - time delay of respective subprocess.

II. THE ALGORITHM OF TUNING CONTROLLERS

The procedure of tuning controllers in the control system with three loops, consists from three steps. In the first step it is tuning the *P* and *PI* controller in the first internal contour, after at the second step it is tuning *P* and *PI* controller in the second internal contour and after in the third step it is tuning *P, PI* and *PID* controller in the external contour.

I Step: **The tuning of the controllers in the first internal contour.**

It is implementing the tuning of controller with transfer function (t.f.) $H_{R3}(s)$ from internal contour to the subproccess with t.f. $H_{F3}(s)$. Suppose that *P* and *PI* controllers are used.

- ***P* controller** is tuning to the object with transfer function (3), applied the maximal stability degree (MSD) method and tuning parameters of controller are determinated from relations [4,5,6]

$$k_{p3}=\frac{e^{-\tau J}}{k_3}(T_3 J-1)\cdot \quad (4)$$

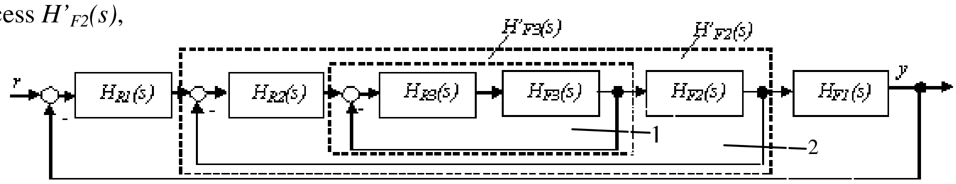

Fig. 1 The structure of multiple-loop feedback control system

In the relation (4) J is the maximal stability degree and which is chosen from the following condition $J>0$.

For determine the t.f of internal contour with P or PI controller, the value $e^{-\tau s}$ it is approximated with Pade approximant:

$$e^{-\tau s} = \frac{1}{\tau s + 1}. \qquad (5)$$

The t.f. of the first internal contour with P controller is :

$$H'_{F2}(s) = \frac{H_{R3}(s) H_{F3}(s)}{1 + H_{R3}(s) H_{F3}(s)} = \frac{k'}{l_0 s^2 + l_1 s + l_2}, \qquad (6)$$

where $k' = \frac{k_{p3} k_3}{1 + k_{p3} k_3}; l_0 = \frac{\tau T_3}{1 + k_{p3} k_3}; \; l_1 = \frac{\tau + T_3}{1 + k_{p3} k_3}; \; l_2 = 1.$

- ***PI* controller** is tuning to the object with the transfer function (3), applied the MSD method and tuning parameters of controller are determinated from relations [4,5,6]

$$k_{p3} = \frac{e^{-\tau J}}{k_3} (-\tau T_3 J^2 + (\tau + 2T_3) J - 1), \qquad (7)$$

$$k_{i3} = \frac{e^{-\tau J}}{k_3} J^2 (-\tau T_3 J + \tau + T_3). \qquad (8)$$

We can obtain the values of parameters k_{p3}, k_{i3}, changing the $J>0$ value, for that the performances of control system are predefined.

The t.f. of the first internal contour with *PI* controller is

$$H'_{F2}(s) = \frac{H_{R3}(s) H_{F3}(s)}{1 + H_{R3}(s) H_{F3}(s)} = \frac{d_0 s + d_1}{c_0 s^3 + c_1 s^2 + c_2 s + c_3}, \qquad (9)$$

where $d_0 = \frac{k_{p2}}{k_{i2}}; \; d_1 = 1; \; c_0 = \frac{\tau T_2}{k_{i2} k_2};$

$$c_1 = \frac{(\tau + T_2)}{k_{i2} k_2}; \; c_2 = \frac{1 + k_{p2} k_2}{k_{i2} k_2}; c_3 = 1.$$

II Step: **The tuning of the controllers in the second internal contour.**

The structure block scheme of the second internal contour is represented in the Fig. 2.

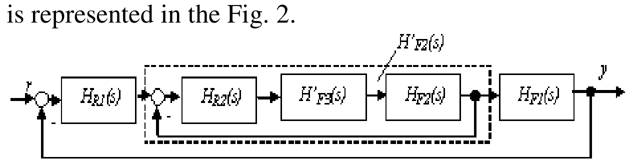

Fig. 2 Structure of control system with second internal contour

It is implementing the tuning of controller with t.f. $H_{R2}(s)$ from the second internal contour to the subproccess with t.f. $H'_{F3}(s)$ and $H_{F2}(s)$. For tuning of *P, PI* controllers in the second internal contour it is necessary to determine the equivalent transfer function of the object (6) with *P* controller in the first internal contour and transfer function of subprocess $H_{F2}(s)$ (2)

$$H_{\Sigma F2}(s) = H'_{F3}(s) H_{F2}(s) = \frac{k}{a_0 s^3 + a_1 s^2 + a_2 s + a_3}, \qquad (10)$$

where $k = \frac{k_{p3} k_3 k_2}{1 + k_{p3} k_3}; \; a_0 = \frac{\tau T_2 T_3}{1 + k_{p3} k_3};$

$$a_1 = \frac{(\tau + T_3) T_2}{1 + k_{p3} k_3} + \frac{\tau T_3}{1 + k_{p3} k_3}; \; a_2 = T_1 + \frac{(\tau + T_3)}{1 + k_{p3} k_3}; \; a_3 = 1.$$

- ***P* controller** is tuning to the object with transfer function (10), applied the MSD method and tuning parameters of controller are determinated from relations [5,6]

$$k_{p2} = \frac{1}{k} (a_0 J^3 - a_1 J^2 + a_2 J - a_3). \qquad (11)$$

- ***PI* controller** is tuning to the object with the transfer function (10), applied the MSD method and tuning parameters of controller are determinated from relations [5,6]

$$k_{p2} = \frac{1}{k} (4 a_0 J^3 - 3 a_1 J^2 + 2 a_2 J - a_3); \qquad (12)$$

$$k_{i2} = \frac{1}{k} (-a_0 J^4 + a_1 J^3 - a_2 J^2 + a_3 J) + k_{p1} J. \qquad (13)$$

For the case when we have in the first internal contour *PI* controller we will have the follow equivalent transfer function of the object (9) with *PI* controller in the first internal contour and t.f. of subprocess $H_{F2}(s)$ (2)

$$H_{\Sigma F2} = H'_{F3}(s) H_{F2}(s) = \frac{b_0 s + b_1}{a_0 s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4}, \qquad (14)$$

where $b_0 = \frac{k_2 k_{p3}}{k_{i3}}; \; b_1 = k_2; \; a_0 = \frac{\tau T_2 T_3}{k_{i3} k_3}; \; a_1 = \frac{T_2 (\tau + T_3)}{k_{i3} k_3} + \frac{\tau T_3}{k_{i3} k_3};$

$$a_2 = \frac{(1 + k_{p3} k_3) T_2}{k_{i3} k_3} + \frac{(\tau + T_3)}{k_{i3} k_3}; \; a_3 = T_2 + \frac{(1 + k_{p3} k_3)}{k_{i3} k_3}; a_4 = 1.$$

- ***P* controller** is tuning to the object with transfer function (14), applied the MSD method and tuning parameters of controller are determinated from relations [4,5,6]

$$k_{p2} = \frac{-a_0 J^4 + a_1 J^3 - a_2 J^2 + a_3 J - a_4}{b_1 - b_0 J}. \qquad (15)$$

- ***PI* controller** is tuning to the object with the transfer function (14), applied the MSD method and tuning parameters of controller are determinated from relations [4,5,6]

$$k_{p2} = \frac{d_0 J^5 - d_1 J^4 + d_2 J^3 - d_3 J^2 + d_4 J - d_5}{(b_1 - b_0 J)^2}, \qquad (16)$$

where $d_0 = 4 a_0 b_0$, $d_1 = 5 a_0 b_1 + 3 a_1 b_0$, $d_2 = 4 a_1 b_1 + 2 a_2 b_0$,

$d_3 = 3 a_2 b_1 + a_3 b_0$, $d_4 = 2 a_3 b_1, d_5 = a_4 b_1$.

$$k_{i2} = \frac{a_0 J^5 - a_1 J^4 + a_2 J^3 - a_3 J^2 + a_4 J}{b_1 - b_0 J} + k_{p2} J. \qquad (17)$$

III Step: **The tuning of the controllers in the external contour.**

The control object consists from three subprocess with inertia and time delay, therefore the procedure of tuning controllers in the third contour becomes difficult, for solving this problem first of all it was obtained the transition process of transfer function $H'_{F2}(s)$ for the case when in the first internal contour it was tuning *P* controller using expression (4) and in the second internal contour it was tuning *PI* controller using expressions (16) and (17), after, this process was identifying and it was obtained the transfer function:

$$H'_{F2}(s) = \frac{d_0 s + d_1}{c_0 s^3 + c_1 s^2 + c_2 s + c_3}. \qquad (18)$$

For the tuning of *P, PI, PID* controllers in the external contour it was necessary to determine the equivalent t.f of object (18) and transfer function of subprocess (1).

$$H_{\Sigma}(s) = H'_{F2}(s) H_{F1}(s) = \frac{b_0 s + b_1}{a_0 s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4}, \qquad (19)$$

where

$$b_0 = k_1 d_0,\ b_1 = k_1 d_1,\ a_0 = c_0 T_1,\ a_1 = c_1 T_1 + c_0,$$
$$a_2 = c_2 T_1 + c_1,\ a_3 = c_3 T_1 + c_2,\ a_4 = 1.$$

The structure of the *external contour* is represented in the Fig. 2 *a, b*.

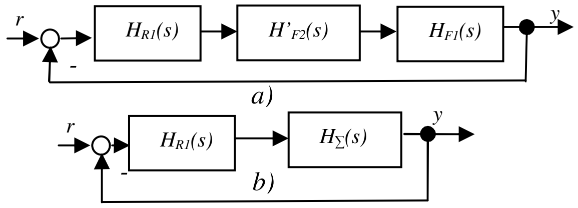

Fig. 3 Structure of external contour

For object with t.f. (19) *P, PI, PID* controllers can be tune applied the MSD method using the relation from [4, 5,6]:

- Control system with *P* controller:

$$k_{p1} = \frac{-a_0 J^4 + a_1 J^3 - a_2 J^2 + a_3 J - a_4}{b_1 - b_0 J}. \qquad (20)$$

- Control system with *PI* controller:

$$k_{p1} = \frac{d_0 J^5 - d_1 J^4 + d_2 J^3 - d_3 J^2 + d_4 J - d_5}{(b_1 - b_0 J)^2}, \qquad (21)$$

where $d_0 = 4a_0 b_0^3;\ d_1 = 5a_0 b_1 + 3a_1 b_1;\ d_2 = 4a_1 b_1 + 2a_2 b_0;$
$d_3 = 3a_2 b_1 + a_3 b_0;\ d_4 = 2a_3 b_1; d_5 = b_1 a_4;$

$$k_{i1} = \frac{a_0 J^5 - a_1 J^4 + a_2 J^3 - a_3 J^2 + a_4 J}{b_1 - b_0 J} + k_{p1} J. \qquad (22)$$

- Control system with *PID* controller:

$$k_{d1} = \frac{-d_0 J^6 + d_1 J^5 - d_2 J^4 + d_3 J^3 - d_4 J^2 + d_5 J - d_6}{2(b_1 - b_0 J)^4}, \qquad (23)$$

where $d_0 = 12 a_0 b_0^3;\ d_1 = 42 a_0 b_0^2 b_1 + 6 a_1 b_0^3;$
$d_2 = 50 a_0 b_0 b_1^2 + 22 a_1 b_0^2 b_1 + 2 a_2 b_0^3;$
$d_3 = 20 a_0 b_1^3 + 28 a_1 b_0 b_1^2 + 8 a_2 b_0^2 b_1; d_4 = 12 a_1 b_1^3 + 12 a_2 b_0 b_1^2;$
$d_5 = 6 a_2 b_1^3 + 2 a_3 b_0 b_1^2 - 2 b_0^2 b_1;\ d_6 = 2 a_3 b_1^3 - 2 b_0 b_1^2;$

$$k_{p1} = \frac{\left(d_0 J^5 - d_1 J^4 + d_2 J^3 - d_3 J^2 + d_4 J - d_5\right)}{(b_1 - b_0 J)^2} + 2 k_{d1} J, \qquad (24)$$

where $d_0 = 4 a_0 b_0;\ d_1 = 5 a_0 b_1 + 3 a_0 b_1;\ d_2 = 4 a_1 b_1 + 2 a_2 b_0;$
$d_3 = 3 a_2 b_1 + a_3 b_0;\ d_4 = 2 a_3 b_1;\ d_5 = a_4 b_1;$

$$k_{i1} = \frac{a_0 J^5 - a_1 J^4 + a_2 J^3 - a_3 J^2 + J}{b_1 - b_0 J} - k_{d1} J^2 + k_{p1} J. \qquad (25)$$

From expressions (20), (21), (22), (23), (24), (25) were determined the optimal values of parameters k_p for *P* controller, k_p and k_i for *PI* controller and k_p, k_i and k_d parameters for *PID* controller.

III. APPLICATION AND COMPUTER SIMULATION

To show the efficiency of the proposed algorithm for tuning of the typical controllers in the multiple-loop feedback control system with inertia (third order) and time delay using the presented relations was examine an example with the object model which has the following parameters: *$H_{F3}(s)$: k_3=0.5, T_3=0.8, τ = 0.1; $H_{F2}(s)$: k_2=2, T_2=2; $H_{F1}(s)$: k_1=1, T_1=3.*

The procedure of tuning controllers it was begun with identification of the transition process of first and second internal contour, the obtained transfer process is presented in the Fig.5. The computer simulation has been made in MATLAB and the simulation diagram of multiple-loop feedback control system with first and second inertial contour is presented in Fig. 4.

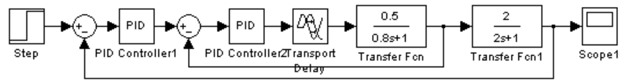

Fig. 4 Simulation diagram of the control system

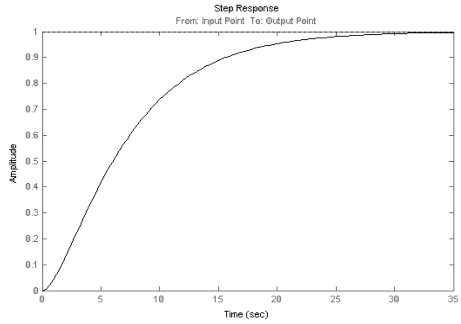

Fig. 5 The transition process of control system

Using the procedure of identification transition process from MATLAB, it was made the identification the transfer process from Fig. 5, the equivalent transfer function which was obtained it is presented in the relation (26).

$$H'_{F2}(s) = \frac{1.498s + 1}{0.2139s^3 + 1.0687s^2 + 1.9788s + 1}. \qquad (26)$$

After when it was obtained the transfer function (26) it was made the tuning *P, PI, PID* controllers in the external contour in conformity with proposed algorithm.

Doing the respectively calculations *were* obtained the following results in case of tuning *P, PI* and *PID* in the external contour for the *P* controller: *J_{opt}=0.61, k_p=1.004;* for the *PI* controller: *J_{opt}=0.34, k_p=0.6187, k_i=0.106;* for the *PID* controller: *J_{opt}=0.91, k_p=4.559, k_i=2.331, k_d=1.982.*

To check the obtained results in case of the tuning controllers *P, PI* and *PID* using the MSD method, it was using the Ziegler-Nichols (ZN) method. In conformity of this method it was obtained the follow critical parameters of the control system: *k_{cr}=14.5,* *T_{cr}= 2.18 s.* Using this values it was determinate the optimal values of *P* controller: *k_{popt}=7.25,* for *PI* controller: *k_{popt}=6.525* and *k_i=0.5734 (T_{iopt}=1.744 s)* and for *PID* controller: *k_{popt}=10.875; k_i=0.7645 (T_{iopt}=1.308 s); k_{dopt}=0.218.*

For verify the obtained results in case of tuning controllers *P, PI, PID* using the MSD and ZN methods, it was made the computer simulation of the control system in MATLAB.

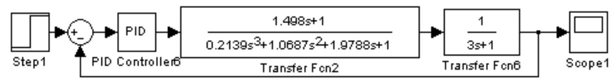

Fig. 6 Simulation scheme of the control system

The obtained transition processes of the computer simulation are presented in Fig. 7 and 8. In Fig. 7 it was made the tuning *P, PI*, and *PID* controllers using ZN method and in Fig. 8 it was made the tuning of *P, PI* and *PID* controller using MSD method, where: curve *1* present the transition process in case of tuning *P* controller in the external contour; curve *2* present the transition process in case of tuning *PI* controller in the external contour; curve *3* present the transition process in case of tuning *PID* controller in the external contour.

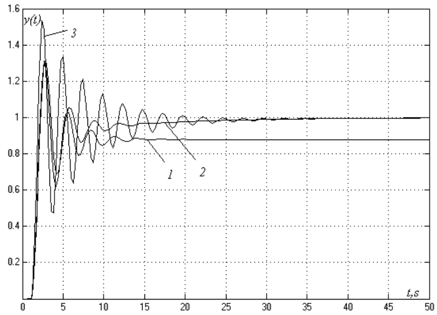

Fig. 7 The transient process of control systems with *P, PI, PID* controllers

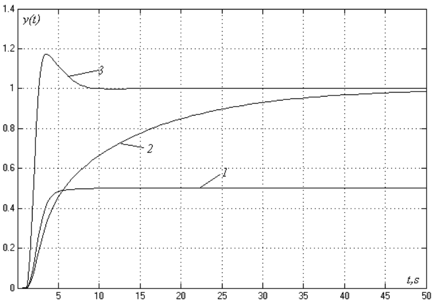

Fig. 8 The transient process of control systems with *P, PI, PID* controllers

The curve *3* obtained in case of tuning *PID* controller using the MSD it was optimized using the NCD Outport block from MATLAB and the obtained optimized curve is presented in Fig. 9, curve 2, curve 1 presents the transition process of tuning *PID* controller using MSD method and curve 3 presents the transition process of tuning *PID* controller using ZN method.

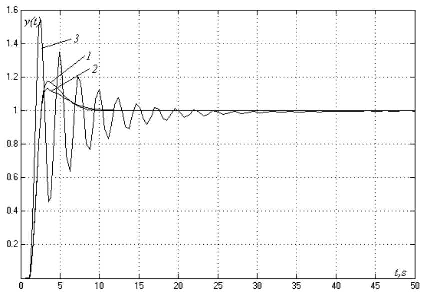

Fig. 9 The transient process of control systems with *P, PI, PID* controllers

For the control system with *P, PI* and *PID* controller tuning by MSD and ZN method, were analyzed the distribution of poles of characteristic equation of the control system in the complex plan which were calculated in the MATLAB and presented in Fig. 10.

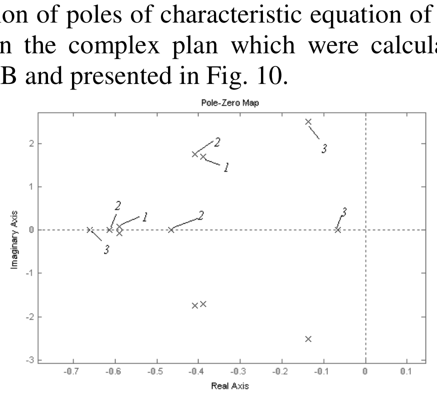

Fig. 10 The distribution of the characteristic equation's poles

In Fig. 10 are presented the domination poles: 1 – for the control system with *PID* controller tuning by MSD method; 2 - for the control system with *PID* controller optimized in MATLAB ; 3 - for the control system with PID tuning by ZN method.

Analyzing the distribution of poles of characteristic equations of control system with *P, PI* and *PID* controller tuning by MSD and ZN method it can be observed that the relative stability of the control system with *PID* controllers tuning by MSD method and the case of optimization have the reserve of stability much higher than the reserve of stability of the control system with *PID* controller tuning by ZN method.

IV. CONCLUSION

As a result of analysing of the obtained results after tuning the *P, PI, PID* controller to the multiple-loop feedback control system with there contours to the models object (1), (2), (3) with known parameters, the following conclusion can be made:

1. For the cases when it is difficult to tune the *P, PI, PID* controllers to the given model objects it can be used the procedure of identification the transfer process and after to apply the MSD method that can give very good performance of the control system.
2. The tuning of *P, PI, PID* controllers in the external contour using the maximal stability degree method permitted to obtain the high results varying the value of the *J>0* and obtained the optimal value and the suboptimal values of the *J*, and choosing the set of the values of regulator's parameters for obtain the predefined performances.
3. The relative stability of the control system with *PID* controllers tuning by MSD method and the case of optimization have the reserve of stability much higher than the reserve of stability of the control system with *P* and *PI* controller tuning by MSD method

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