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THE TUNING OF THE PID AND PIDD^2 ALGORITHMS TO THE MODEL OBJECTS WITH INERTIA AND IDENTICAL ELEMENTS AND TIME DELAY

BY

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Abstract. In this paper it were analysed the possibilities of using the additional component in the standard structure of PID controller. This component represents the second derivatives that corresponds to acceleration of error’s control value. The standard algorithm PID and modified PIDD^2 are tuned to the model object with inertia with identical elements and time delay by the maximal stability degree method. There is made a comparative analyse of the obtained results use the different methods of controller tuning.

Key words: algorithms PID, PIDD^2; the control object with inertia identical elements; transfer function; the tuning values; the tuning algorithm; transition process; the performances.

1. Introduction

In the recent decades were obtained outstanding results in the development of theory system and optimal control direction. Actually the most widely used controllers in the industry are the simple structures that present the mathematical model with reduced order and fixed structure. The phenomenon of using the simple structure is due to the following factors:

a) The controllers with simple structures have the advantages over the controllers with complex structures, because they are well accepted by the users

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classically trained in the field of control systems.

b) These structures of controllers contain fewer components which may cause failures.

c) These structures of controllers don't have severe requirements for the calculation procedures and have the lower probability of errors apparition in process of programming.

For these reasons it is necessary to develop the methods of synthesis (tuning) the controllers with reduced-order structure compared to the object model order.

The most used structure of controller becomes practically universal in the automated systems industry and there is the structure with proportional, integral and derivative action (PID) (Dorf *et al.*, 2004, Sheinberg *et al.*, 2004; Gudvin *et al.*, 2004; Rotach, 2004; Boichenko *et al.*, 2007; Sidorova, 2012; Smirnov *et al.*, 2005).

This type of algorithm with simple structure of realization allows obtaining high performance and good robustness of the control system.

We consider that the model of control object is presented as a third-order inertia model with identical elements and time delay being described by the following transfer function:

$$H(s)=\frac{k\mathrm{e}^{-\tau s}}{(Ts+1)^n}, \tag{1}$$

where: k is the transfer coefficient, T, τ – time constant and time delay, respectively.

To the model object (1) it will be tuned the PID controller and PIDD^2 which are described by the following transfer functions:

$$H(s)=k_p+\frac{k_i}{s}+k_d s=\frac{k_d s^2+k_p s+k_i}{s}, \tag{2}$$

$$H(s)=k_p+\frac{k_i}{s}+k_{d1}s+k_{d2}s^2=\frac{k_{d2}s^3+k_{d1}s^2+k_p s+k_i}{s}, \tag{3}$$

respectively, where k_p , k_i , k_d are the tuning parameters of the standard PID algorithm and k_p , k_i , k_{d1} , k_{d2} – the tuning parameters of the modified PIDD^2 algorithm. In the modified algorithm was introduced the second derivative and with additional component introduction in the algorithm, the number of tuning values are increased and as result the calculations became complicated. The classical methods of tuning controllers became unacceptable in these cases.

There are many methods of tuning the standard PID algorithm (Dorf *et al.*,2004; Sheinberg *et al.*, 2004; Gudvin *et al.*, 2004; Rotach, 2004; Boichenko *et al.*, 2007; Sidorova, 2012; Zagarii *et al.*, 1998; Smirnov *et al.*, 2005).

Some of existing methods of tuning the PID controller have difficult procedures of tuning, other methods guarantee stability, but does not guarantee the quality of operating mode. The algorithms (2) and (3) will be tune to the model object (1) using the maximal stability degree method with iteration (Izvoreanu *et al.*, 1997), and the obtained results will be compared with results obtained using the integral criteria method presented by Smirnov *et al*., (2005).

In this paper will be analysed the effectiveness of tuning the standard algorithm PID and modified PIDD^2 algorithm to the performance and robustness of the control system under the action of the reference signal and perturbation step signal.

2. The Tuning Algorithm of PID and PIDD^2 Controllers

It is presented the structure of control system consist of model object (1) and the controller PID type (2) in the closed loop (view Fig. 1).

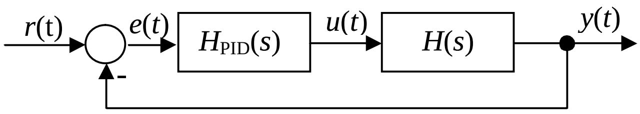

Fig. 1 – The control system structure schema.

The methods are presented below in the reduced form.

2.1. The Maximal Stability Degree Method

For tuning the PID controller by the maximal stability degree (MSD) method with iteration to the model object (1) is used the algebraic eqs. system of fourth degree, with the unknown parameters J, k_p , k_i , k_d that were obtained in the author's previous work (Izvoreanu *et al.*, 1997), which are the following form:

$$(1-TJ)^n(-c_0J^4+c_1J^3-c_2J^2+c_3J-c_4)=0, \tag{4}$$

where:

$$c_0=\tau^3T^3;\ c_1{=}3\tau^2T^2[T(n{+}1){+}\tau];\ c_2=3\tau T[\tau^2+\tau T(2n+3){+}T^2n(n{+}1)];$$
$$c_3=t^3{+}3t^2T^2(n+3)+tT^2 3n(n+3)+T^3n(n^2-1);$$
$$c_4=3t^2+6ntT+3n(n-1)T^2,$$

$$k_p=\left[\exp(-tJ)/k\right](1-Ts)^{n-1}\left\{-tTJ^2+J\left[T(n+1)+t\right]-1\right\}+2k_dJ, \tag{5}$$

$$k_i=\left[\exp(-\tau J)/k\right]J(1-Ts)^n-k_dJ^2+k_pJ, \tag{6}$$

$$\begin{aligned}k_d=&\left[\exp(-tJ)/(2k)\right](1-Ts)^{n-2}\left\{t^2T^2J^3-2\tau TJ^2\left[t+T(n+1)\right]+J\left[t^2+\right.\right.\\&\left.\left.+2tT(n+2)+T^2n(n+1)\right]\right\}-2(t+nT).\end{aligned} \tag{7}$$

For tuning the parameters of the modified PIDD^2 controller by the maximal stability degree method with iteration to the model object (1) is used the algebraic eqs. system of fourth degree having the five unknown parameters J, k_p, k_i, k_{d1}, k_{d2} that were obtained analogically from eqs. (5),…,(7) and are the following forms:

$$k_p = [\exp(-tJ)/k](1-Ts)^{n-1}\{-tTJ^2 + J[T(n+1)+t]-1\} - 3k_{d2}J^2 + 2k_{d1}J, \tag{8}$$

$$k_i = [\exp(-tJ)/k]J(1-Ts)^n + k_{d2}J^3 - k_{d1}J^2 + k_pJ, \tag{9}$$

$$k_{d1} = [\exp(-tJ)/(2k)](1-Ts)^{n-2}\{t^2T^2J^3 - 2tTJ^2[t+T(n+1)] + J[t^2 + 2tT(n+2) + T^2n(n+1)]\} - 2(t+nT) + 3k_{d2}J. \tag{10}$$

$$k_{d2} = (\exp(-tJ)/(6k)(1-Ts)^{n-2}(-c_0J^4 + c_1J^3 - c_2J^2 + c_3J - c_4), \tag{11}$$

where coefficients c_0, c_1, c_2, c_3, c_4 were determined from (4).

Solved eq. (4) it is determined the optimal value of stability degree J_{opt}, which presents the smallest real root or the real part of the complex root.

From expressions (5),…,(7) were determined the numerical optimal values of tuning parameters k_p, k_i and k_d of the PID controller. In this proceeding of maximal stability degree method the values of tuning parameters of the PID controller, k_p, k_i, k_d are taken the maximal values (view the curves from Fig. 2). The performance of control system is verified by the computer simulation in the MATLAB (Fig. 3) and the transient curve is presented in the Fig. 4 (curve *1*).

If the performances of the control system are not satisfing the imposed requirements when is using the maximal stability degree method with iteration, which is able to optimize the performance of control system. In thise case the expressions (5),…,(7) are represented as functions of J, namely $k_p=f(J)$, $k_i=f(J)$ and $k_d=f(J)$ known parameters of the control object and unknown value J. The variable J varies from zero up to a certain value J_x (this value is chosen) and it is constructed the curves $k_p=f(J)$, $k_i=f(J)$, $k_d=f(J)$. There are chosen the sets of suboptimal values of J_i and at the respective slope of the curves are determined obtaining the suboptimal values of the tuning parameters values $k_{pi}=f(J_i)$, $k_{ii}=f(J_i)$, $k_{di}=f(J_i)$ of the PID controller, assuming that the value J_i is lower or greater than J_{opt}. For the chosen sets of tuning values of PID controller is simulated the control system and it is determined the highest possible performance of the automatis system designed by the proposed method. The analogical procedure is used for the system of functions (8),…,(11) obtaining the curves $k_p=f(J)$, $k_i=f(J)$, $k_{d1}=f(J)$, $k_{d2}=f(J)$, are chosen the sets of values of tuning parameters for the modified PIDD^2 controller and after all, subsequently, is performed the computer simulation of the control system and it is determined the performance of control system.

2.2. The Integral Criteria Method

The mathematical model of control object is presented by the transfer function

$$H(s) = \frac{k\mathrm{e}^{-ts}}{(1+T_1 s)(1+T_2 s)...(1+T_n s)} = H_0(s)\mathrm{e}^{-ts}, \tag{12}$$

where k is the transfer coefficient, T_1, T_2, ..., T_n, t – time constants and, respectively, time delay of the control object. An approximation of the transfer function of the controller is

$$H_R(s) = \frac{1}{H_0(s)ts} = k_p + \frac{k_i}{s} + k_{d1}s + k_{d2}s^2 + ..., \tag{13}$$

where k_p, k_i, k_{d1}, k_{d2} are the tuning parameters of the modified PIDD^2 controller. According to relation (13) the tuning parameters of the PID controller and PIDD^2 controller are determined using the object parameters (Smirnov *et al.*, 2005):

a) for the object with $n = 2$ the optimal algorithm will be the PID and its parameters are determined with relations

$$k_p = \frac{T_1 + T_2}{kt},\ k_i = \frac{1}{kt},\ k_d = \frac{T_1 T_2}{kt}; \tag{14}$$

b) for the object with $n = 3$ the optimal algorithm will be the PIDD^2 and its parameters are determined with relations

$$k_p = \frac{T_1 + T_2 + T_3}{kt},\ k_i = \frac{1}{kt},\ k_{d1} = \frac{T_1 T_2 + T_1 T_3 + T_2 T_3}{kt},\ k_{d2} = \frac{T_1 T_2 T_3}{kt}. \tag{15}$$

The tuning parameters calculated by the above eqs. are approximate values and these values are not optimal (Smirnov *et al.*, 2005).

To determine the optimal values of the tuning parameters' PID and PIDD^2 algorithms to the model object (1), Smirnov *et al.*, (2005), have proposed to use the integral criteria based on the assessment of model's transient process of the control system applying restrictions on some quality indicators. The calculations of these criteria are difficult and in the indicated paper the authors use a numerical method with help of a computer.

Below is presented the example of tuning parameters of PID algorithm and modified PIDD^2 algorithm of the model object (1) according to the maximal stability degree method and integral criteria method.

3. Application and Computer Simulation

Smirnov *et al.*, (2005), have analysed the control object (1) with third order inertia and identical elements and time delay with numerical values:

k = 0.41, T = 1.9, t = 0.52, n = 3. At this model object were tuned the PID controller and modified PIDD^2 controller, according to the integral criteria method.

It is proposed to tune the PID parameters using the maximum stability degree method with optimum degree and maximum stability degree method with iteration. In case of maximum stability degree method with optimal degree are used the algebraic eqs. systems (4),…,(6) and for the model object with known parameters are obtained the possible optimum values of the PID parameters, which are presented in the Table 1, line 1. According to the maximum stability degree method with iteration are obtained the curves k_p = = $f(J)$, k_i = $f(J)$, k_d = $f(J)$ having the expressions (5),…,(7), which are presented in the Fig. 2.

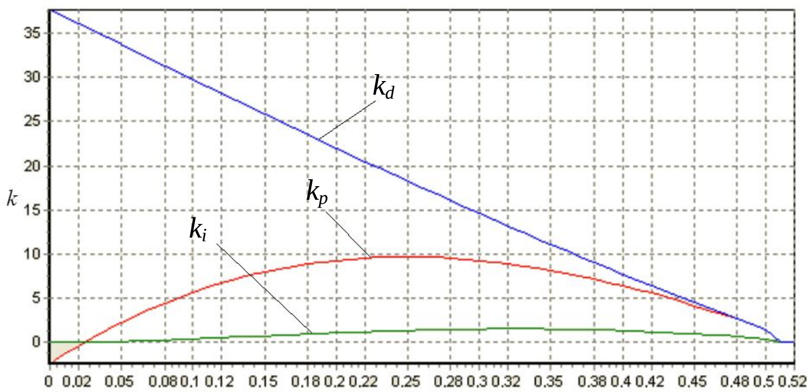

Fig. 2 – The dependence of the PID controller *vs. J*.

Table 1

The Tuning Parameters of the PID Algorithm Tuned by the Maximum Stability Degree Method with Iteration

No. iter.	J	k_p	k_i	k_d
1	0.33	8.672	1.54	12.459
2	0.4	6.407	1.34	7.71
3	0.45	4.166	0.755	3.291
4	0.45	3.666	0.863	3.901
5	0.47	3.151	0.755	3.291
6	0.48	2.621	0.639	2.688
7	**0.49**	**2.077**	**0.515**	**2.095**

The chosen sets of J-values k_p, k_i, k_d from the corresponding curves are presented in the Table 1, rows 2,…,7. The computer simulation of the control system with PID algorithm was made in MATLAB (Fig. 4) and the obtained transient processes are presented in the Fig. 5 (the numbering of processes 1,…,7 corresponds to the numbering from the Table 1) and the performance of control system are presented in the Table 4. The highest performance (row 7 from the Table 3) was obtained for the tuning parameters from row 7 of Table 1. For the control system with modified PIDD^2 algorithm tuned by the maximum stability degree method with iteration is used the system of algebraic eqs. (8),…,(11) and constructed the curves $k_p = f(J)$, $k_i = f(J)$, $k_{d1} = f(J)$, $k_{d2} = (J)$, which are presented in the Fig. 3.

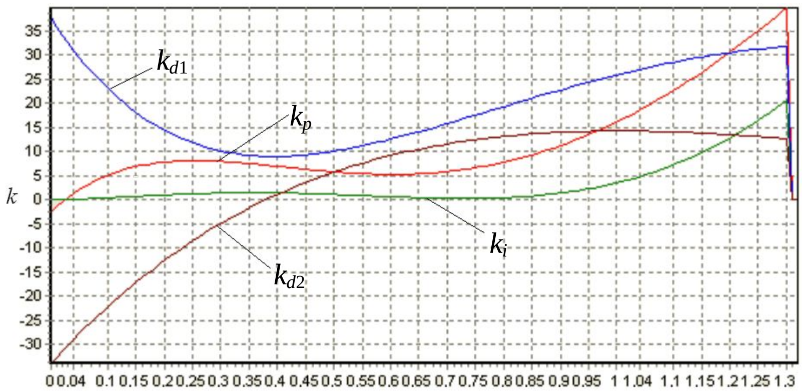

Fig. 3 – The dependence of the PIDD^2 controller *vs. J*.

The chosen sets of J-values k_p, k_i, k_{d1}, k_{d2} from the obtained curves are presented in the Table 3, rows 1,…,7. The chosen sets of J-values k_p, k_i, k_{d1}, k_{d2} from the corresponding curves (Fig. 3) are presented in the Table 2. The computer simulation of the control system with PIDD^2 algorithm was made in MATLAB and the obtained transient processes are presented in the Fig. 6 (the numbering of processes 1,…,7 corresponds with the numbering from the Table 2) and the performance of control system is presented in the Table 4.

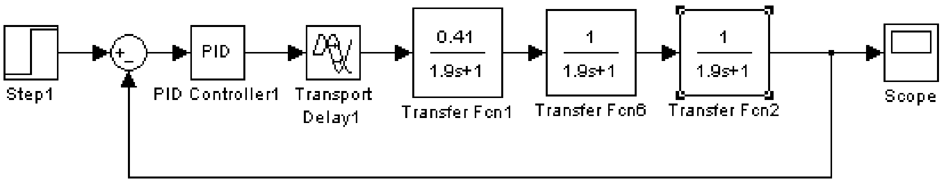

Fig. 4 – The control system simulation diagram.

The highest performance of control system (row 3 from the Table 4) was obtained for the tuning parameters from row 3, Table 2.

Table 2
The Tuning Parameters of the PIDD^2 Algorithm Tune by the Maximal Stability Degree Method with Iteration.

No. iter.	J	k_p	k_i	k_{d1}	k_{d2}
1	0.45	6.31	1.28	9.28	3.53
2	0.47	6.09	1.21	9.53	4.43
3	**0.48**	**5.98**	**1.18**	**9.69**	**4.86**
4	0.49	5.88	1.14	9.85	5.28
5	0.5	5.78	1.09	10.03	5.68
6	0.51	5.69	1.05	10.23	6.08
7	0.52	5.61	1.01	1.44	6.46

As comparison, in the Table 5 are presented the numerical values of the tuning parameters of PID and PIDD^2 tuned algorithms: by the maximum stability degree method the optimum variants (in the respectively tables they are highlighted), the row 1 – the PID algorithm, row 2 – the PIDD^2 algorithm; the obtained data by Smirnov *et al*., (2005). (Table 2, the column 3, 4) by the integral criteria method: the row 3, (column 3), for the PID controller and the row 4, (column 4), for the PIDD^2 controller; the row 5, (column 5), for the PID controller and the row 6, (column 7), for the PIDD^2 controller established by Smirnov *et al*., (2005) (the Table 3, column 5, 7); in the row 7 are presented the tuning values for the PIDD^2 established by Smirnov *et al*., (2005) (Table 2, column 5, 7); in the row 7 are presented the tuning values PIDD^2 established by Smirnov *et al*., (2005) (the Table 2, column 8) calculated according to relations (15). The transient processes of control system with tuning parameters for the PID and PIDD^2 controllers from the Table 5 are presented in the Fig. 7 (the numbering of curves corresponds with the numbering from the Table 5), and the performances are presented in the Table 6.

Table 3
The Performance of the Control System with PID Algorithm Tune by the Maximum Stability Degree Method

No. iter.	t_c , [s]	σ, [%]	t_r , [s]	λ
1	2.82	30.4	12.07	3
2	3.59	24.7	11.91	2
3	4.96	15.42	10.65	1
4	5.48	12.8	11.44	1
5	6.24	9.7	12.27	1
6	7.34	6.6	13.05	1
7	**9.16**		**9.16**	

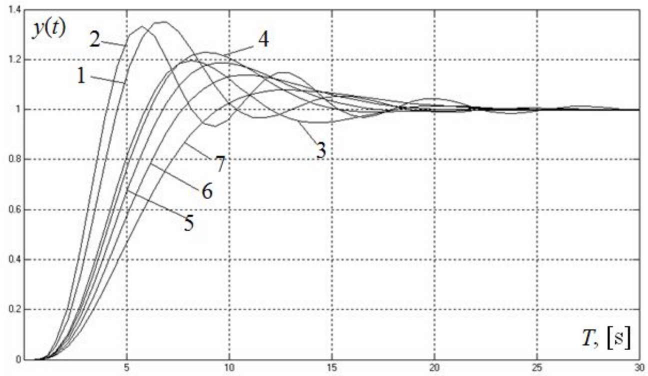

Fig. 5 – The transient processes of control system with PID controller tuned by the maximum stability degree method.

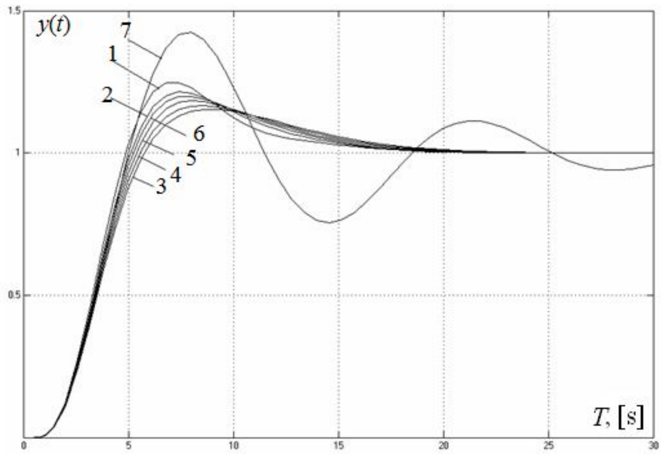

Fig. 6 – The transient processes of control system with PIDD^2 controller tuned by the maximum stability degree method.

Table 4
The Performance of the Control System with PIDD^2 Algorithm Tuned by the Maximum Stability Degree Method

No. iter.	t_c	σ	t_r	λ
1	3.87	10.12	8.08	1
2	4.17	6.3	7.71	1
3	**4.35**		**4.35**	
4	4.59		4.59	
5	4.91		4.91	
6	5.3		5.3	
7	5.8		5.8	

Table 5
The Tuning Parameters of the PID and PIDD² Algorithms Tuned by the Maximum Stability Degree Method and Integral Criteria Method

No.	Method	k_p	k_i	k_{d1}	k_{d2}
1	The MSD method with PID	2.077	0.515	2.095	
2	The MSD with PIDD^2	5.98	1.18	9.69	4.86
3	Integral criteria PID	5.58	1.04	12.8	
4	Integral criteria PIDD^2	12.64	2.12	24.63	18.47

Table 6
The Performance of the Control System with PID and PIDD² Algorithms from the Table 5

No.	t_c	σ	t_r	λ
1	9.16		9.16	
2	4.35		4.35	
3	3.36		9.13	
4	1.71		1.71	

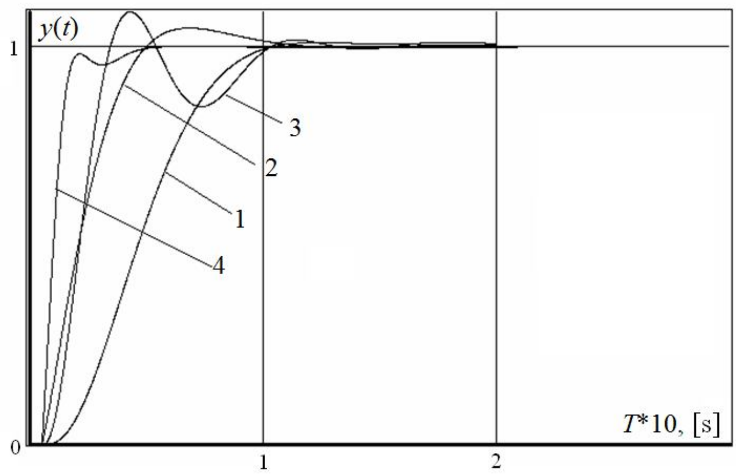

Fig. 7 – The transient processes of control system with PID and PIDD^2 controllers with values tuned from the Table 5.

Analysing the performance of control system with PID controller tune by the maximum stability degree method with iteration and the control system with controller tuned by integral criteria method proposed by Smirnov *et al.*, (2005), (the rows 1 and 3 from the Table 5), it can be observed that the time control is practically the same, but the rapidity of system control is different – at the maximum stability degree method the process is slow but in the case of using the integral criteria method the process is faster (by 2.73 times).

Analysing the performance of control system with PIDD^2 controller tune by the maximum stability degree method with iteration (the row 2 from the Table 5) it can be observed the rapidity and time control increase by twice in comparison with control system with PID controller. The performance of control system with PIDD^2 controller tune by the integral criteria method proposed by Smirnov *et al*., (2005), (row 4), increased the rapidity by 2.87 times and the time control by 5.34 times in comparison with control system with PID controller tuned by the integral criteria method.

The performance of control system with PID controller and the control system with PIDD^2 controller tune by the integral criteria method have the other values in the Tables 2, 3 (Smirnov *et al*., 2005) and characterize the processes with high overshooting, which does not present interest.

In the Fig. 8 is presented the equation poles distribution characteristic for control system with performance from Table 6 (the rows 1,…,4). Analysing the control system with PID controller and PIDD^2 tune by the analysed methods of poles distribution, the better robustness has the control system with PID controller (p = –0.19) and control system with PIDD^2 (p = –0.104) controller tuned by maximum stability degree method.

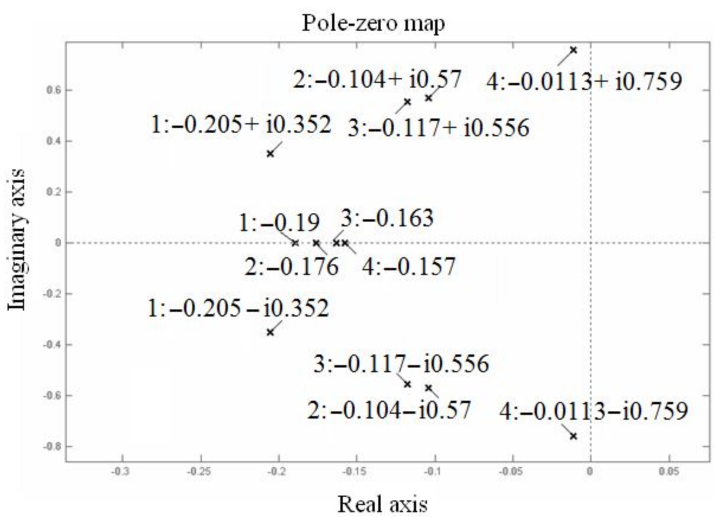

Fig. 8 – The distribution of the eq.'s poles characteristic for control system.

It was applied the unitary step disturbance using the first order inertia element with values k = 0.3 and T = 0.6, under the control system with PID and PIDD^2 controllers with parameters from the Table 5 and it was obtained the transient processes, their performance being presented in the Table 7.

Table 7
The Performance of Control System with PID and PIDD^2 Controllers at the Disturbance Action

No.	t_c	σ	t_r	λ
1	9.68		9.68	
2	4.67	5.9	8.17	1
3	3.65		9.54	
4	3.64		3.64	

Analysing the performance of control system with PID and PIDD^2 controller tune by the maximum stability degree method the rapidity practically remains the same like at the system without perturbation signal but appears a small overshooting at the control system with PIDD^2 controller and it was increased the control time (by 1.88 times), but at the control system with controller tuned by the integral criteria method: the performance of the control system with PID controller practically remain the same as the system without perturbation signal; at the controls system with PIDD^2 controller the rapidity decreasing by twice and control time increasing also by twice.

4. Conclusions

Analysing the obtained results it can be obtained the following conclusions:

1. The control system with modified algorithm PIDD^2 has the performance of the control system better than the control system with PID algorithm.

2. Under the perturbation action the control system with PID controller keeps the performances and the control system with modified algorithm PIDD^2 tuned use the maximum stability degree method is keeps the rapidity and the control time increases approximately by twice in comparison with control system with PIDD^2 controller without perturbation action. In the control system with PIDD^2 controller tuned which uses the integral criteria has the speed by twice lower and the control time is increasing by twice.

3. Analysing the control system poles distributions it can be observed that best robustness have the control system with PID and PIDD^2 controllers tuned by the maximum stability degree method.

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ACORDAREA ALGORITMILOR PID ŞI PIDD^2 LA OBIECTE CU INERȚIE CU ELEMENTE IDENTICE ŞI TIMP MORT

(Rezumat)

Se analizează posibilitățile utilizării în structura regulatorului standard PID a unei componente suplimentare, proporționale derivatei a doua, care corespunde accelerației abaterii mărimii reglate. Algoritmul standard PID şi cel modificat PIDD^2 sunt acordate la modelul obiectului cu inerție de ordinul trei cu elemente identice şi timp mort după metoda GMS cu iterații. Se efectuează o analiză comparativă a rezultatelor obținute la acordarea algoritmilor propuşi la obiectul dat în comparație cu acordarea acestor algoritmi după alte metode cunoscute.