WEAK REFLEXIVE SUBCATEGORY

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In the subcategory of topological vector locally convex spaces Hausdorff are built proper classes of weakly reflective and weakly coreflective subcategories, respectively. We proved that the right product of two subcategories often leads to a weakly reflective subcategory, which is not reflective.

A full subcategory $\mathcal{R}$ of category $\mathcal{C}$ is called *weakly reflective*, if for any $X \in |\mathcal{C}|$ there are an object $rX \in |\mathcal{R}|$ and a morphism $r^X : X \to rX$ with the property: for an object $A \in |\mathcal{R}|$ any morphism $f : X \to A$ extends through $r^X : f = g \cdot r^X$ for some g. If the extension is always unique, then $\mathcal{R}$ is called the *reflective subcategory.*

In universal algebra, reflective subcategories are realized as factorizations of objects, but extensions are also known, for example localizations in torsion theories. In the general topology, reflective subcategories are more common as extensions, but there are also some as factorizations. In the subcategory of topological vector locally convex spaces Hausdorff $\mathcal{C}_2\mathcal{V}$ are known proper classes of reflective, coreflective and bireflective subcategories. In $\mathcal{C}_2\mathcal{V}$ we construct weakly reflective subcategories and weakly coreflective subcategories and study their properties.

We denote by $\mathbb{R}$ (respectively $\mathbb{K}$) the class of non-zero subcategories of category $\mathcal{C}_2\mathcal{V}$. For $\mathcal{K} \in \mathbb{K}$ and $\mathcal{R} \in \mathbb{R}$ with the respective functors $k : \mathcal{C}_2\mathcal{V} \to \mathcal{K}$ and $r : \mathcal{C}_2\mathcal{V} \to \mathcal{R}$, either $\mu\mathcal{K} = \{m \in \mathcal{M}ono | k(m) \in \mathcal{I}so\}$, $\varepsilon\mathcal{R} = \{e \in \mathcal{E}pi | r(e) \in \mathcal{I}so\}$. Both $\mu\mathcal{K}$ and $\varepsilon\mathcal{R}$ are classes of bimorphisms.

If $\mathcal{B}$ is a class of bimorphisms and $\mathcal{A}$ a subcategory, then $S_{\mathcal{B}}(\mathcal{A})$ (respec-tively $Q_{\mathcal{B}}(\mathcal{A})$) is the full subcategory of all $\mathcal{B}$-subobjects (respectively: $\mathcal{B}$-factorobjects) of the objects of subcategory $\mathcal{A}$. We examine the following two conditions: a) $\mathcal{R}$ contains the subcategory $\mathcal{S}$ of spaces with weak topology; b) $\mathcal{K}$ contains the subcategory $\tilde{\mathcal{M}}$of spaces with Mackey topology.

Theorem 1. *Let be $r^X = u^X \cdot v^X$ is the $((\mu\mathcal{K})^\top,\ \mu\mathcal{K})$-factorization of morphism r^X , $k^X = w^X \cdot t^X$ is the $((\varepsilon\mathcal{R}),\ \varepsilon\mathcal{R}^\perp)$-factorization of morphism k^X .* 1. *$S_{\mu\mathcal{K}}(\mathcal{R})$ is a weak reflective subcategory of the category $\mathcal{C}_2\mathcal{V}$ and $v^X : X \to$*

vX is the weak replique of object X.
1^*. *$Q_{\varepsilon\mathcal{R}}(\mathcal{K})$ is a weak coreflective subcategory of the category $\mathcal{C}_2\mathcal{V}$ and w^X : $wX \to X$ is the weak coreplique of object X.*
2. *$S_{\mu\mathcal{K}}(\mathcal{R})$ is a reflective subcategory if it meets one of the conditions a) or b).*
2^*. *$Q_{\varepsilon\mathcal{R}}(\mathcal{K})$ is a coreflective subcategory if it meets one of conditions a) or b).*

Theorem 2. *Let $\mathcal{R} \in \mathbb{R}$ and let Σ be the coreflective subcategory of the spaces with the most powerful locally convex topology and $\sigma : \mathcal{C}_2\mathcal{V} \to \Sigma$. The following statements are equivalent:*

1. *$\mathcal{S}_{\mu\Sigma}(\mathcal{R})$ is a reflective subcategory of the category $\mathcal{C}_2\mathcal{V}$.*
2. *$\mathcal{S} \subset \mathcal{R}$.*

Theorem 3. *Let $\mathcal{K} \in \mathbb{K}$ and let Π be the reflective subcategory of the complete spaces with weak topology and $\pi : \mathcal{C}_2\mathcal{V} \to \Pi$. The following statements are equivalent:*

1. *$\mathcal{Q}_{\varepsilon\Pi}(\mathcal{K})$ is a coreflective subcategory of the category $\mathcal{C}_2\mathcal{V}$.*
2. *$\tilde{\mathcal{M}} \subset \mathcal{K}$.*