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TIME AND FREQUENCY MEASUREMENTS

ASSESSMENT OF INFLUENCE OF SYSTEMATIC ERRORS ON THE PRECISION WITH WHICH THE NORMALIZED FREQUENCY OF A SINUSOIDAL SIGNAL IS DETERMINED BY MEANS OF A DISCRETE FOURIER TRANSFORMATION WITH INTERPOLATION

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The influence of systematic errors caused by the infleunce of the imaginary part in the spectrum of a sinusoidal signal on the estimator of its normalized frequency is analyzed by means of a discrete Fourier transformation with interpolation and a window with maximal rate of descent of the side lobes. An expression for the absolute error of the normalized frequency is presented and a condition for the minimal integral number of cycles of the sine curve is found; note that computation of the actual integral number of cycles enables us to assure that this error will be less than some specific value. The reliability of the expressions that are obtained is confirmed by a computer simulation.

Key words: *systematic errors, normalized frequency, discrete Fourier transformation (DFT) method with interpolation, computer simulation.*

The normalized frequency of a sinusoidal signal is one of the most important parameters in time sampling of a signal. It is equal to the ratio of the frequency of a sinusoidal signal to the frequency of its sample. In practice, the frequencies of a sinusoidal signal and its sample do not satisfy the coherence ratio of a sample, one consequence of which is the well-known effect of blurring or "leakage" of the components of a spectrum. This means that the energy of the spectral lines propagate along the entire axis of the frequencies. Weighting is used to reduce the errors associated with blurring of the spectrum. This technique presupposes a priori multiplication of a discrete-time sine curve by an appropriate sequence, called a window [1, 2].

A discrete Fourier transformation with interpolation (IDFT), in particular, is used to determine the normalized frequency of a sinusoidal signal. This method is a frequency-based method and yields a very precise estimator of the normalized frequency [3–5]. The indicators of the IDFT method depend on the type of window employed. The best results are achieved in the case of windows with maximal rate of descent of the side lobes, to which Class I Raiffa–Vincent windows belong [5, 6], and with the use of corresponding analytic formulas [5]. In particular, polynomial approximations, which are required in the IDFT method, are thus eliminated.

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The systematic errors introduced by the imaginary part of the spectrum of a sinusoidal signal affect the precision with which the normalized frequency is calculated according to the IDFT method. Unfortunately, published results in this area do not provide any sort of mathematical expression for use in estimating the errors of the normalized frequency associated with the influence of the systematic errors. In the present article, an analysis of the influence of these errors will be presented. An estimator of the normalized frequency is obtained by the IDFT method with the use of a window with maximal rate of descent of the side lobes. The objective of the analysis is to find an expression for the absolute error of the normalized frequency that occurs when there are systematic errors present along with conditions when these errors become less than some specified level.

Determination of Normalized Frequency of Sinusoidal Signal by the DFT Method with Interpolation. Let us consider a sinusoidal signal with sample frequency f_s:

$$x(m) = A\sin\left(2\pi \frac{f_{in}}{f_s} m + \varphi\right) = A\sin(2\pi f m + \varphi), \quad m = 0, 1, ..., M-1,$$

where A, f_{in}, and φ are the amplitude, frequency, and phase, respectively, of the sinusoidal signal; $f = f_{in}/f_s$, normalized frequency in accordance with the Nyquist criterion $f_{in} < f_s/2$; and M, number of discrete readings.

The relationship between the frequencies f_{in} and f_s is expressed by the relation

$$f_{in}/f_s = \lambda_0/M = (l + \delta)/M, \tag{1}$$

where λ_0 is the number of recorded sinusoidal cycles and l and δ the integral and fraction part of the number λ_0; $-0.5 \le \delta < 0.5$.

In the case $\delta = 0$, the process of sampling of discrete values is coherent to the input sinusoid (coherent mode), hence (1) represents the ratio of coherence between the frequencies f_{in} and f_s. In the case $\delta \ne 0$, the quantization process is not coherent to the input sinusoid (noncoherent mode), consequently, the coherence ratio is not satisfied. In this mode, the spectrum of a sinusoidal signal is distorted due to blurring of the spectral components. Weighting is used to reduce the error associated with blurring of the spectrum. Such an approach leads to spectral analysis of the signal and $x_w(m) = x(m)w(m)$, where $w(m)$ is a weighting function (window) [1, 2]. A discrete-time Fourier transformation of the signal $x_w(m)$ is specified by the formula

$$X_w(\lambda) = \frac{A}{2\mathrm{j}}\left[W(\lambda - \lambda_0)\,\mathrm{e}^{\mathrm{j}\varphi} - W(\lambda + \lambda_0)\,\mathrm{e}^{-\mathrm{j}\varphi}\right], \quad \lambda \in [0, M), \tag{2}$$

where $W(\lambda)$ is the discrete-time Fourier transform of the sequence $w(m)$; the second term in (2) represents the imaginary part of the spectrum.

The value of δ may be estimated with a high degree of precision by means of the IDFT technique [3–5], where windows of cosine class may be employed. A very precise estimator δ is obtained with the use of the above method together with windows of maximal rate of descent of the side lobes [5]. Such a window is of order H ($H \ge 2$) and will possess the greatest rate of descent of the side lobes, equal to $6(2H - 1)$ dB/octave, of all windows of cosine class [6] and is determined by the function

$$w(m) = \sum_{h=0}^{H-1} (-1)^h a_h \cos\left(2\pi h \frac{m}{M}\right), \quad m = 0, 1, ..., M-1,$$

where, by [7], a_h are the coefficients of the window:

$$a_0 = \frac{C_{2H-2}^{H-1}}{2^{2H-2}}, \quad a_h = \frac{C_{2H-2}^{H-h-1}}{2^{2H-3}}, \quad C_m^p = \frac{m!}{(m-p)!\,p!}, \quad h = 1, 2, ..., H-1.$$